OCT-32226
Paper-II
PHYSICAL SCIENCE

Series

D

Question Booklet Serial No. प्रश्नपत्रिका अनुक्रमांक

Signature and Name of Invigilator

1. (Signature) .....

   (Name) .....

2. (Signature) .....

   (Name) .....

Seat No. | | | | | | | | |

(In figures as in Admit Card)

Seat No. .....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

Time Allowed : 2 Hours] **[Maximum Marks : 200**

Number of Pages in this Booklet : **32** Number of Questions in this Booklet : **100**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **100** objective type questions. **Each** question will carry **two** marks. **All** questions of Paper II will be **compulsory**.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. **Do not accept a booklet without sticker-seal or open booklet.**
   (ii) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. **Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes.** Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.
   (iii) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (B) is the correct response.
   (A) ● (C) (D)
   Following wrong methods should not be used as they are not recognised by scanning machine in digitized assessment. Candidate using such method will be responsible for their loss.

WRONG METHODS
(A) ✓ (C) (D) (A) ✗ (C) (D) (A) ◒ (C) (D) (A) (B) (C) (D)

5. Your responses to the items are to be indicated in the OMR Sheet given inside the Booklet only. If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. Use of any calculator or log table, etc., is prohibited.
12. There is no negative marking for incorrect answers.

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परीक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या OMR उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत **100** बहुविकल्प प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. या प्रश्नपत्रिकेतील **सर्व** प्रश्न सोडविणे अनिवार्य आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनिटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात :
   (i) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. **स्टीकर सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्वीकारू नये.**
   (ii) पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. **पृष्ठे कमी असलेली/कमी प्रश्न असलेली/ प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी.** त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया नोंद घ्यावी.
   (iii) वरीलप्रमाणे सर्व पडताळून पाहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा क्रमांक लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (B) हे योग्य उत्तर असेल तर.
   (A) ● (C) (D)
   खालील चुकीच्या पद्धती वापरू नये, कारण डिजिटाइज्ड (Digitized) मूल्यांकनात स्कॅनिंग मशीन त्यांना ओळखत नाही. त्या पद्धती वापरून नुकसान झाल्यास त्यास विद्यार्थीच जबाबदार असतील.

चुकीची पद्धती
(A) ✓ (C) (D) (A) ✗ (C) (D) (A) ◒ (C) (D) (A) (B) (C) (D)

5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत**. इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. उत्तरपत्रिकेवर नमूद केलेल्या ठिकाणाव्यतिरिक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापि, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.
12. चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.

Physical Science

Paper II

Time Allowed : 120 Minutes] **[Maximum Marks : 200**

Note : This Paper contains **Hundred (100)** multiple choice questions. Each question carrying **Two (2)** marks. Attempt *All* questions.

1. Consider a three dimensional infinite cubic well

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < a, \quad 0 < y < a, \quad 0 < z < a \\ \infty & \text{otherwise} \end{cases}$$

If a perturbation

$$H' = \begin{cases} V_0 & \text{if } 0 < x < \frac{a}{2}, \quad 0 < y < a/2 \\ 0 & \text{otherwise} \end{cases}$$

is applied, then the first order correction to the ground state energy E_0^1 is given by

(A) $\frac{1}{4}V_0$

(B) V_0

(C) $\frac{1}{2}V_0$

(D) $2V_0$

2. Consider $E = <\psi|\hat{H}|\psi>$, where $|\psi>$, is a trial wave function for the variational method. If $|\psi>$ differs from the correct ground state wave function $|\psi_0>$ by terms of the order of ε, where ε is a very small real number, then the variational estimate of the ground state energy differs from the correct ground state energy by

(A) a constant to any order in ε

(B) order of ε^3

(C) order of ε

(D) order of ε^2

3. The energy splitting due to spin-orbit interaction between the $^2p_{3/2}$ and $^2p_{1/2}$ energy levels of a hydrogen atom is given by

(A) $\frac{e^2}{m_e^2c^2} < \frac{1}{r^3} >$

(B) $< \frac{1}{r^3} >$

(C) $- < \frac{1}{r^3} >$

(D) $\frac{3e^2\hbar^2}{4m_e^2c^2} < \frac{1}{r^3} >$

4. Consider two scenarios for a particle of mass m in the ground state of a one-dimensional infinite square well potential of width a. In the first, the right wall of the well is moved slowly to make the width of well 2a and in the second, the wall is abruptly moved to the distance 2a. What is the relation between the two wavefunctions ψ_{slow} and ψ_{sudden} ?

(A) They are identical.

(B) $\psi_{slow} = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$ for $(x \le 2a)$ and $\psi_{sudden} = \sqrt{\frac{1}{a}} \sin\left(\frac{\pi x}{2a}\right)$ for $(x \le a)$

(C) $\psi_{slow} = \sqrt{\frac{1}{a}} \sin\left(\frac{\pi x}{2a}\right)$ for $(x \le 2a)$ and $\psi_{sudden} = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$ for $(x \le a)$

(D) $\psi_{slow} = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$ for $(x \le 2a)$ and $\psi_{sudden} = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$ for $(x \le a)$

5. A particle moves in a potential

$$V(x) = \begin{cases} \frac{1}{2} m\omega^2 x^2 & \text{for } x > 0 \\ \infty & \text{for } x \le 0 \end{cases}$$

Using the WKB quantization condition $\frac{1}{\hbar}\int_{x_1=0}^{x_2} p(x)dx + \frac{\pi}{4} = n\pi$ where n is a positive integer, $p(x) = \sqrt{2m\,(E - V(x))}$ and $x_1 = 0$ and x_2 are the classical turning points, the ground state energy of the particle is given by

(A) $\left(1/2\right)\hbar\omega$

(B) $\left(3/2\right)\hbar\omega$

(C) $2\hbar\omega$

(D) $\left(5/2\right)\hbar\omega$

6. Consider the Green's function for the inhomogeneous wave equation,

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2}\right) G(t - t', x - x') = \delta(t - t')\delta(x - x')$$

Here c is a real constant.
Using the symmetric Fourier transform, let

$$G(t - t', x - x') = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\,\omega \int_{-\infty}^{\infty} d\,k\, F(\omega, k) e^{-i\omega(t-t') + ik(x-x')}$$

Which of the following gives $F(\omega,k)$?

(A) $F(\omega, k) = \frac{1}{(i\omega^2 + c^2k^2)}$

(B) $F(\omega, k) = \frac{1}{(\omega^2 - c^2k^2)}$

(C) $F(\omega, k) = \frac{1}{(c^2k^2 - i\omega^2)}$

(D) $F(\omega, k) = \frac{1}{(c^2k^2 - \omega^2)}$

7. In the Cartesian $x_1 - x_2$ plane, four quantities v_{ij} (where i, j = 1,2) are defined by

$v_{11}=x_1^2,\ v_{12}=x_1x_2,\ v_{21}=x_1x_2,\ v_{22}=x_2^2$.

Which one of the following statements about v_{ij}, is correct ?

(A) v_{ij} is a second-rank Cartesian tensor and its trace $v_{11}+v_{22}$ is invariant under rotations in $x_1 - x_2$ plane.

(B) v_{ij} is a second-rank Cartesian tensor, but its trace changes under rotations in $x_1 - x_2$ plane.

(C) v_{ij} is not a second-rank Cartesian tensor, although $v_{11} + v_{22}$ is invariant under rotations in $x_1 - x_2$ plane.

(D) v_{ij} is not a second-rank Cartesian tensor and $v_{11} + v_{22}$ is not invariant under rotations in $x_1 - x_2$ plane.

8. The area under the curve $x^3 + 3x^2 - 1$ from x = 1 to x = 2 is obtained using Simson's 1/3rd method. The error in the estimated numerical value

(A) depends on the step size used

(B) depends on the number of steps used

(C) is zero

(D) depends on the curve used to estimate the given function

9. The differential equation $\frac{dy}{dx} = y^3 - 10$ is to be solved to get the solution y(x) with the initial condition y(0) = 0 using the Runge-Kutta method. The error in the estimated value of y

(A) can be made exceedingly small by reducing the step size used for the calculation.

(B) will decrease with step size initially but will start to increase again if step size is reduced further.

(C) has two parts depending upon the truncation of the Taylor series and numerical errors, both of which decrease with the step size.

(D) does not depend on the step size beyond a certain limit.

10. Consider the Lagrangian of a particle of mass m moving in one dimension, $L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2$. Which one of the following terms, if we add to the Lagrangian (L), keeps equation of motion unchanged ?

(A) $\frac{1}{2}mq + \frac{1}{2}m\dot{q}t$

(B) $\frac{1}{2}mq^2 + \frac{1}{2}m\dot{q}$

(C) $\frac{1}{2}mqt + \frac{1}{2}m\dot{q}t^2$

(D) $mq + m\ddot{q}t$

11. Auger electron emission can not be obtained from

(A) Carbon

(B) Sodium

(C) Nickel

(D) Hydrogen

12. Which of the following statement best describes the difference between a quasicrystal and an ordinary crystal:

(A) Quasicrystal has periodic atomic arrangement with only 2-fold rotational symmetry while an ordinary crystal can have periodic atomic arrangement with 2–, 3–, 4–,6– fold rotational symmetry.

(B) Quasicrystals do not have translational periodicity.

(C) Quasicrystals have completely random atomic arrangement with no long-range order.

(D) Quasicrystals have periodic atomic arrangement with cubic symmetry.

13. A semiconductor sample carries a current density J along the +x direction. A magnetic field $\vec{B}$ is applied along the +z direction. The measured Hall coefficient is $R_H = -3.1\times10^{-4}m^3C^{-1}$. Which of the following statement is correct for the situation ?

(A) The Semiconductor is p-type and the carrier concentration is approximately $2\times10^{22}m^{-3}$.

(B) The Semiconductor is n-type and the carrier concentration is approximately $2\times10^{22}m^{-3}$.

(C) The Semiconductor is n-type and the carrier concentration is approximately $2\times10^{23}m^{-3}$.

(D) The Semiconductor is p-type and the carrier concentration is approximately $2\times10^{23}m^{-3}$.

14. Imagine a one-dimensional crystal with two different types of atoms in its each primitive unit cell. Which of the following statements is correct for this crystal ?

(A) Only acoustic phonons exist because the crystal is periodic.

(B) Only optical phonons exist because there are two atoms per unit cell.

(C) The crystal has one acoustic branch and one optical branch.

(D) The crystal has two acoustic branches and no optical branch.

15. The magnitude and direction of displacement of a crystalline plane in otherwise perfect crystal are defined by burgers vector. Which of the following is correct statement ?

(A) Burger vector is perpendicular to the edge dislocation.

(B) Burger vector is parallel to the edge dislocation.

(C) Burger vector is perpendicular to the screw dislocation.

(D) Burger vector moves around the screw dislocation.

16. A point charge q is placed at the origin. The flux of the electric field that passes through a square described by the vertices (0, 0, 1), (1, 0, 1), (1, 1, 1) and (0, 1, 1) is

(A) $\frac{q}{\varepsilon_0}$

(B) $\frac{q}{3\varepsilon_0}$

(C) $\frac{q}{6\varepsilon_0}$

(D) $\frac{q}{24\varepsilon_0}$

17. If $\vec{A}$ and V are the vector and scalar potentials, respectively, then the Lorentz gauge in time-varying electromagnetic case is given by

(A) $\vec{\nabla} \times \vec{A} = \mu_0 \varepsilon_0 \frac{\partial v}{\partial t}$

(B) $\vec{\nabla} . \vec{A} = \mu_0 \varepsilon_0 \frac{\partial v}{\partial t}$

(C) $\vec{\nabla} . V = \mu_0 \varepsilon_0 \frac{\partial \vec{A}}{\partial t}$

(D) $\vec{\nabla} . \vec{A} = -\mu_0 \varepsilon_0 \frac{\partial V}{\partial t}$

18. In electrodynamics, the $\vec{\nabla} \times \vec{B}$ is more appropriately expressed as

(A) a bound magnetization current density, $\vec{J}_M$

(B) a free current density, $\vec{J}$

(C) stationary electric charges

(D) a free current density $\vec{J}$ and a time-varying electric field $\vec{D}$

19. A linearly polarized monochromatic plane electromagnetic wave is incident on a plane interface at the Brewster angle. The reflected wave is

(A) unpolarized

(B) linearly polarized with polarization direction parallel to the interface

(C) circularly polarized

(D) linearly polarized with polarization direction perpendicular to the interface

20. The total instantaneous power, P radiated by an accelerated charged particle at non-relativistic speed follows the relation

(A) $P \propto \left|\frac{d\vec{v}}{dt}\right|$

(B) $P \propto \left|\vec{v}.\frac{d\vec{v}}{dt}\right|^2$

(C) $P \propto \left|\frac{d\vec{v}}{dt}\right|^2$

(D) $P \propto \left|\vec{v}\right|^2$

21. A metal crystallizes in a cubic structure. Its atomic packing factor is approximately 0.68 and each atom has 8 nearest neighbours. If the atomic radius is r, which expression gives the lattice constant a ?

(A) $a = 2r$

(B) $a = 4r/\sqrt{3}$

(C) $a = 2\sqrt{2}\, r$

(D) $a = 4r/\sqrt{2}$

22. The dispersion relation of electrons in the lowest energy band of a one-dimensional crystal near the Brillouin-zone boundary is approximated by

$$E(k) = E_0 + \frac{E_g}{2} - \sqrt{\left(\frac{E_g}{2}\right)^2 + 4A^2 \cos^2\left(\frac{ka}{2}\right)}$$

Where a is the lattice constant, E_g is the band gap opened at the zone boundary and A is a positive constant.
Which one of the following statements is correct ?

(A) The effective mass is positive everywhere in the first Brillouin zone.

(B) The group velocity is maximum at the Brillouin zone boundary.

(C) Near $k = \pi/a$, the curvature of the lower band is negative, so the charge transport is conveniently described by holes with positive effective mass.

(D) The density of states becomes zero at the Brillouin-zone boundary because the group velocity is maximum there.

23. A three-dimensional crystal has a Face-Centred Cubic (FCC) direct lattice with lattice constant a. Let the first Brillouin zone be constructed as the Wigner-Seitz cell of its reciprocal lattice. Which one of the following statements is correct ?

(A) The first Brillouin zone has 8 hexagonal faces and 6 square faces and its nearest zone boundary from Γ lies along the $\langle 111 \rangle$ direction.

(B) The first Brillouin zone has 12 identical rhombic faces and the shortest reciprocal lattice vectors belong to the 110 family.

(C) The first Brillouin zone is a truncated octahedron and the distance from Γ to the centre of a hexagonal face is π/a.

(D) The first Brillouin zone is a rhombic dodecahedron and the nearest zone boundary from Γ lies along the $\langle 100 \rangle$ direction.

24. A proton with a rest mass m (= 1.007 u) is bombarded onto a stationary target nucleus of rest mass M (= 8.0559 u) to induce a nuclear reaction. If the Q- value of this endothermic nuclear reaction is -1.60 MeV, the threshold kinetic energy of the incident proton in the laboratory system is closest to

(A) 1.8 MeV

(B) 2.8 MeV

(C) 3.2 MeV

(D) 3.8 MeV

25. Which of the following expression correctly represents the standard Gell-Mann–Nishijima formula for defining electric charge Q of a particles in terms of I_3, S and B. meaning of the terms; I_3 -Third component of Isospin; S-Strangeness ; B- Baryon number ; Q- Electric Charge

(A) $Q = I_3 + 1/2 . (B + S)$

(B) $Q = I_3 - 1/2 . (B + S)$

(C) $Q = 1/2 . I_3 + (B + S)$

(D) $Q = I_3 + (B - S)$

26. Operational amplifiers are widely used for various electronics-related applications. Which of the following sets of parameters correctly describe the characteristics of an ideal operational amplifier ?

(A) Zero input impedance, infinite output impedance, and zero open-loop voltage gain.

(B) Infinite input impedance, zero output impedance and infinite open-loop voltage gain.

(C) Infinite input impedance, infinite output impedance and zero Common Mode Rejection Ratio(CMRR).

(D) Zero input impedance, zero output impedance and finite bandwidth.

27. Analog to Digital converters are used for various electronics related applications. Which of the following Analog to Digital converter architectures offers the fastest conversion speed ?

(A) Successive Approximation Register (SAR) ADC

(B) Dual – Slop integrating ADC

(C) Flash ADC

(D) Counter – Ramp or Digital-ramp ADC

28. What is the most commonly minimized function in standard nonlinear least-squares curve fitting ?

(A) The Sum of Squared Residuals (SSR) between the observed data and the model predictions

(B) The sum of absolute differences between the data and the model

(C) The maximum error between any single data point and the model prediction

(D) The product of the residuals across all data points

29. An instrumentation amplifier has two stages, constructed using three operational amplifiers and precision resistors. Which of the following is a unique feature of the instrumentation amplifier that allows it to accurately measure small, weak signals in noisy environments ?

(A) A circuit design that minimizes the differential-mode gain at low input signal levels while keeping the common-mode gain high, with a moderate loading effect.

(B) The ability to decrease the second-stage amplifier gain from a very high value by adjusting the gain-resistor value under high loading conditions.

(C) Controlling the Common-Mode Rejection Ratio (CMRR) solely by applying the input signals directly to the non-inverting terminals.

(D) A dual op-amp configuration at the input stage that provides exceptionally high input impedance, preventing the loading and loss of weak signals.

30. In high-frequency RF transmission systems, a load matching condition is achieved when the load impedance is perfectly matched to the characteristic impedance of the transmission line. Which of the following is a direct result of this ideal matching condition ?

(A) The voltage reflection coefficient becomes nearly equal to one, allowing all power to enter the load.

(B) The Voltage Standing Wave Ratio (VSWR) reaches a very high value and therefore all power enters the load.

(C) The voltage reflection coefficient reduces to almost zero, and therefore all power to enter the load.

(D) A large fraction of the total power gets reflected at the load boundary and therefore the remaining power enters the load.

31. Consider the transformation

$Q_1 = q_1 + \alpha p_2$, $Q_2 = q_2 + \alpha p_1$, $P_1 = p_1$, $P_2 = p_2$, where α is a constant. Which of the following statements is correct ?

(A) The transformation is canonical for every α

(B) The transformation is canonical only for $\alpha=0$

(C) The transformation is canonical only for $\alpha=1$

(D) The transformation is never canonical

32. Two identical masses move on a straight line and are connected to each other by a spring, but neither mass is attached to a fixed support. In the small-oscillation approximation, one of the normal-mode frequencies is zero. What does this mode represent ?

(A) The two masses move with equal displacements in opposite directions

(B) The spring undergoes a static deformation

(C) The equilibrium configuration is necessarily unstable

(D) The two masses undergo a common translation without changing the spring length

33. An electromagnetic wave of angular frequency ω, propagates through a medium with conductivity σ and permittivity ε. If the medium satisfies the condition, $\frac{\sigma}{\varepsilon\omega} \ll 1$, the material behaves as

(A) a good conductor

(B) a good insulator

(C) a semiconductor

(D) plasma medium

34. A constant current is flowing through a cylindrical wire. Then the Poynting vector at the surface of the cylindrical wire is

(A) in the direction of the current

(B) in the radially outward direction

(C) in the radially inward direction

(D) in the opposite direction of the current

35. The Poynting vector, $\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$ has the dimension of

(A) energy

(B) energy/area

(C) power

(D) power/area

36. Two identical spinless, non-interacting bosons can occupy two non-degenerate single-particle states of energies 0 and ε. Which expression is the canonical partition function of the two-particle system ?

(A) $Z_2 = 1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$

(B) $Z_2 = 1 + 2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$

(C) $Z_2 = 1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon} + e^{-3\beta\varepsilon}$

(D) $Z_2 = 2 + 2e^{-\beta\varepsilon}$

37. Which of the following is an intensive thermodynamic variable ?

(A) Heat capacity

(B) Entropy

(C) Density

(D) Volume

38. A two-level quantum system is in state $|0\rangle$ with probability 3/4 and in the state $|1\rangle$ with probability 1/4. In the orthonormal basis $\{|0\rangle,|1\rangle\}$, which density matrix represents the mixture ?

(A) $\rho = \begin{pmatrix} 3/4 & 0 \\ 0 & 1/4 \end{pmatrix}$

(B) $\rho = \begin{pmatrix} 1/4 & 0 \\ 0 & 3/4 \end{pmatrix}$

(C) $\rho = \begin{pmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{pmatrix}$

(D) $\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

39. A very large system at temperature $T = 30$ K receives heat $Q = 10^{-7}$ J without an appreciable change in temperature. Using $k_B = 1.38 \times 10^{-23}$ J K^{-1}, the ratio of the final to initial numbers of accessible microstates is approximately

(A) 2.4×10^{14}

(B) $e^{2.4 \times 10^{14}}$

(C) 1

(D) 2.4×10^{7}

40. In how many ways can three distinguishable, non-interacting particles be distributed among seven distinct single-particle energy levels, if each particle may occupy any level ?

(A) 3^7

(B) 21

(C) $\frac{7!}{3!4!}$

(D) 7^3

41. A xyz coordinate system is rotating with respect to a fixed inertial XYZ coordinate system (both having same origin). At a particular instance of time, the position vector $\vec{r} = 5\hat{i} + 6\hat{j} + 7\hat{k}$, the velocity in xyz coordinate is $\vec{v}_{app} = \hat{i} + 2\hat{j} + 5\hat{k}$, and the angular velocity vector $\vec{\omega} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

The true velocity vector $\vec{v}_{true}$ in XYZ frame is

(A) $-2\hat{i} + 8\hat{j} + 2\hat{k}$

(B) $4\hat{i} + 3\hat{j}$

(C) $3\hat{i} + 4\hat{j}$

(D) $4\hat{i} - 3\hat{j} + \hat{k}$

42. A particle of mass m moves in the attractive central potential

$$V(r) = -\frac{\alpha}{r} + \frac{\beta}{2r^2}, \alpha, \beta > 0.$$

If the magnitude of the angular momentum is l, which of the following gives the radius r_0 of a stable circular orbit ?

(A) $r_0 = \frac{l^2}{m\alpha}$

(B) $r_0 = \frac{l^2 - m\beta}{m\alpha}$

(C) $r_0 = \frac{l^2 + m\beta}{m\alpha}$

(D) $r_0 = \frac{m\alpha}{l^2 + m\beta}$

43. Consider a Lagrangian that depends on generalized coordinates, generalized velocities and generalized accelerations $L(q, \dot{q}, \ddot{q}\ ;\ t)$. For such a system, the correct equation of motion is given by the Euler-Poisson equation

$$\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\partial L}{\partial q} = 0$$

If

$$L(q, \dot{q}, \ddot{q}\ ;\ t) = -\frac{m}{2}q\ddot{q} - \frac{k}{2}q^2 + \frac{d}{dt}\left(\frac{m}{4}\dot{q}^2\right)$$

Then, which one of the following equations of motion is true ?

(A) $m\ddot{q} + kq^2 = 0$

(B) $m\ddot{q} + kq = 0$

(C) $m\ddot{q} + k\dot{q} = 0$

(D) $m\ddot{q} - k\dot{q} = 0$

44. Consider a double pendulum in which the length of the first pendulum varies as $l_1 = l_0 e^{-rt}$ and the length of the second pendulum varies as $l_2 = e^{l_0}$. What type of constraint do they fall under ? (where, l_0 is constant)

(A) First: Holonomic-Scleronomic and Second: Holonomic-Scleronomic

(B) First: Holonomic-Scleronomic and Second: Holonomic-Rheonomic

(C) First: Holonomic-Rheonomic and Second: Holonomic-Scleronomic

(D) First: Holonomic- Rheonomic and Second: Holonomic- Rheonomic

45. Consider a function $A(q_1, q_2) = \frac{1}{2} a\, q_2^2 + V(q_1)$, Where a is some constant. Let $B(q_1, q_3)$ be its Legendre transform. Which one of the following expressions about $B(q_1, q_3)$ is correct ?

[Hint: you may use Legendre transform $B(q_1, q_3) = q_2 q_3 - A(q_1, q_2)$]

(A) $-V(q_1) + \frac{1}{2a} q_3^2$

(B) $V(q_1) + \frac{1}{2a} q_1 q_3^2$

(C) $V(q_1) + \frac{1}{2a} q_3 q_1^2$

(D) $V(q_1) - \frac{1}{2a} q_3 q_1^2$

46. The one-dimensional diffusion equation is

$$\frac{\partial n(x,t)}{\partial t} = D \frac{\partial^2 n(x,t)}{\partial x^2},$$

where n(x,t) is the particle number density and D is the diffusion coefficient. To obtain a unique solution over a finite spatial interval, which set of conditions is generally required ?

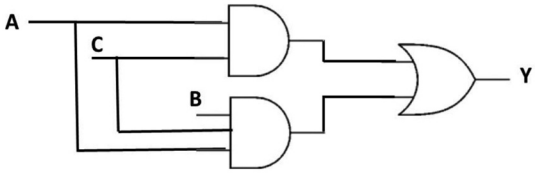
(A) One initial condition and two boundary conditions

(B) One initial condition and one boundary condition

(C) Only one initial condition

(D) Only two boundary conditions

47. Three inputs, A, B and C, are given to a digital circuit as shown below and Y is the output.



Which one is the correct output Y of the digital circuit ?

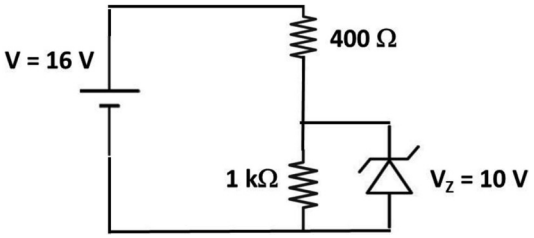
(A) A + B + C + A.C

(B) (A + B).C + A.C

(C) A.B.C + A.C

(D) (A + C) + A.B

48. In the following electronic circuit, the estimated current flowing through the Zener diode having V_Z equal to 10 V is



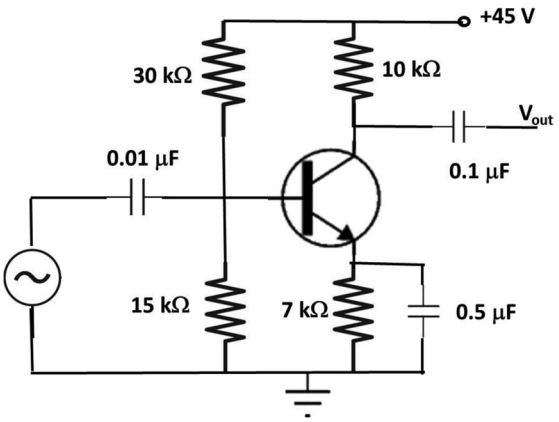
(A) 3 mA

(B) 5 mA

(C) 7 mA

(D) 9 mA

49. The following electronic transistor circuit is analysed by using the DC values of I_E, V_E and V_C which represent the emitter current, emitter voltage and collector voltage respectively. (Assume V_{BE} = 0.7 V, Current gain β = 100)



Considering these D.C. voltages, the collector- emitter voltage V_{CE} is close to

(A) 09 V

(B) 11 V

(C) 13 V

(D) 15 V

50. MOSFETs of enhancement mode and depletion mode have standard terminals, drain, source and gate. Which of the following statement is correct related to operation of these MOSFETs ?

(A) In a depletion mode MOSFET, n-channel or p–channel is fabricated by diffusing n or p type of impurities into doped silicon substrates.

(B) In a n–channel depletion mode MOSFET, applying a positive gate voltage depletes the n-channel of its electrons due to the induce negative charges in it.

(C) An enhancement mode MOSFET can be operated in both depletion mode and enhancement mode by applying the appropriate biasing.

(D) Under proper biasing, an n-channel enhancement MOSFET is operated by applying positive voltage to gate and the drain current flows due to formation of electron inversion layer in the P-region.

51. The ladder operators for the one-dimensional harmonic oscillator are given by $\hat{a}_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}}\left(\mp i\hat{p} + m\omega\hat{x}\right)$. Given the exact ground state wavefunction for the harmonic oscillator as $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$, the first excited state wavefunction $\psi_1(x)$ is given by

(A) $\psi_1(x) = \sqrt{\frac{2m\omega}{\hbar}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} x\psi_0(x)$

(B) $\psi_1(x) = \left(x + \frac{\hbar}{m\omega}\right)\psi_0(x)$

(C) $\psi_1(x) = \sqrt{\frac{2m\omega}{\hbar}}\left(x + \frac{\hbar}{m\omega}\right)\psi_0(x)$

(D) $\psi_1(x) = \sqrt{\frac{2m\omega}{\hbar}}\, x\psi_0(x)$

52. The Hamiltonian for a certain two-level system is

$\hat{H} = E\left(|1><1| - |2><2| + |1><2| + |2><1|\right)$, where $|1>$ and $|2>$ form an orthonormal basis and E is a number with the dimensions of energy. The eigenvalues of the matrix representing $\hat{H}$ with respect to this basis are

(A) $+\sqrt{2}\,E$ and $-\sqrt{2}\,E$

(B) +2E and –2E

(C) Eigenvalues are degenerate

(D) +E and +E/2

53. Two operators $\hat{A}$ and $\hat{B}$ have two orthonormalized eigenstates ψ_1 and ψ_2 and ϕ_1 and ϕ_2 with eigenvalues a_1 and a_2 and b_1 and b_2 respectively. The eigen states are related as $\psi_1 = \frac{\phi_1 + 2\phi_2}{\sqrt{5}}$ and $\psi_2 = \frac{2\phi_1 - \phi_2}{\sqrt{5}}$. To start with, it is assumed that the system is in such a state that both the eigenvalues of $\hat{A}$ are equally likely if a measurement of observable A is done. Observable A is measured first but the value obtained is not known. Then a measurement of observable B is done. What are the probabilities of getting the values b_1 or b_2 ?

(A) $\frac{1}{5}$ and $\frac{4}{5}$

(B) Both the eigenvalues b_1 and b_2 are equally probable.

(C) $\frac{16}{25}$ and $\frac{1}{25}$

(D) $\frac{4}{5}$ and $\frac{1}{5}$

54. An electron is in the spin state $\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$. The expectation value of operator $\widehat{S_y} = \frac{\hbar}{2}\widehat{\sigma_y}$ in the state χ is

(A) 0

(B) $\frac{5}{2}\hbar$

(C) $-\frac{12}{25}\hbar$

(D) $-\frac{1}{2}\hbar$

55. A particle of mass m is moving in a finite potential well of height V_0(V_0>0) and width 2a such that $k^2 + l^2 = \frac{2mv_0}{\hbar^2}$, where $k^2 = 2mE/\hbar^2$ and $l^2 = 2m(V_0 - E)/\hbar^2$ with the energy E of the bound state satisfying $0<E<V_0$. The bound state energy further satisfies one of the two equations $l = k\tan(ka)$ or $l = -k\cot(ka)$. Which of the following statement is true ?

(A) There may not be any bound state if the well is very narrow and very shallow.

(B) There are two bound states if $\left(\frac{\pi}{2}\right)^2 < \frac{2mV_0a^2}{\hbar^2} < \pi^2$

(C) There are four bound states if $\pi^2 < \frac{2mV_0a^2}{\hbar^2} < \left(\frac{3\pi}{2}\right)^2$

(D) There is only one bound state if $\left(\frac{\pi}{2}\right)^2 < \frac{2mV_0a^2}{\hbar^2} < \pi^2$

56. Let a be a nonzero real constant. Consider the integral

$$I(a) = \frac{1}{2\pi i}\lim_{\in\to 0^+}\int_{-\infty}^{\infty}\frac{e^{iax}}{x + i\in}dx\cdot$$

Which one of the following gives the correct value of $I(a)$?

(A) $I(a)$ =1 for a > 0 and $I(a)$ = 0 for a < 0.

(B) $I(a)$ = 0 for a > 0 and $I(a)$= –1 for a < 0.

(C) $I(a)$= –1 for a > 0 and $I(a)$ = 0 for a < 0.

(D) $I(a)$ = 0 for both a > 0 and a < 0.

57. Consider the scalar function

$$F(x,y,z) = \frac{|z|}{R}$$

defined on the surface of the sphere $x^2 + y^2 + z^2 = R^2$. It is expanded in Legendre polynomials as

$$F(\theta) = \sum_{\ell=0}^{\infty} a_\ell P_\ell(\cos\theta)$$

where θ is the polar angle. Which of the following correctly identifies the Legendre polynomials that may have nonzero coefficients in this expansion ?

(A) $P_0, P_1, P_2, P_3, \ldots$

(B) $P_1, P_3, P_5, P_7, \ldots$

(C) $P_0, P_2, P_4, P_6, \ldots$

(D) $P_0, P_1, P_3, P_5, \ldots$

58. Consider the integral

$$I = \int_{-\infty}^{\infty} x\,\delta(x^2 - 5x + 6)dx\,.$$

The value of I is

(A) 1

(B) 5

(C) $\frac{5}{2}$

(D) 6

59. Consider the second-order linear differential equation

$$(x-1)^3\frac{d^2y}{dx^2} + (x-1)^2\frac{dy}{dx} + y = 0\,.$$

Which of the following correctly classifies the point x = 1 ?

(A) x = 1 is an irregular singular point.

(B) x = 1 is a regular singular point.

(C) x =1 is an ordinary point.

(D) x = 1 is not a singular point of the differential equation.

60. A discrete random variable X has the probability distribution

$$P(X = x) = \frac{x}{10},\ x = 1,2,3,4\,.$$

Given that X > 1, the probability that X > 2 is

(A) $\frac{2}{3}$

(B) $\frac{7}{9}$

(C) $\frac{3}{5}$

(D) $\frac{5}{9}$

61. Considering the pure rotational spectrum of carbon monoxide, one observes the effect of isotope substitution. The ratio of reduced mass (μ'/μ) of $^{12}C^{16}O$ (reduced mass μ) to $^{13}C^{16}O$ (reduced μ') = 1.04. What will be the ratio of B/B′, where B and B′ stand for rotational constants?

(A) 1.04

(B) 2.08

(C) 0.96

(D) 4.16

62. Which of the following molecule is microwave active ?

(A) CO_2

(B) OCS

(C) CH_4

(D) N_2

63. At what frequency is the Electron Spin Resonance observed with an applied magnetic field of 0.3 T ? (given $\beta e = 9.2 \times 10^{-24}$ J T^{-1} and g factor = 2)

(A) 9.5×10^6 Hz

(B) 9.5×10^9 Hz

(C) 9.5×10^3 Hz

(D) 9.5×10^{15} Hz

64. A proton NMR spectrometer operates with a radiation of frequency 300 MHz. Hence, the resonance of free proton is expected to be observed approximately at :

(A) 0.7 T

(B) 70 T

(C) 700 T

(D) 7 T

65. Helium gas with different partial pressure is used in both CO_2 as well as He-Ne Lasers. The purpose of using He in He-Ne laser and in CO_2 laser is _____ of He atoms and _____ of CO_2 molecules respectively.

(A) Excitation, Excitation

(B) De-Excitation, DE-Excitation

(C) Excitation, De-Excitation

(D) De Excitation , Excitation

66. What is the expression for the characteristic impedance (Z_0) of an ideal lossless transmission line with series inductance per unit length L and shunt capacitance C ?

(A) $\sqrt{\frac{L}{C}}$

(B) $\sqrt{\frac{C}{L}}$

(C) $\frac{L}{C}$

(D) $\sqrt{LC}$

67. The fundamental dispersion relation for an electromagnetic wave propagating through low density unmagnetized plasma is given by $\omega^2 = \omega_p^2 + c^2k^2$, where ω_p is plasma frequency, and c is the speed of light, k is wavenumber. What is the relation between phase velocity (v_p) and group velocity (v_g) ?

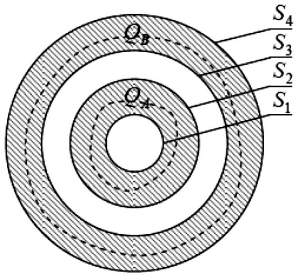
(A) $v_p = v_g$

(B) $v_p.v_g = c^2$

(C) $v_p = v_g^2$

(D) $v_p.v_g = \omega_p^2$

68. Electric charges Q_A and Q_B have been put on two conducting concentric spherical shells – see the figure.



The charges of each of the surfaces S_1, S_2, S_3, S_4 respectively are

(A) 0, Q_A , $-Q_A$, $Q_A + Q_B$

(B) 0, Q_A , Q_A , $Q_A + Q_B$

(C) Q_A , Q_A , Q_B , $Q_A + Q_B$

(D) $Q_A/2$, $Q_A/2$, $Q_B/2$, $Q_B/2$

69. In an extended region of space, there are vector fields, expressed in Cartesian coordinates as $\vec{E}_1 = E_0\{yz, xz, xy\}$ and $\vec{E}_2 = E_0\{xy, xy, yz\}$, where E_0 is an appropriate constant. Then

(A) Both $\vec{E}_1$ and $\vec{E}_2$ can be electrostatic fields

(B) $\vec{E}_1$ can be an electrostatic field but not $\vec{E}_2$

(C) $\vec{E}_2$ can be an electrostatic field but not $\vec{E}_1$

(D) Both $\vec{E}_1$ and $\vec{E}_2$ cannot be electrostatic fields

70. Two parallel infinite wires are at a distance d from each other. The force per unit length on one wire carrying current I_1 due the other carrying current I_2 in opposite direction is

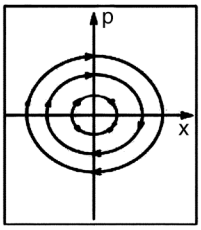
(A) Attractive and inversely proportional to d

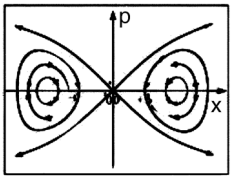
(B) Repulsive and inversely proportional to d^2

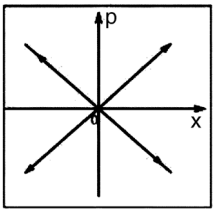
(C) Repulsive and inversely proportional to d

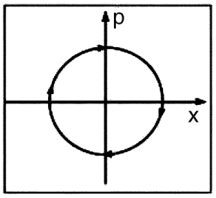
(D) Attractive and inversely proportional to d^2

71. Consider a particle of mass m moving in one dimensional potential $V(x)=x^4 - 2x^2$. The phase trajectory (x,p) of the particle is best represented by

(A) 

(B) 

(C) 

(D) 

72. The Hamiltonian of a particle in cylindrical coordinate is given by

$$H = \frac{1}{2m}\left(P_\rho^2 + \frac{P\varphi^2}{\rho^2} + P_z^{\,2}\right) - \frac{k}{\sqrt{\rho^2+z^2}},$$

then its equivalent Hamilton-Jacobi equation is

(Where, P_ρ,P_φ and P_z are conjugate momenta and $S(\rho, \varphi, z, t)$ is Hamilton's Principle function.)

(A) $$\frac{1}{2m}\left\{\left(\frac{\partial s}{\partial \rho}\right)^2 + \frac{1}{\rho^2}\left(\frac{\partial s}{\partial \varphi}\right)^2 + \left(\frac{\partial s}{\partial z}\right)^2\right\} + \frac{\partial s}{\partial t} - \frac{k}{\sqrt{\rho^2+z^2}} = 0$$

(B) $$\frac{1}{2m}\left\{\left(\frac{\partial s}{\partial \rho}\right)^2 + \frac{1}{\rho^2}\left(\frac{\partial s}{\partial \varphi}\right)^2 + \left(\frac{\partial s}{\partial z}\right)^2\right\} + \frac{\partial s}{\partial t} - \frac{k}{\rho^2+z^2} = 0$$

(C) $$\frac{1}{2m}\left\{\left(\frac{\partial s}{\partial \rho}\right)^2 + \rho^2\left(\frac{\partial s}{\partial \varphi}\right)^2 + \left(\frac{\partial s}{\partial z}\right)^2\right\} + \frac{\partial s}{\partial t} - \frac{k}{\rho^2+z^2} = 0$$

(D) $$\frac{1}{2m}\left\{\left(\frac{\partial s}{\partial \rho}\right)^2 + \frac{1}{\rho^2}\left(\frac{\partial s}{\partial \varphi}\right)^2 + \left(\frac{\partial s}{\partial z}\right)^2\right\} - \frac{\partial s}{\partial t} - \frac{k}{\sqrt{\rho^2+z^2}} = 0$$

73. A number of degrees of freedom of an ellipsoid free to rotate about fixed point is

(A) 2

(B) 3

(C) 5

(D) 6

74. Two particles of masses m_1 and m_2 are connected by a rigid massless rod of length l, and move freely in a plane. The moment of inertial of the system about an axis perpendicular to the plane and passing through the center of mass is

(A) $\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} l^2$

(B) $\frac{1}{3}\frac{m_1 m_2}{m_1 + m_2} l^2$

(C) $\frac{1}{12}\frac{m_1 m_2}{m_1 + m_2} l^2$

(D) $\frac{m_1 m_2}{m_1 + m_2} l^2$

75. Let $\vec{L} = \vec{r} \times \vec{p}$ be the angular momentum of a particle. Using the standard canonical Poisson brackets, evaluate

$$\left\{L_x, L_y^2 + L_z^2\right\}.$$

(A) 0

(B) $2L_yL_z$

(C) $-2L_yL_z$

(D) $L_y^2 - L_z^2$

76. Let A be a 2×2 complex matrix satisfying

$$A^2 = \text{II}, \qquad A^3 = A$$

Here II denotes 2×2 identity matrix and θ is a real number. Which of the following expressions for $e^{i\theta A}$ is correct for $\theta = \pi/2$?

(A) II

(B) $-$ II

(C) A

(D) iA

77. Let A be a linear operator on a finite-dimensional complex vector space satisfying

$A^4 = A$.

If λ is an eigenvalue of A, which of the following sets contains all possible values of λ ?

(A) $\{0,1,-1,i\}$

(B) $\{0,1, e^{2\pi i/3}, e^{4\pi i/3}\}$

(C) $\{1, e^{2\pi i/3}, e^{4\pi i/3}\}$

(D) $\{1, i, -i\}$

78. Let A be a real antisymmetric $n \times n$ matrix. Let S be a real $n \times n$ orthogonal matrix. Under the similarity transformation

$A \rightarrow A' = S^{-1} AS$

which of the following statements is correct ?

(A) A' is always antisymmetric.

(B) A' is always symmetric.

(C) A' is antisymmetric only if A is diagonal.

(D) A' need not be either symmetric or antisymmetric.

79. Consider the complex function

$$f(z) = \frac{e^z - 1 - z}{z^3}.$$

What is the nature of the singularity of f(z) at z = 0 ?

(A) Removable singularity

(B) Pole of order two

(C) Simple pole

(D) Essential singularity

80. The residue of the function

$$f(z) = \frac{1}{z^2 \sin z}$$ at z = 0 is

(A) 0

(B) $\frac{1}{2}$

(C) $-\frac{1}{6}$

(D) $\frac{1}{6}$

81. Which type of fundamental force binds quarks together to form protons and neutrons, and what gauge boson mediates this interaction ?

(A) Weak nuclear force, mediated by W and Z bosons

(B) Electromagnetic force, mediated by photons

(C) Strong nuclear force, mediated by gluons

(D) Strong nuclear force, mediated by pions

82. According to the CPT [C = Charge Conjugation, P = Parity, T = Time Reversal] theorem in physics, a particle and its antiparticle must always have the exact same

(A) Electric charge

(B) Rest mass

(C) Spin direction

(D) Spatial parity

83. To test parity conservation in weak interactions, an experiment was conducted at temperatures approaching absolute zero, utilizing a polarized cobalt-60 radioactive source that emitted negative beta particles.

During this experiment, which of the following observations directly confirmed the violation of parity symmetry in the beta decay process?

(A) Beta particles were emitted symmetrically in equal numbers both parallel and antiparallel to the direction of nuclear spins.

(B) In the mirror image of the experimental system, the beta particles were emitted antiparallel to the direction of nuclear spins.

(C) The emission rate of beta particles from Cobalt-60 changed depending on whether the applied external magnetic field was turned on or off.

(D) In the mirror image of the experimental system, the beta particles were emitted parallel to the direction of nuclear spins.

84. Alpha particles ^{4}He (A=4 ,Z=2) bombard a stationary target of Selenium ^{76}Se (A=76 and Z =34) to induce the $^{76}Se(\alpha, 2n)^{78}$ Kr nuclear reaction.

The coulomb barrier between the ^{76}Se nucleus and the alpha particle when they just touch is ;

(A) 10 MeV

(B) 12 MeV

(C) 14 MeV

(D) 16 MeV

85. In a typical decay process, X nucleus can either decay via β^+ decay process through, ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + \beta^+ + \nu,$, and/or in an electron capture process through ${}^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y + \nu$. If the Q value of the reaction is 0.9 MeV, which of the following statements will hold true ?

(A) Decay will occur through purely β^+ decay process

(B) Decay will occur through electron capture process

(C) Both decay processes are possible

(D) Both are forbidden

86. Consider a relativistic spin 1/2 particle with rest mass m in its rest frame. Let

$$\Psi_1(t) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{imc^2 t/\hbar} \text{ and } \Psi_2(t) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{imc^2 t/\hbar}$$

be two wave functions at time t for this particle. c is the speed of light. Which of the following statements is true ?

(A) Both Ψ_1 and Ψ_2 are valid wavefunctions.

(B) Ψ_1 is a valid wavefunction but Ψ_2 is not a valid wavefunction.

(C) Ψ_1 is not a valid wavefunction but Ψ_2 is a valid wavefunction.

(D) Both Ψ_1 and Ψ_2 are not valid wavefunctions.

87. Consider the free Klein-Gordon equation. Let

$$\varphi_1 = e^{i\vec{p}.\vec{x}/\hbar}\, e^{+iE_1 t/\hbar} \text{ and}$$
$$\varphi_2 = e^{i\vec{p}.\vec{x}/\hbar}\, e^{-iE_2 t/\hbar}$$

where

$$E_1 = \sqrt{\vec{p}^{\,2} c^2 + m^2 c^4} \text{ and } E_2 = \vec{p}^{\,2}/2m$$

(m is the rest mass and c is the speed of light). Which of the following statements is true ?

(A) φ_1 and φ_2 are both solutions of the free Klein-Gordon equation.

(B) φ_1 is a solution of the free Klein-Gordon equation but φ_2 is not a solution of the free Klein-Gordon equation.

(C) φ_1 is not a solution of the free Klein-Gordon equation but φ_2 is a solution of the free Klein-Gordon equation.

(D) Both φ_1 and φ_2 are not solutions of the free Klein-Gordon equation.

88. Consider scattering from a potential

$$V(r) = \begin{cases} V_0 & \text{for } r \le a \\ 0 & \text{for } r > a \end{cases}$$

For low-energy scattering, the total scattering cross-section σ under first order Born approximation is given by

(A) $\sigma = 2\pi\left(\frac{h^2}{V_0 a^2}\right)^{1/2}$

(B) $\sigma = 2\pi\left(\frac{V_0 a^2}{h^2}\right)^{1/2}$

(C) $\sigma = 4\pi\left(\frac{mV_0}{h^2 a^2}\right)$

(D) $\sigma = 4\pi\left(\frac{2mV_0 a^3}{3h^2}\right)^2$

89. A system in thermal and diffusive equilibrium with a reservoir can exist in only two states: an empty state with particle number $N_0=0$ and energy $E_0=0$, and an occupied state with particle number $N_1=1$, energy $E_1=\varepsilon$, and degeneracy g. Using the grand canonical ensemble, what is the mean number of particles $\langle N \rangle$ in the system ? Here $\beta = \frac{1}{k_B T}$ and $z=e^{\beta\mu}$ is the fugacity, where μ is the chemical potential.

(A) $\langle N \rangle = \frac{ge^{-\beta(\varepsilon-\mu)}}{1+ge^{-\beta(\varepsilon-\mu)}}$

(B) $\langle N \rangle = \frac{ge^{-\beta\varepsilon}}{1+ge^{-\beta\varepsilon}}$

(C) $\langle N \rangle = \frac{ge^{-\beta(\varepsilon+\mu)}}{1-ge^{-\beta(\varepsilon+\mu)}}$

(D) $\langle N \rangle = -\frac{ge^{-\beta(\varepsilon-\mu)}}{1+ge^{-\beta(\varepsilon-\mu)}}$

90. A crystalline one-dimensional solid contains N identical molecules. Even very close to absolute zero, each molecule may remain frozen independently in either of two equally probable $\rightarrow$ or $\leftarrow$ orientations. What is the residual entropy S_{res} of the solid ?

(A) $S_{res}=0$

(B) $S_{res}=Nk_B \ln 2$

(C) $S_{res}=2Nk_B$

(D) $S_{res}=k_B \ln N$

91. In a Klystron oscillator the oscillations are maintained by

(A) Plate – to – cathode external feedback.

(B) Bunches of electrons passing the cavity grids.

(C) A helical slow – wave structure.

(D) Cross field magnetic field.

92. In a Linear Variable Differential Transformer (LVDT) displacement transducer _____ is varied according to displacement.

(A) Capacitance

(B) Resistance

(C) Inductance

(D) Permittivity

93. Which one of the following options describes the basic phenomenon involved in a Pirani gauge for pressure measurement ?

(A) Ionization of gas molecules.

(B) Variation of thermal conductivity of a gas with pressure.

(C) Deflection of a mechanical diaphragm.

(D) Hydrostatic pressure of a mercury column.

94. Which one of the following material compositions is used in a Type K thermocouple ?

(A) Chromel / Alumel

(B) Copper / Constantan

(C) Iron / Constantan

(D) Chromel / Constantan

95. The Zeeman pattern of a line consists of six equidistant components. If the upper state term is $^2P_{3/2}$, then the lower state term is

(A) $^2S_{1/2}$

(B) $^2P_{1/2}$

(C) $^3P_{3/2}$

(D) $^2P_{3/2}$

96. In how many ways can three indistinguishable fermions be distributed among seven distinct single-particle energy levels, assuming each level can hold at most one particle ?

(A) $\frac{7!}{3!4!}$

(B) 3^7

(C) $\frac{9!}{3!6!}$

(D) 7^3

97. A system has three nondegenerate energy levels E = 0, E = 1.38×10^{-23} J, and E = 2.76×10^{-23} J.
If $k_B = 1.38 \times 10^{-23}$ J K^{-1} and T = 1 K, what is the probability that the system is in the level E ?

(A) $\frac{1}{e}$

(B) $\frac{1}{1+e^{-1}+e^{-2}}$

(C) $\frac{e^{-1}}{1+e^{-1}+e^{-2}}$

(D) $\frac{e^{-2}}{1+e^{-1}+e^{-2}}$

98. An initially uncharged capacitor of capacitance C is fully charged to potential difference V by an ideal battery at constant temperature T. If ΔQ is the heat released to the surroundings and ΔS_{univ} is the entropy change of the universe, which relation is correct ?

(A) $\Delta Q = \frac{CV^2}{2}, \Delta S_{univ} = \frac{CV^2}{2T}$

(B) $\Delta Q = \frac{CV^2}{2}, \Delta S_{univ} = \frac{CV^2}{T}$

(C) $\Delta Q = CV^2, \Delta S_{univ} = CV^2T$

(D) $\Delta Q = CV^2, \Delta S_{univ} = \frac{CV^2}{T}$

99. Photon cannot undergo Bose-Einstein condensation. This is due to

(A) Non-conservation of photon number

(B) Photon has zero mass

(C) Photon has spin 1

(D) Photons travel with speed of light

100. For a two-spin one-dimensional Ising model with energy given by

$$E = -J\sum_{K=1}^{2} S_k S_{k+1} - H\sum_{k=1}^{2} S_k$$

The partition function Z with periodic boundary condition is given by

$$Z = 2e^{2\beta J}\cosh(2\beta H) + 2e^{-2\beta J}$$

where $\beta = \frac{1}{k_B T}$

If M denotes the total magnetization and m = M/2 is the magnetization per spin, which expression is correct ?

(A) $m = \frac{\sinh(2\beta H)}{\cosh(2\beta H) + e^{-4\beta J}}$

(B) $m = \frac{\cosh(2\beta J)}{\sinh(2\beta J) + e^{-4\beta H}}$

(C) m=0

(D) $m = \frac{\sinh(2\beta J)}{\cosh(2\beta J) + e^{-4\beta H}}$

ROUGH WORK