OCT-30226
Paper-II
MATHEMATICAL SCIENCES

Series

A

Question Booklet Serial No. प्रश्नपत्रिका अनुक्रमांक

Signature and Name of Invigilator

1. (Signature) .....

   (Name) .....

2. (Signature) .....

   (Name) .....

Seat No. | | | | | | | | |

(In figures as in Admit Card)

Seat No. .....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

Time Allowed : 2 Hours] **[Maximum Marks : 200**

Number of Pages in this Booklet : **48**

Number of Questions in this Booklet : **180**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **180** objective type questions. Each question will carry **two** marks. Candidates should attempt **all** questions either from Sections I and II or from Sections I and III only.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. **Do not accept a booklet without sticker-seal or open booklet.**
   (ii) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. **Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes.** Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.
   (iii) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (B) is the correct response.
   (A) ● (C) (D)
   Following wrong methods should not be used as they are not recognised by scanning machine in digitized assessment. Candidate using such method will be responsible for their loss.

WRONG METHODS
(A) (B) (C) (D) (A) (B) (C) (D) (A) (C) (D) (A) (B) (C) (D)

5. Your responses to the items are to be indicated in the OMR Sheet given inside the Booklet only. If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. Use of any calculator or log table, etc., is prohibited.
12. There is no negative marking for incorrect answers.

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परीक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या OMR उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत **180** बहुविकल्प प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. विद्यार्थ्यांनी खण्ड I व II किंवा खण्ड I व III मधील **सर्व** प्रश्न सोडविणे अनिवार्य आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनिटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात :
   (i) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. **स्टीकर सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्वीकारू नये.**
   (ii) पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. **पृष्ठे कमी असलेली/कमी प्रश्न असलेली/ प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी.** त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया नोंद घ्यावी.
   (iii) वरीलप्रमाणे सर्व पडताळून पाहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा क्रमांक लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (B) हे योग्य उत्तर असेल तर.
   (A) ● (C) (D)
   खालील चुकीच्या पद्धती वापरू नये, कारण डिजिटाइज्ड (Digitized) मूल्यांकनात स्कॅनिंग मशीन त्यांना ओळखत नाही. त्या पद्धती वापरून नुकसान झाल्यास त्यास विद्यार्थीच जबाबदार असतील.

चुकीची पद्धती
(A) (B) (C) (D) (A) (B) (C) (D) (A) (C) (D) (A) (B) (C) (D)

5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत**. इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. उत्तरपत्रिकेवर नमूद केलेल्या ठिकाणाव्यतिरिक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापि, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.
12. चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.

Mathematical Sciences

Paper II

Time Allowed : 120 Minutes] **[Maximum Marks : 200**

Note : This paper contains **One Hundred Eighty (180)** multiple choice questions in THREE (3) Sections, each question carrying **TWO (2)** marks. Attempt ***all*** questions either from **Sections I and II** only or from **Sections I and III** only. The OMR Sheets with questions attempted from both the Sections viz. **II and III, will not be assessed**.

Number of questions, sectionwise :

Section I : Q. Nos. 1 to 20, **Section II : Q. Nos. 21 to 100,**
Section III : Q. Nos. 101 to 180.

SECTION I

1. Let $F : u \subset \mathbb{R}^3 \to \mathbb{R}^2$ be a continuous function. Which of the following statements is true ?

   (A) If all the directional derivatives of F at $p \in u$ exist then F is differentiable at p.

   (B) If all the partial derivatives of F at $p \in u$ exist then F is differentiable at p.

   (C) F is differentiable at p then the directional derivative of F exists at p in all directions $\vec{v} \in \mathbb{R}^3$.

   (D) F is differentiable at p but there exists a vector $\vec{v} \in \mathbb{R}^3$ such that the directional derivative of F in direction of $\vec{v}$ does not exist.

2. Consider the sequence of functions $\{f_n\}$ on $\mathbb{R}$ defined as

$$f_n(x) = \begin{cases} 2 & \text{for } |x| \le n \\ 1 & \text{for } |x| > n \end{cases}, \text{ then}$$

   (A) $\{f_n\}$ converges pointwise to a discontinuous function

   (B) $\{f_n\}$ converges pointwise to a continuous function

   (C) $\{f_n\}$ does not converge pointwise on $\mathbb{R}$

   (D) $\{f_n\}$ converges uniformly to a continuous function

3. Let $1 < b$. Then the sequence $(1 + b^n)^{1/n}$ converges to

   (A) 1

   (B) 2b

   (C) b

   (D) ∞

4. Let $f : [0, 1] \to \mathbb{R}$ be defined as $f(x) = \sqrt{x - x^2}$. Then

   (A) There exists $x_0 \in (0, 1)$ such that $f(x_0) = 0$

   (B) There exists $x_0 \in (0, 1)$ such that $f'(x_0) = 0$

   (C) There exists $x_0 \in (0, 1)$ such that $f(x_0) < 0$

   (D) There exists distinct numbers x_0 and $x_1 \in (0, 1)$ such that $f'(x_0) = 0$ and $f'(x_1) = 0$

5. Let $n \geq 2$. Which of the following statements is always true about $n \times n$ matrices with real entries ?

   (A) Every triangulable matrix is diagonalizable.

   (B) Every symmetric matrix is diagonalizable.

   (C) Every invertible matrix is diagonalizable.

   (D) If a matrix has one eigenvalue repeated n times then it is not diagonalizable.

6. Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in M_3(\mathbb{R})$ and W be eigenspace corresponding to eigenvalue 0. Then

   (A) dim W = 2 and A is diagonalizable

   (B) dim W = 2 and A is not diagonalizable

   (C) dim W = 1 and A is diagonalizable

   (D) dim W = 1 and A is not diagonalizable

7. Let $V = M_3(\mathbb{R})$ be the vector space of all 3×3 matrices with real entries and

$$W = \left\{ A = [a_{ij}] \in M_3(\mathbb{R}) \middle| \sum_{j=1}^{3} a_{1j} = 0 \right\}$$

be the subspace of V consisting of matrices with sum of entries in the first row is equal to zero. Then

(A) dim W = 9 and dim V/W = 8

(B) dim W = 8 and dim V/W = 1

(C) dim W = 7 and dim V/W = 2

(D) dim W = 6 and dim V/W = 3

8. Let A be 5×6 matrix with real entries. Let V be a subspace of $\mathbb{R}^6$ spanned by rows of A and W be a subspace of $\mathbb{R}^5$ spanned by columns of A. Then

(A) dim V < dim W

(B) dim V > dim W

(C) dim V = dim W

(D) dim V = dim W + 1

9. Let m denote Lebesgue measure on $\mathbb{R}$. Consider the sequence of sets $E_n = [n, n + 1]$ for $n \in \mathbb{N}$. Then

(A) $\lim_{n\to\infty} m(E_n) = m\left(\bigcap_{n=1}^{\infty} E_n\right)$

(B) $\lim_{n\to\infty} m(E_n) \neq m\left(\bigcap_{n=1}^{\infty} E_n\right)$

(C) $\bigcap_{n=1}^{\infty} E_n$ is not measurable

(D) $\lim_{n\to\infty} m(E_n)$ does not exist

10. Which of the following sets does *not* have measure zero ?

(A) $\{(x, y, z) \in \mathbb{R}^3 / 2x - 3y + 2z = 1\}$

(B) $\left\{(x,y) \in \mathbb{R}^2 / \frac{x^2}{4} + \frac{y^2}{9} = 1\right\}$

(C) $\left\{(x,y,z) \in \mathbb{R}^3 / \frac{x^2}{4} + \frac{y^2}{9} = 1\right\}$

(D) $\left\{(x,y) \in \mathbb{R}^2 / \frac{x^2}{4} + \frac{y^2}{9} \geq 1\right\}$

11. Let f be a non-negative measurable function defined on a set $A \subset \mathbb{R}$. Such that $\int_A f = 0$ is false, if

(A) f(x) = 1 on $Q \cap A$
= 0 on $A \setminus Q$
where Q denotes the set of rational numbers in $\mathbb{R}$

(B) A is subset of set of integers in $\mathbb{R}$

(C) A is the set of irrational numbers and f is bounded below by 1

(D) A is a subset of rational numbers and f is bounded above by 1

12. For the map F(x, y) = (1 + x, 3) which of the following statements is false ?

(A) F is continuous

(B) F is differentiable

(C) F is not one-one

(D) Inverse function theorem holds for F at (1, 1)

13. Let $I : \mathbb{R}^2 \to \mathbb{R}^2$ be the identity operator defined by I(v) = v for all $v \in \mathbb{R}^2$. Let $B_1 = \{u_1 = (1, 1), u_2 = (1, -1)\}$ be a basis of the domain space and $B_2 = \{v_1 = (1, 0), v_2 = (0, 1)\}$ be a basis of co-domain space. Then matrix representation of the identity operator I is

(A) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

14. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation giving the reflection about the line given by x + y = 0. Then which of the following are eigenvalues of T ?

(A) 1 and 2

(B) 0 and 1

(C) 1 occuring with multiplicity two

(D) 1 and –1

15. Determine which of the following defines an inner product on $\mathbb{C}^2$. For $u = (x_1, y_1)$ and $v = (x_2, y_2)$ in $\mathbb{C}^2$.

(A) $\langle u, v \rangle = x_1 x_2$

(B) $\langle u, v \rangle = y_1 y_2$

(C) $\langle u, v \rangle = x_1 x_2 + y_1 y_2$

(D) $\langle u, v \rangle = x_1 \overline{x}_2 + y_1 \overline{y}_2$

16. Read the statements carefully and select the correct option.

Let V be a finite dimensional vector space over $\mathbb{R}$ with dim V $\geq$ 2. Let T : V $\rightarrow$ V be a linear transformation. Consider the following statements :

I : Every vector in V can be written as linear combination of eigenvectors of T.

II : T always has eigenvectors in V.

Then

(A) I is true and II is false

(B) I is false and II is true

(C) Both I and II are false

(D) Both I and II are true

17. Let f : [a, b] $\rightarrow \mathbb{R}$ be a continuous function and suppose that $\int_a^b f(x)dx = 0$. Then

(A) $f(x) \neq 0$ for all $x \in (a, b)$

(B) $f(x) > 0$ for all $x \in (a, b)$

(C) $f(x) = 0$ for some $x \in (a, b)$

(D) $f(x) < 0$ for all $x \in (a, b)$

18. Let C [0, 1] denote the space of continuous functions on [0, 1] with sup. norm $||.||_\infty$ and P denote the subset of all polynomials in C [0, 1]. Then

(A) P is an open subset of C [0, 1]

(B) P is a closed subset of C [0, 1]

(C) P is a dense subset of C [0, 1]

(D) P is a compact subset of C [0, 1]

19. Let A be n $\times$ n matrix with real entries and B be a row echelon form of A. Then

(A) A is similar to B

(B) det A = det B

(C) A and B have same eigenvalues

(D) rank A = rank B

20. Let $n \geq 2$. Let A be n $\times$ n matrix with real entries having all its eigenvalues real and let J be a Jordan Canonical form of A. Which of the following statements is false about A and J ?

(A) rank J = rank A

(B) det J = det A

(C) J and A are similar matrices

(D) J is a row echelon form of A

SECTION II

21. Let S be a commutative ring with unity. Let R be a subring of S. Which of the following statements is true ?

(A) If R is a field, then every ideal of R is an ideal of S

(B) If R is an integral domain and S is its field of fractions, then every ideal of R is an ideal of S

(C) If R is an integral domain and S is not an integral domain, then every ideal of R is an ideal of S

(D) The intersection of an ideal of S with R is an ideal of R

22. Let F be a field of 49 elements. Then F has an element of multiplicative order

(A) 7

(B) 49

(C) 24

(D) 14

23. Let K/F be a finite Galois extension. Let the Galois group G of K/F be a cyclic group of order 20. Then the number of intermediate fields between F and K (excluding F and K) is

(A) 5

(B) 6

(C) 3

(D) 4

24. Consider the three topologies on $\mathbb{R}^J$, where $\mathbb{R}$ is the space of reals with standard topology and J be an index set, namely τ_1-uniform topology. τ_2-box topology and τ_3-the product topology. Then which of the following is true ?

(A) $\tau_1 \geq \tau_2 \geq \tau_3$

(B) $\tau_3 \geq \tau_1 \geq \tau_2$

(C) $\tau_2 \geq \tau_3 \geq \tau_1$

(D) $\tau_1 \geq \tau_3 \geq \tau_2$

25. Consider the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ in $\mathbb{R}$.

Then which of the following is true ?

(A) The sequence converges to 0 with respect to any topology on $\mathbb{R}$

(B) The sequence does not converge to more than one point with respect to arbitrary topology on $\mathbb{R}$

(C) The sequence converges to more than one point with respect to co-finite topology on $\mathbb{R}$

(D) The sequence is not convergent with respect to any topology on $\mathbb{R}$

26. The extremals of the functional

$$I\left(x(t), y(t)\right) = \int_{t_0}^{t_1}\left[\left(\dot{x}^2 + \dot{y}^2\right)^{1/2} + a^2\left(x\dot{y} - y\dot{x}\right)\right]dt,$$

where 'a' is a constant, are

(A) Parabolas

(B) Cycloids

(C) Circles

(D) Catenaries

27. A homogeneous string has mass per unit length ρ and is maintained under a constant tension T. The function u (x, t) that makes the integral

$$I\left(u(x,t)\right) = \int_0^t\int_0^l \frac{1}{2}\left(\rho u_t^2 - T u_x^2\right)dxdt$$

stationary satisfies, which of the following partial differential equations ?

(A) Laplace equation

(B) Heat equation

(C) Diffusion equation

(D) One-dimensional wave-equation

28. The solution of the Volterra integral equation

$u(x) = (1+x) + \int_0^x (x-t)u(t)dt$ is

(A) $u(x) = 1 + x$

(B) $u(x) = \sin x + \cos x$

(C) $u(x) = e^{-x}$

(D) $u(x) = e^{x}$

29. The integral equation

$f(x) = \lambda \int_0^x K(x,t)u(t)dt$, where

$K(x,t) = \sum_{i=1}^{n} a_i(x)b_i(t)$ and f(x) is known function, is of the type

(A) Volterra equation of first kind with degenerate Kernel

(B) Volterra equation of second kind with convolution Kernel

(C) Volterra equation of first kind with singular Kernel

(D) Volterra equation of second kind with symmetric Kernel

30. The eigenvalue of the Fredholm integral equation

$u(x) = \lambda \int_0^1 e^{x+t} u(t)dt$ is

(A) $\lambda = \frac{e^2 - 1}{2}$

(B) $\lambda = \frac{e^2 + 1}{2}$

(C) $\lambda = \frac{2}{e^2 + 1}$

(D) $\lambda = \frac{2}{e^2 - 1}$

31. Let $\mathbb{R}_d$ denote $\mathbb{R}$ in discrete metric. Consider the following functions :

$f : \mathbb{R} \to \mathbb{R}_d$ defined by f(x) = x for all x, and

$g : \mathbb{R}_d \to \mathbb{R}$ defined by g(1) = 1 and g(x) = 0 for any $x \neq 1$

Then

(A) f is continuous, but g is discontinuous

(B) f is discontinuous, but g is continuous

(C) Both f and g are discontinuous

(D) Both f and g are continuous

32. Which of the following topological spaces is *not* path connected ?

(A) An open connected subset of $\mathbb{R}^2$

(B) $A = \{(x, y) : y = x^2\}$ in $\mathbb{R}^2$

(C) $\mathbb{R}^2 \backslash B$ where B = {(x, y) : x, y are rationals}

(D) $\mathbb{R}_K$, where $\mathbb{R}_K$ denotes $\mathbb{R}$ with K-topology

33. Which of the following is true ?

(A) $\mathbb{R}$ is homeomorphic to $\mathbb{R}^2$

(B) A circle is homeomorphic to a triangle in $\mathbb{R}^2$

(C) [0, 1] is homeomorphic to a circle

(D) [0, 1] is homeomorphic to $\mathbb{R}$

34. Which of the following statements is true ?

(A) Every normal space is metrizable

(B) Every regular space is normal

(C) $\mathbb{R}_K$ is a normal space, where $\mathbb{R}_K$ denotes $\mathbb{R}$ in K-topology

(D) $\mathbb{R}_l^2$ is a regular space, where $\mathbb{R}_l$ denotes $\mathbb{R}$ in lower limit topology

35. Let A and B be disjoint closed sets in a metric space (X, d). Suppose $f : X \to \mathbb{R}$ is defined by $f(x) = \dfrac{d(x, A)}{d(x, A) + d(x, B)}$. Then

(A) f(a) = 1 for all $a \in A$

(B) f(b) = 0 for all $b \in B$

(C) f(x) > 1 for some $x \in X$

(D) f is a continuous function

36. The convergence of Picard's iteration method for the initial value problem $\dfrac{dy}{dx} = f(x, y), y(x_0) = y_0$ is always guaranteed on the rectangle R by

(A) continuity of f alone on the rectangle R

(B) a Lipschitz's condition on f in y on the rectangle R

(C) f is bounded on the rectangle R

(D) f has finitely many discontinuities on the rectangle R

37. With h = 1, the value of $\left(\dfrac{\Delta^2}{E}\right)x^2$ is

(A) 2

(B) 4

(C) 2x

(D) 8

38. Consider the following two statements with usual notations.

I. $\frac{\Delta^2 f(x)}{Ef(x)} = \left(\frac{\Delta^2}{E}\right) f(x)$.

II. $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)] = 24 \times 10!$.

Then which of the following is true ?

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) Neither I nor II is true

39. Newton-Raphson formula to obtain the cube root of N is

(A) $x_{n+1} = x_n - \frac{(x_n^3 - N)}{3x_n^2}$

(B) $x_{n+1} = x_n + \frac{(x_n^3 + N)}{3x_n}$

(C) $x_{n+1} = x_n - \frac{x_n^3}{3x_n^2}$

(D) $x_{n+1} = x_n + \frac{3x_n^2}{x_n^3}$

40. With usual notation – : Δ-forward difference operator, δ-central difference operator, E-shift operator, which relation is correct ?

(A) $\delta = \Delta E^{-1/2}$

(B) $\delta = \Delta E^{1/2}$

(C) $\delta = \Delta E$

(D) $\delta^2 = \Delta E^{-1/2}$

41. Suppose f is analytic in $|z| \leq 1$ and $|f(z)| \leq |z|$ for all z. If f has a zero of order k at z = 0, then necessarily

(A) k = 0

(B) k = 1

(C) $k \geq 1$

(D) $k \leq 1$

42. Let γ denote the circle $\left|z-\frac{1}{2}\right|=1$ in $\mathbb{C}$.

Then $\int_{\gamma}\frac{z^2}{(z-1)^2(z+1)}dz =$

(A) $\frac{3\pi i}{2}$

(B) $3\pi i$

(C) $-\frac{3\pi i}{4}$

(D) 0

43. The function $f(z)=\frac{\sin z - z}{z^4}$ has at $z = 0$ a

(A) removable singularity

(B) pole of order 1

(C) pole of order 3

(D) essential singularity

44. The number of zeros of $z^5 - 2z + 8$ in the disc $|z| < 1$ is

(A) 0

(B) 1

(C) 2

(D) 5

45. The number of non-negative integer solutions of $x + y + z + t = 8$ is

(A) $\binom{11}{3}$

(B) $\binom{9}{3}$

(C) $\binom{8}{4}$

(D) $\binom{11}{4}$

46. For the initial value problem

$y''(x)+ y(x) = \sin x$, $y(0) = 0$, $y'(0) = 0$,

the equivalent Volterra integral equation is

(A) $u(x)=\sin x-\int_0^x (x-t)u(t)dt$

(B) $u(x)=\sin x+\int_0^x (x+t)u(t)dt$

(C) $u(x)=\sin x-\int_0^x u(t)dt$

(D) $u(x)=\sin x+\int_0^x u(t)dt$

47. Consider the following two statements for the integral operator K.

$(Ku)(x) = \int_a^b K(x,t)u(t)dt$, where

K(x, t) = K(t, x).

I. The integral operator K is self-adjoint.

II. Eigen functions corresponding to two distinct eigenvalues of K are orthogonal.

Then

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) Both I and II are false

48. The m^{th} iterated Kernel $K_m(x, t)$ of the integral equation

$u(x) = f(x) + \int_0^x e^{x^2-t^2}u(t)dt$ is

(A) $K_m(x,t) = \frac{(x-t)^{m-1}e^{x^2-t^2}}{(m-1)!}$

(B) $K_m(x,t) = \frac{(x-t)^{m-1}}{(m-1)!}$

(C) $K_m(x, t) = \frac{(x^2-t^2)^{m-1}}{(m-1)!}$

(D) $K_m(x,t) = \frac{(x^2-t^2)}{(m-1)!}e^{x^2-t^2}$

49. A particle of mass 'm' is projected with initial velocity 'u' making an angle θ with the horizontal, when gravity and air resistance (proportional to the particle's velocity) are present. Then the horizontal component of the velocity, at any time t is

(A) $\dot{x} = u\cos\theta$

(B) $\dot{x} = u\cos\theta e^{-\frac{k}{m}t}$

(C) $\dot{x} = \frac{mu\cos\theta}{k}\left(1 - e^{-\frac{k}{m}t}\right)$

(D) $\dot{x} = u\sin\theta e^{-\frac{k}{m}t}$, where k is a constant

50. Two particles fall freely from the same point at a distance 'h' from the ground. One is sliding down the smooth inclined plane of length 'l' and inclination θ with the horizontal, while the other falls vertically down through the height 'h', under a constant gravity. Then both the particles hit the ground

(A) simultaneously

(B) at the same velocity

(C) simultaneously with the same velocity

(D) at different times and with different velocities

51. The solution of the initial value problem $\dot{x} = Ax$, $x(0) = (0, 1)^T$ where $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is

(A) $x(t) = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$

(B) $x(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$

(C) $x(t) = \begin{bmatrix} \sin t \\ -\cos t \end{bmatrix}$

(D) $x(t) = \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix}$

52. Which of the following differential equations is exact ?

(A) $(2xy + y)dx + (x^2 + x)dy = 0$

(B) $(x + y)dx + (x - y)dy = 0$

(C) $(2xy + y^2)dx + (2xy + x^2)dy = 0$

(D) $(y^2 + x)dx + (x^2 + y)dy = 0$

53. Consider the following two statements.

I. All surfaces such as spheres with their centres on the z-axis and radius 'a', right circular cylinders with z-axis as their axis of symmetry and ingeneral, all surfaces of revolution about z-axis are represented by the first order partial differential equation of the form $yp - xq = 0$.

II. The partial differential equation satisfied by all surfaces of the form $F(u, v) = 0$, where u and v are functions of x, y, z is of the form $Pp + Qq = R$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ and P, Q, R are functions of x, y, z.

Then which one of the following is true ?

(A) Both I and II are true

(B) Neither I nor II is true

(C) Only I is true

(D) Only II is true

54. Normals to the surfaces represented by equations $yzdx + xzdy + xydz = 0$ and $yzp + xzq = xy$ are

(A) Collinear

(B) Orthogonal

(C) Parallel

(D) Non-collinear

55. Let D be a bounded domain in $\mathbb{R}^2$ and u be a non-constant solution of $\Delta u = 0$ in D

$u = c$ on ∂D, where ∂D denotes the boundary of D and $c \neq 0$ is a constant. Then

(A) If $c > 0$ then there exists $x_0 \in D$ such that $u(x_0) = 0$

(B) If $c < 0$ then there exists $x_0 \in D$ such that $u(x_0) = 0$

(C) There exists $x_0 \in D$ such that $u(x_0) = c$

(D) There does not exist $x_0 \in D$ such that $u(x_0) = c$

56. Let G be a group of order 35 and let H be the center of G. Then

(A) $|H| = 1$

(B) $|H| = 5$

(C) $|H| = 7$

(D) $|H| = 35$

57. Let G denote the group $GL_2(\mathbb{R})$ of 2×2 invertible real matrices. Which of the following statements is false ?

(A) The center of G is infinite

(B) Every finite group is a subgroup of G

(C) Every finite cyclic group is a subgroup of G

(D) G has a non-trivial, proper, normal subgroup

58. The number of zero-divisors in $\mathbb{R}[x]/(x^2)$ is

(A) One

(B) Two

(C) Countably infinite

(D) Uncountably infinite

59. Which of the following polynomials is irreducible over $\mathbb{Q}$?

(A) $x^5 + 3x^2 + 300$

(B) $x^4 + x^2 + 1$

(C) $x^7 + 1$

(D) $x^5 + x^4 + x^3 + x^2 + x + 1$

60. Which of the following rings has every ideal principal ?

(A) $\mathbb{Z}[x]$

(B) $\mathbb{Z}[i]$

(C) $\mathbb{R}[x, y]$

(D) $\mathbb{Z}\left[\sqrt{-5}\right]$

61. A non-identity Möbius transformation

(A) fixes at least one point of $\mathbb{C}$

(B) fixes at most two points of $\mathbb{C}$

(C) always maps lines onto lines

(D) always is an entire function

62. Which of the following power series converges uniformly on the closed unit disc ?

(A) $\sum_{n=0}^{\infty} z^n$

(B) $\sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n$

(C) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{z^n}{n}$

(D) $\sum_{n=0}^{\infty} z^{n!}$

63. Suppose f is analytic in the unit disc D and $f(z) = 1$ for all $z = x + iy \in D$ with $y = x^2$. Then

(A) f is constant

(B) f has a zero in D

(C) $f(z) = \lambda z$ for some $\lambda \neq 0$

(D) $f\left(\frac{1}{2}\right) = \frac{1}{4}$

64. If f is an entire one-one function, then

(A) $f'(z_0) = 0$ for some z_0

(B) f has an essential singularity at ∞

(C) f is not a polynomial

(D) f′ is a constant function

65. Let $D = \{z \in \mathbb{C} : 0 < |z| < 1\}$ and let $f(z) = e^{1/z}$ for $z \in D$. Then

(A) f is one-one in D

(B) f is not analytic in D

(C) f is bounded

(D) there exists a sequence $\{z_n\}$ in D such that $\lim_{n \to \infty} f(z_n) = 100i$

66. Solution to which of the following problems may not be unique ?

(A) $\Delta u = 0$ in D, u = constant on ∂D where $D \subset \mathbb{R}^2$ is a bounded domain and ∂D is boundary of D.

(B) $u_{tt} - 4u_{xx} = 0,\ 0 < x < 1,\ t > 0$

$u(x, 0) = x(1 - x),\ 0 \leq x \leq 1$

(C) $u_t = u_{xx},\ 0 < x < 1,\ t > 0$

$u(0, t) = u(l, t) = 0$

$u(x, 0) = x(1 - x),\ 0 \leq x \leq 1$

(D) $\Delta u = 0$ in D, $\frac{\partial u}{\partial \vec{n}} = f$ on ∂D and $u(0) = P_0$ for some $P_0 \in D$, where $D \subset \mathbb{R}^2$ is a bounded domain and ∂D is boundary of D

67. Let a flexible string be stretched to the length l and fixed at the end points 0 and A(l, 0), which lies along x-axis in its equilibrium position. Let ρ be the mass per unit length of the string. If the string is set vibrating in the xy plane and T is the tension in the string, the potential energy of the string is

(A) $\frac{1}{2}\int_0^l \rho u_t^2 dx$

(B) $\frac{T}{2}\int_0^l \left(u_x^2 + \frac{1}{c^2}u_t^2\right) dx$

(C) $\frac{1}{2}\int_0^l T u_x^2 \, dx$

(D) $\int_0^l \left(\rho u_t^2 + T u_x^2\right) dx$

68. The characteristic equations of the partial differential equation $yu_{xx} + (x + y)u_{xy} + xu_{yy} = 0$, for $y \neq x$ are

(A) $\xi = \frac{x^2}{2}, \eta = \frac{y^2}{2}$

(B) $\xi = x + y, \eta = x^2 + y^2$

(C) $\xi = y - x, \eta = y^2 - x^2$

(D) $\xi = x^2 + y^2, \eta = x^2 - y^2$

69. The complete integral of the equation $p + 2q = pq$ is

(A) $z = \frac{ax}{a-1} + ay + b$

(B) $z = \frac{ax}{a-1} + 2ay + b$

(C) $z = ax + \frac{a}{a-2}y + b$

(D) $z = 2ax + \frac{a}{a-1}y + b$

70. Consider the following two statements for a real-valued continuous function f(x) on the interval [a, b].

I. If f(a) and f(b) are of opposite signs then the equation f(x) = 0 has at least one root in the interval [a, b].

II. If f(a) and f(b) are of opposite signs then the equation f(x) = 0 has at most one root in the interval [a, b].

Then which of the following is true ?

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) Neither I nor II is true

71. Let $\overline{P}$ and P° be denote the closure and interior of a set P in a given topological space respectively. Then which of the following is true ?

(A) $P^\circ \geq \overline{P}$

(B) $P \geq \overline{P} \geq P^\circ$

(C) $P^\circ \geq P \geq \overline{P}$

(D) $\overline{P} \geq P \geq P^\circ$

72. The characteristic function of a subset P in a topological space X, $f : X \to \mathbb{R}$ is continuous if and only if P is

(A) closed

(B) finite

(C) open

(D) open and closed

73. Let A be a connected subspace of a topological space X and A_0 be the set of limit points of A. Then which of the following sets is always connected ?

(A) $A \cup A_0$

(B) $A \cap A_0$

(C) (Interior of A) $\cup A_0$

(D) (Interior of A) $\cap A_0$

74. Let X be a non-empty compact Hausdorff space. Then which of the following is always true ?

(A) X has no isolated points $\Rightarrow$ X is countable

(B) X is countable $\Rightarrow$ X has no isolated points

(C) X is uncountable $\Rightarrow$ X has no isolated points

(D) X has no isolated points $\Rightarrow$ X is uncountable

75. Let $X \times Y$ be the product space of the topological spaces X and Y. Then which of the following is *not* true ?

(A) $X \times Y$ is second countable whenever X and Y are second countable

(B) $X \times Y$ is separable whenever X and Y are separable

(C) $X \times Y$ is Lindelöf whenever X and Y are Lindelöf

(D) $X \times Y$ is first countable whenever X and Y are first countable

76. The second approximate solution of the differential equation $\frac{dy}{dx} = 1 + xy$, $y(0) = 2$ by Picard's method is

(A) $2 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

(B) $2 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{4}$

(C) $2 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}$

(D) $2 + x + \frac{x^2}{2} + \frac{x^3}{3}$

77. The extremals of the functional

$I\left(x(t), y(t)\right) = \int_{t_0}^{t_1} \left(\dot{x}^2 + \dot{y}^2 + \dot{x}\dot{y}\right) dt,$

$\dot{x} = \frac{dx}{dt}$, $\dot{y} = \frac{dy}{dt}$, are

(A) $x(t) = C_1 t + C_2$, $y(t) = C_3 t + C_4$

(B) $x(t) = \frac{t^2}{2} + C_1 t + C_2$,

$y(t) = \frac{-t^2}{2} + C_3 t + C_4$

(C) $x(t) = C_1 t^2 + C_2$, $y(t) = C_3 t^2 + C_4$

(D) $x(t) = \frac{-t^2}{2} + C_1 t + C_2$,

$y(t) = \frac{t^2}{2} + C_3 t + C_4$

78. The extremal of the functional

$I\left(y(x)\right) = \int_0^{x_1} \frac{\sqrt{1 + y'^2}}{y} dx$, $y(0) = 0$, and the point (x_1, y_1) can vary along the line $y = x - 5$, is

(A) $x^2 - 10x + y^2 = 0$

(B) $x^2 + 10x + y^2 = 0$

(C) $x^2 + 8x + y^2 = 0$

(D) $x^2 - 8x + y^2 = 0$

79. The extremal of the functional

$I\left(y(x)\right) = \int_0^{\frac{\pi}{2}} \left(y'^2 - y^2 + 2xy\right) dx,$

$y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$, satisfies the condition

(A) $y(0) = \frac{\pi}{2}$

(B) $y(\pi) = \pi$

(C) $y\left(\frac{\pi}{2}\right) = \frac{-\pi}{2}$

(D) $y(2\pi) = 2\pi$

80. The extremal of the functional $\iint_R \left(1+z_x^2+z_y^2\right)^{1/2} dxdy$, where z(x, y) must assume prescribed value on the boundary of the region R, is the solution of the equation

(A) $z_{xx}\left(1+z_y^2\right)+z_{yy}\left(1+z_x^2\right)-2z_xz_yz_{xy}=0$

(B) $z_{xx}\left(1+z_x^2\right)+z_{yy}\left(1+z_y^2\right)-2z_xz_yz_{xy}=0$

(C) $z_{xx}\left(1+z_y^2\right)+z_{yy}\left(1+z_x^2\right)+z_x^2+z_y^2-2z_xz_yz_{xy}=0$

(D) $z_x^2z_{xx}+z_y^2z_{yy}-2z_xz_yz_{xy}=0$

81. Let f = u + iv be analytic in $\mathbb{C}$ with $v = e^x\sin y + 3y$. Then u satisfies

(A) $u_{xx} = u_{yy}$

(B) u = v

(C) u is an unbounded function

(D) $u = e^x\cos x + 3x + C$

82. Let $A_n = \{z \in \mathbb{C} : z^n = 1\}$ for n = 1, 2, 3, ... and let $T = \bigcup_{n=1}^{\infty} A_n$. Then

(A) T is an uncountable set

(B) T is a connected set

(C) T is the unit circle

(D) $|1 + z + z^2 + ... + z^{n-1}| \leq n$ for any $z \in T$

83. Which of the following functions maps the first quadrant onto the upper half plane ?

(A) $f(z) = e^z$

(B) f(z) = sinz

(C) f(z) = 2z

(D) $f(z) = z^2$

84. Which of the following functions is nowhere analytic ?

(A) $f(z) = e^{iy}\cdot e^x$

(B) $f(z) = e^y\cdot e^{ix}$

(C) f(z) = 3x – y + i (x + 3y)

(D) $f(z) = -2xy + i(x^2 - y^2)$

85. A function $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = 1 + \sin z$ is

(A) bounded

(B) one-one

(C) onto

(D) differentiable but not analytic

86. Consider two uniform solid spheres of equal radii r placed so that one is directly above the other. The lower sphere is fixed and the upper sphere initially at rest, rolls down without slipping. Then the point at which the two spheres lose contact is

(A) $\cos\theta = \frac{10}{17}$

(B) $\cos\theta = \frac{5}{7}$

(C) $\cos\theta = \frac{2}{3}$

(D) $\cos\theta = \frac{7}{15}$

87. The velocity of a particle, falling freely from the north pole and traveling through a central tunnel, at a distance 'r' from the centre of the Earth of radius R is

(Where 'g' is the gravitational acceleration)

(A) $\dot{r} = gR - \frac{gR^2}{r}$

(B) $\dot{r}^2 = gR - \frac{gr^2}{R}$

(C) $\dot{r}^2 = \frac{gr^2}{R} - gR$

(D) $\dot{r} = \frac{gR^2}{r} - gR$

88. A particle is projected from the North Pole through a tunnel passing through the centre of the Earth of radius R with initial velocity $\sqrt{2gR}$, where 'g' is the gravitational acceleration. Assuming the earth is spherically symmetric and of uniform density, the particle then

(A) executes oscillatory motion under gravity along the tunnel from North Pole to South Pole

(B) executes oscillatory motion under gravity along the tunnel with amplitude of $\frac{3R}{2}$ from the centre

(C) exists from the South Pole with velocity $\sqrt{2gR}$ and escapes the earth's gravity, moving to infinity without returning

(D) reach the centre of the Earth with zero velocity and remain stationary there, without moving any further

89. The motion of a particle falling freely under Earth's gravity 'g', from infinity is described by the differential equation $\ddot{r} + \frac{gR^2}{r^2} = 0$, where R is the radius of the Earth. Then the total energy E of the particle is

(A) $E = \frac{1}{2}m\dot{r}^2 - \frac{mgR}{r^2}$

(B) $E = \frac{1}{2}m\dot{r}^2 + \frac{mgR^2}{r}$

(C) $E = \frac{1}{2}m\dot{r}^2 + \frac{mgr^2}{R}$

(D) $E = \frac{1}{2}m\dot{r}^2 - \frac{mgR^2}{r}$

90. Let the Eulerian angles ϕ, θ, ψ be respectively given by $\phi = \pi$, $\theta = \frac{\pi}{2}$, $\psi = \pi$, then the matrix of transformation from space set of axes to body set of axes is

(A) $A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(B) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$

(C) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

(D) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

91. Suppose that the function f satisfies $|f(x) - f(y)| \leq 2026 \mid x - y \mid$ for all $x, y \in D$, where $D \subseteq \mathbb{R}$. Which of the following is also necessarily true ?

(A) f is bounded on D

(B) f is uniformly continuous on D

(C) f is differentiable on D

(D) f is constant on D

92. Consider the Sturm-Liouville boundary value problem $-y'' = \lambda y$, $0 < x < \pi$, subject to the boundary conditions $y(0) = 0$, $y(\pi) = 0$. The eigenvalues of this problem are

(A) $\lambda_n = n$

(B) $\lambda_n = n^2$

(C) $\lambda_n = -n^2$

(D) $\lambda_n = 2n$

93. Let $f(x) = x^2$ and $g(x) = 2x|x|$, $x \in (-1, 1)$. Then

(A) g is not differentiable

(B) g is differentiable and the Wronskian $W(f, g)(x) = 0$ for all x

(C) f and g are linearly dependent on $(-1, 1)$

(D) f and g are solutions of some second-order linear differential equation with constant coefficients

94. The general solution of the Bernoulli's differential equation $y' + y = xy^2$ is

(A) $y(x) = \dfrac{1}{x + 1 + ce^x}$

(B) $y(x) = \dfrac{1}{x - 1 + ce^{-x}}$

(C) $y(x) = x + 1 + ce^x$

(D) $y(x) = x - 1 + ce^{-x}$

95. Which of the following first-order systems in matrix form is equivalent to the differential equation $y'' + y' + y = 0$?

(A) $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} . \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

(B) $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

(C) $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

(D) $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

96. Consider $\mathbb{Z}^3 = \{(a, b, c) : a, b, c \in \mathbb{Z}\}$. Let S be a finite subset of $\mathbb{Z}^3$. What is the least number of elements of S so that S has two elements whose arithmetic mean is also in S ?

(A) 8

(B) 9

(C) 10

(D) 11

97. The number of positive divisors of 1200 is

(A) 15

(B) 20

(C) 24

(D) 30

98. For which of the following values of n, $\left(\mathbb{Z}/n\mathbb{Z}\right)^x$ is *not* cyclic ?

(A) 100

(B) 81

(C) 1331

(D) 98

99. The common solution to $2x \equiv 3 \pmod 5$ and $3x \equiv 5 \pmod 7$ is

(A) $x \equiv 1 \pmod{35}$

(B) $x \equiv 2 \pmod{35}$

(C) $x \equiv 4 \pmod{35}$

(D) $x \equiv 8 \pmod{35}$

100. Let A_5 be the alternating group on 5 letters. Let H be a quotient group of A_5. Which of the following is a possible order of H ?

(A) 20

(B) 30

(C) 45

(D) 60

SECTION III

101. Suppose X is a continuous random variable with probability density function (pdf) f(x) and median θ. Suppose W denotes the sample median based on a random sample of size n from the distribution of X. The large sample distribution of W is $N(\theta, v)$, where variance v is given by

(A) $4f^2(\theta)/n$

(B) $[4nf^2(\theta)]^{-1}$

(C) $f^2(\theta)/(4n)$

(D) $1/(nf^2(\theta))$

102. Suppose $\{X_1, X_2, \ldots, X_n\}$ is a random sample of size n from a distribution with finite variance σ^2 and $S^2 = \sum(X_i - \bar{X})^2$. Then

(A) S^2 is an unbiased estimator of σ^2

(B) S^2/n is an unbiased estimator of σ^2

(C) $S^2/(n-1)$ is an unbiased estimator of σ^2

(D) $X_{(n)} - X_{(1)}$ is an unbiased estimator of σ^2

103. On the basis of a random sample from Poisson $P(\theta)$, $\theta > 0$ distribution, which of the following parametric functions are *not* estimable ?

(A) θ

(B) θ^2

(C) $\exp(-\theta)$

(D) $1/\theta$

104. Suppose $\{X_1, X_2, \ldots, X_n\}$ are independent random variables, where X_i follows uniform $U(0, i\theta)$ for i = 1, 2, . . ., n $\theta > 0$. Then the maximum likelihood estimator of θ is

(A) $\sum_{i=1}^{n} X_i / n$

(B) $\max\{X_1, X_2, \ldots, X_n\}$

(C) $\max\left\{X_1, \frac{X_2}{2}, \frac{X_3}{3}, \ldots \frac{X_n}{n}\right\}$

(D) $\left(1/n\right)\sum_{i=1}^{n}(X_i / n)$

105. Suppose X follows normal $N(\theta, \sigma^2)$ distribution, $\theta \in R$, $\sigma^2 > 0$. To construct a confidence interval for θ based on a random sample of size n, when σ^2 is unknown, the pivotal quantity is

(A) Sample mean $\bar{X}_n$

(B) $(\bar{X}_n - \theta)/\sigma$

(C) $(\bar{X}_n - \theta)/S$, where $S^2 = \sum(X_i - \bar{X})^2/(n-1)$

(D) $(\bar{X}_n - \theta)^2/\sigma^2$

106. Mean square error for regression estimator of population mean is given by

(A) $S_y^2\left(1-\rho^2\right)$

(B) $\left(1-\frac{n}{N}\right)S_y^2\left(1-\rho\right)$

(C) $\left(\frac{1-f}{n}\right)S_y^2\left(1-\rho^2\right)$, where f is a sampling fraction

(D) $\left(\frac{N-n}{n}\right)S_y^2\left(1-\rho\right)$

107. Consider a 2^6 factorial design with n replications. As experimenter is interested in only main effects and two factor interactions. Hence, such a model is fitted. Then error degrees of freedom for this model are

(A) 64(n –1)

(B) 64n – 42

(C) 64n – 22

(D) 64n – 21

108. Consider a connected block design. Then which of the following statements is true for the C-matrix of the design ?

(A) It must be positive definite

(B) It is always positive semi-definite

(C) It may be positive definite or positive semi-definite

(D) Definiteness of C-matrix will change from design to design

109. Let $\hat{L}_1$ and $\hat{L}_2$ be the BLUEs of two treatment contrasts $L_1 = \underline{l}'\underline{\tau}$ and $L_2 = \underline{m}'\underline{\tau}$ respectively in a BIBD. Consider the following two statements.

I. $\text{var}\left(\hat{L}_1\right) = \text{var}\left(\hat{L}_2\right)$.

II. $\text{cov}\left(\hat{L}_1, \hat{L}_2\right) = 0$, if $\underline{l}'\underline{m} = 0$.

Then

(A) I is true, but II is false

(B) II is true, but I is false

(C) I and II both are true

(D) I and II both are false

110. A single replicate of a 2^5 factorial experiment is performed in 8 blocks using three block defining effects ABC, ADE and BCE. Which of the following statements is *not* correct ?

(A) In all, 7 treatment combinations are confounded with blocks

(B) The effect AE, BD, ABCD are completely confounded with block effects

(C) No main effect is completely confounded with block effects

(D) All two-factor interactions are estimable independently of block effects

111. Let $X_1, X_2, ..., X_n$ be a random sample from U(0, θ) distribution. Then the $100(1-\alpha)\%$ shortest length confidence interval for parameter θ is given by

(A) $\left(X_{(n)}, \frac{X_{(n)}}{\alpha}\right)$

(B) $\left(\frac{X_{(n)}}{\alpha}, X_{(n)}\right)$

(C) $\left(X_{(n)}, \frac{X_{(n)}}{\alpha^{1/n}}\right)$

(D) $\left(X_{(1)}, X_{(n)}\right)$

112. Non-parametric test for testing goodness of fit is

(A) Run test

(B) Kolmogorov-Smirnov test

(C) Anderson-Darling test

(D) Sign test

113. Suppose $S_n(x)$ is an empirical distribution function with respect to a sample of size n from a population distribution function F(x), then which of the following is correct for fixed $x \in R$?

(A) $S_n(x)$ does not converge to F(x) in probability

(B) $S_n(x)$ follows binomial B(n, F(x))

(C) $nS_n(x)$ follows binomial B(n, F(x))

(D) $\frac{S_n(x) - F(x)}{\sqrt{F(x)\,(1 - F(x))}}$ converges in distribution to N(0, 1)

114. A company manufactures rods having population median 10 cm. A random sample of 10 rods was taken and the lengths were measured. The sorted observations are as follows : 9.7, 9.7, 9.8, 9.8, 9.8, 9.9, 9.9, 9.9, 10.1, 10.1. It is decided to use Wilcoxon signed rank test to test the null hypothesis H_0 : Median length of rod is 10 cm. What is the value of test statistic in this situation ?

(A) 7

(B) 6

(C) 48

(D) 55

115. First kind of beta distribution is the most appropriate prior distribution for

(A) Normal (θ, 1)

(B) Poisson (θ)

(C) Exponential with mean θ

(D) Bernoulli (θ)

116. The multiple correlation coefficient $\rho_{1.23\ ...\ p}$ is the

(A) Simple correlation between X_1 and linear combination of $X_2, X_3, ..., X_p$

(B) Maximum correlation between X_1 and linear combination of $X_2, X_3, ..., X_p$

(C) Correlation between X_1 and the residual $X_1 - \hat{X}_1$ where $\hat{X}_1$ is the best linear predictor of X_1 on $X_2, X_3, ..., X_p$

(D) max {corr(X_1, X_i), i = 2, 3, ..., P}

117. Variances of the principal components computed using correlation matrix on the basis of n observations on p variables, belong to which of the following intervals ?

(A) (0, p)

(B) (0, n)

(C) (0, 1)

(D) (p, n), $p < n$

118. Suppose $\underline{X}$ is a p-dimensional random vector with dispersion matrix $\Sigma = [\sigma_{ij}]$. Suppose $\lambda_1 > \lambda_2 > ... > \lambda_j > \lambda_{j+1} ... > \lambda_p$ are p eigenvalues of Σ. In Principal Component (PC) analysis of $\underline{X}$ based on n observations, the variance of j-th PC

(A) is always σ_{jj}

(B) is always λ_j

(C) is always $\sigma_{jj}\ \lambda_j$

(D) may depend on the weights of principal components

119. Reshuffling of clusters at each stage occurs in which of the following clustering methods ?

(A) Only in agglomerative hierarchical

(B) Only in divisive hierarchical

(C) K-means

(D) Both the hierarchical methods

120. The first two canonical correlations between two bivariate random vectors $\underline{X}$ and $\underline{Y}$ are 0.9 and 0.6 respectively. Further, U_1 and U_2 are the canonical variates based on $\underline{X}$ and V_1 and V_2 are the canonical variates based on $\underline{Y}$. If we fit a simple linear regression model to predict U_1 based on V_1, what would be the unexplained variation in this model ?

(A) 0.19

(B) 0.36

(C) 0.10

(D) 0.40

121. A biologist wants to estimate the total population of deer in a forest. She captures and marks 50 deer and releases them. One week later, she captures 80 deer and finds 10 are marked. If we assume that no emigration/immigration occurs and that marks don't affect survival, what is the estimate of deer population based on this information ?

(A) 400

(B) 65

(C) 120

(D) 260

122. A metro ticket counter handles peak morning crowds. On a given morning, 10 passengers arrive at the counter independently. Each passenger's online transaction has a 30% chances of failing. Queue A processes passengers whose transactions succeeded. Queue B processes passengers whose transactions failed. If X_A and X_B denote the numbers of passengers in Queue A and Queue B respectively, find $E(X_A - X_B)$.

(A) 3

(B) 7

(C) 4

(D) 10

123. A circular dartboard has a radius of 10 cm. At its centre, there is a circular bullseye with a radius of 2 cm. Assuming every dart thrown hits the dartboard uniformly at random, what is the probability of hitting the bullseye ?

(A) 0.02

(B) 0.04

(C) 0.2

(D) 0.1

124. Which of the following statements is true ? For a zero-mean random variable X, with finite variance, the expectation of $|X|$.

(A) May not exist

(B) Is positive and smaller than the standard deviation

(C) Exists but may not be smaller than the standard deviation

(D) Exists and may not be non-zero

125. Let X follow uniform distribution on [0, 1].

Define $X_n := n^{3/2} \max\left\{0, 1/n - X\right\}$.

Find the incorrect option as n tends to infinity.

(A) $X_n \to 0$ almost surely

(B) $X_n \to 0$ in probability

(C) $EX_n \to 0$

(D) $EX_n^2 \to 0$

126. Which of the following expressions is correct about failure rate r(t) ?

(A) $F(x) = \exp\left\{-\int_0^x r(t)\,dt\right\}$

(B) $F(x) = \exp\left\{-\int_x^\infty r(t)\,dt\right\}$

(C) $F(x) = 1 - \exp\left\{-\int_0^x r(t)\,dt\right\}$

(D) $F(x) = 1 - \exp\left\{-\int_x^\infty r(t)\,dt\right\}$

127. A life length random variable X has the following pdf

$f(x, \theta) = (1+\theta x)^{-\left(\frac{1}{\theta}+1\right)}$, $x > 0, \theta > 0$

This distribution has

(A) increasing failure rate when $\theta > 1$

(B) increasing failure rate for any θ

(C) decreasing failure rate when $\theta > 1$

(D) decreasing failure rate for any θ

128. Reliability function of parallel system of n independent components, with each component reliability is p is given by

(A) p^n

(B) $1 - p^n$

(C) $(1 - p)^n$

(D) $1 - (1 - p)^n$

129. The optimal value of the objective function of the following LPP

Min : x + 2y

Subject to $x - y \geq 1$ and $y \geq 2$

(A) is a positive finite value

(B) is a negative finite value

(C) does not exist

(D) exists and has infinitely many optimal solutions

130. For the LPP

Max : $3x + 2y$

Subject to $x/2 + y/3 \leq 1$,

$x \geq 0$ and $y \geq 0$

(A) the optimal solution does not exist

(B) the optimal value is more than 5

(C) there is a unique optimal solution

(D) the optimal value is zero

131. Consider the model,

$y_{ij} = \mu + \alpha_i + \gamma x_{ij} + \varepsilon_{ij}$, $i = 1, 2, ..., k$

$j = 1, 2, ..., n$

Then BLUE of γ is

(A) $$\frac{\sum_{i=1}^{k}\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_{i.}\right)\left(y_{ij} - \bar{y}_{i.}\right)}{\sum_{i=1}^{k}\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_{i.}\right)^2}$$

(B) $$\frac{\sum_{i=1}^{k}\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_{i.}\right)\left(y_{ij} - \bar{y}_{i.}\right)}{\sum_{i=1}^{k}\sum_{j=1}^{n}\left(y_{ij} - \bar{y}_{i.}\right)^2}$$

(C) $$\frac{\sum_{i=1}^{k}\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_{..}\right)\left(y_{ij} - \bar{y}_{..}\right)}{\sum_{i=1}^{k}\sum_{j=1}^{n}\left(y_{ij} - \bar{y}_{..}\right)^2}$$

(D) $$\frac{\sum_{i=1}^{k}\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_{..}\right)\left(y_{ij} - \bar{y}_{..}\right)}{\sum_{i=1}^{k}\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_{..}\right)^2}$$

132. Consider a Latin Square Design with P treatments. How many maximum linearly independent estimable parametric functions can be estimated in this model ?

(A) $3P - 1$

(B) $3(P - 1)$

(C) $3P - 2$

(D) $3P - 3$

133. Consider a CRD with K treatments and n observations per treatment as,

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}, \quad \tau_i \overset{iid}{\sim} N\left(0, \sigma_\tau^2\right), \quad \varepsilon_{ij} \overset{iid}{\sim} N\left(0, \sigma^2\right)$$

with τ_i's independent of ε_{ij}'s. If MST and MSE are mean treatment and error sum of respectively, then which of the following is correct ?

(A) $E(MST) = \sigma^2 + \sigma_\tau^2, E(MSE) = \sigma^2$

(B) $E(MST) = K\sigma_\tau^2, E(MSE) = \sigma^2 + \sigma_\tau^2$

(C) $E(MST) = \sigma^2 + \sigma_\tau^2, E(MSE) = \sigma^2 + K\sigma_\tau^2$

(D) $E(MST) = \sigma^2 + K\sigma_\tau^2, E(MSE) = \sigma^2$

134. Suppose a new predictor X_2 is added to a model containing X_1. X_1 and X_2 are uncorrelated. Thus, the original model was $Y = \beta_0 + \beta_1 X_1 + \varepsilon$ and the new model is $Y = r_0 + r_1 X_1 + r_2 X_2 + \delta$. Which of the following statements is correct about the ordinary least square estimators in the two models ?

(A) $\hat{\beta}_1 = \hat{r}_1$

(B) $\hat{\beta}_1 - \left|\hat{r}_2\right| = \hat{r}_1$

(C) $\hat{r}_1$ is a biased estimate of r_1

(D) $\left|\hat{r}_1\right| < \left|\hat{\beta}_1\right|$

135. Two separate regressions of height (Y) and age (X) for boys and girls yield identical intercepts, but significantly different slopes. What does this imply ?

(A) Both groups grow at identical rates, but start at different birth heights

(B) Gender has no impact on height growth trends over time

(C) Mean heights are identical across all age groups for both genders

(D) Growth rates differ by gender, but the expected height at birth is the same for both genders

136. Following are two statements.

I. Poisson process is a stationary process.

II. Poisson process is a process with stationary and independent increments.

Which of the following options is correct ?

(A) Both I and II are false

(B) Both I and II are true

(C) I is true but II is false

(D) I is false but II is true

137. Which of the following options is correct ?

For a linear birth-death process $\{X(t), t \geq 0\}$ with birth rate λ, death rate μ and $X(0) = a$, the mean function $M(t) = E(X(t))$ is

(A) $ae^{\lambda\mu t}$

(B) $ae^{-(\lambda-\mu)t}$

(C) $ae^{(\lambda-\mu)t}$

(D) $ae^{(\lambda+\mu)t}$

138. Let z_1, z_2, z_3 be three independent and identically distributed standard normal random variables. Define $Y = (z_1 + 2)^2 + (z_2 + 3)^2 + z_3^2$. What distribution does Y follow ?

(A) Central χ_3^2

(B) Non-central χ^2 (3, 5)

(C) Non-central χ^2 (3, 13)

(D) Non-central χ^2 (3, 10)

139. Suppose $X_{(r)}$ is the r^{th} order statistic based on a sample of size n from a continuous probability distribution. Let Q_2 be the population median. Then, the confidence coefficient of the confidence interval $(X_{(1)}, X_{(n)})$ of Q_2 is

(A) $1-\frac{1}{2^n}$

(B) $\frac{1}{2^n}$

(C) $1-\frac{1}{2^{n-1}}$

(D) $\frac{1}{2^{n-1}}$

140. Suppose X and Y are independent and identically distributed uniform U(0, 1) random variables. We define $Z = \max\{X, Y\}$ and $W = \min\{X, Y\}$.

Then, $P\left(Z - W > \frac{1}{2}\right)$ is

(A) 1/2

(B) 3/4

(C) 2/3

(D) 1/4

141. In how many distinct ways can the letters of the word "STATISTICS" be arranged ?

(A) 36,28,800

(B) 6,04,800

(C) 50,400

(D) 1,51,200

142. Consider the following two variables related to a task :

X = Time at which the work starts (e.g. 9:00 AM)

Y = Duration required to complete the work (e.g. 3 hours)

Which of the following statements is correct ?

(A) X is measured on a nominal scale and Y is measured on an interval scale

(B) Both X and Y are measured on a ratio scale

(C) X is measured on an interval scale and Y is measured on a ratio scale

(D) Both X and Y are measured on an interval scale

143. Suppose N follows Poisson $P(\lambda)$, $(\lambda > 0)$ distribution and conditional distribution of X given N = n is binomial B(n, p), $0 < p < 1$. Then P[x = i] is

(A) $\frac{e^{-\lambda}\lambda^i}{i!}$, i = 0, 1, 2, ...,

(B) $\frac{e^{-\lambda p}(\lambda p)^i}{i!}$, i = 0, 1, 2, ...,

(C) $p(1-p)^i$, i = 0, 1, 2, ...,

(D) $(1-p)p^i$, i = 0, 1, 2, ...,

144. Suppose X and Y are two random variables. Which of the following statements is false ?

(A) If X and Y are both ordinal variables, then we can use Spearman's rank correlation coefficient to check whether X and Y are related

(B) If X and Y are both measured on a nominal scale, then we can use Spearman's rank correlation coefficient to check whether X and Y are related

(C) If X and Y are both measured on a nominal scale, then we can use the coefficient of association to check whether X and Y are related

(D) If X and Y are both measured on a ratio scale, then we can use the product moment correlation coefficient to check whether X and Y are related

145. A bank analyses loan data. For approved loans with amount X (in lakhs) and borrower risk category $R \in$ {Low, Medium, High}, following information is available.

Risk Category (R)	Low	Medium	High
P(R)	0.5	0.3	0.2
E(X/R)	8	14	20

What is the expected loan amount (in lakhs) across all risk categories ?

(A) 14

(B) 12.2

(C) 9.8

(D) 15

146. In a simple linear regression model, $Y = \beta_0 + \beta_1 X + \varepsilon$, let $\hat{y}_1$ and $\hat{y}_2$ be fitted values for $x_1 = 15$ and $x_2 = 5$ respectively. If $\bar{x} = 10$, what can we say about $\hat{y}_1 - \hat{y}_2$?

(A) It is equal to $2\bar{Y}$

(B) It is equal to $10\bar{Y}$

(C) It is equal to $5\hat{\beta}_1$

(D) It is equal to $10\hat{\beta}_1$

147. A researcher fits a simple logistic regression model

$$\log\left(\frac{p(x)}{1-p(x)}\right) = \beta_0 + \beta_1 X + \varepsilon$$

If increasing x caused p(x) to decrease from 1 to 0, which of the following statements is true ?

(A) The curve of p(x) vs. x is s-shaped and odds ratio > 1

(B) The curve of p(x) vs. x is inverted s-shaped and odds ratio < 1

(C) The curve of p(x) vs. x is s-shaped and odds ratio < 1

(D) The curve of p(x) vs. x is inverted s-shaped and odds ratio > 1

148. Let $\underline{X} \sim Np(\underline{\mu}, \Sigma), \Sigma > 0$ and S_1 and S_2 be MLE and unbiased estimate of Σ respectively based on random sample of size n. What we can say about $|S_2| - |S_1|$?

(A) It is positive

(B) It is non-negative

(C) It can be negative or positive but cannot be equal to zero

(D) Its sign would depend on sample

149. Let $\underline{x}_1, \underline{x}_2, \underline{x}_3, \underline{x}_4$ be a random sample from $N_2\left(\underline{0}, I_2\right)$ and $D = \sum_{i=1}^{4} \underline{x}_i \underline{x}_i'$.
If $D = (d_{ij})$ then $var(d_{11} + 4d_{12} + 4d_{22})$ is

(A) 40

(B) 100

(C) 200

(D) 250

150. If $\underline{X} \sim Np\left(\underline{\mu}, \Sigma\right)$ and rank $(\Sigma) = r < p$, then which of the following is *not* correct ?

(A) The distribution of $\underline{X}$ is concentrated on an r-dimensional subspace of $\mathbb{R}^P$

(B) Components of $\underline{X}$ are linearly dependent

(C) There is at least one $r \times r$ submatrix of Σ which is positive definite

(D) The characteristic function of $\underline{X}$ does not exist

151. In a three way contingency table for testing the null hypothesis of independence of attributes against the alternative that the attributes are *not* independent, the expected frequency eijk of $(i, j, k)^{th}$ cell is given by

(Where nijk is the frequency of $(i, j, k)^{th}$ cell

$$ni.. = \sum_j \sum_k nijk \qquad n.j. = \sum_i \sum_k nijk$$
$$n..k = \sum_i \sum_j nijk \quad \text{and} \quad n = \sum_i \sum_j \sum_k nijk)$$

(A) nijk/n

(B) (ni../n)(n.j./n)(n..k)

(C) (ni..) (n.j.) (n..k)/n^3

(D) (ni..) (n.j.) (n..k)/n

152. Probability of accepting H_0 when H_0 is true is

(A) Probability of Type-I error

(B) 1 – [Probability of Type-I error]

(C) Probability of Type-II error

(D) 1 – [Probability of Type-II error]

153. From the uniform distribution over the interval $(0, \theta)$ two independent observations X and Y are randomly selected to test $H_0 : \theta = 1$ against $H_1 : \theta = 2$.

Find the size of the test for the test function.

$$\phi(X, Y) = \begin{cases} 1 & \text{if } X + Y > 1 \\ 0 & \text{o. w.} \end{cases}$$

(A) 0.25

(B) 0.50

(C) 0.75

(D) 0.95

154. Let $f(x) = \lambda e^{-\lambda x}$, $x > 0$, $\lambda > 0$. This family of distributions has Monotone Likelihood Ratio (MLR) property in

(A) $X_{(1)}$

(B) $X_{(n)}$

(C) $-\sum_{i=1}^{n} X_i$

(D) $\sum_{i=1}^{n} X_i$

155. Suppose the family of distribution has Monotone Likelihood Ratio (MLR) property in $T(\underline{x})$. Suppose the test $\phi(T)$ is UMP of size α for testing.

$H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$. Then

(A) $\phi(T)$ is also UMP for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$ of size smaller than α.

(B) $\phi(T)$ is also UMP of size α for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

(C) $\phi(T)$ is also UMP for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$ of size greater than α.

(D) $\phi(T)$ is not UMP for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$ of size α.

156. The Mahalanobis squared sample distance between the centroids of two groups is given by
(Where $\underline{\bar{X}}$ and $\underline{\bar{Y}}$ are sample mean vectors and D is a pooled sample dispersion matrix)

(A) $\left(\underline{\bar{X}} - \underline{\bar{Y}}\right)' D \left(\underline{\bar{X}} - \underline{\bar{Y}}\right)$

(B) $\left(\underline{\bar{X}} - \underline{\bar{Y}}\right)' D^{-1} \left(\underline{\bar{X}} - \underline{\bar{Y}}\right)$

(C) $\left(\underline{\bar{X}} - \underline{\bar{Y}}\right)' D^{-1} \left(\underline{\bar{X}} + \underline{\bar{Y}}\right)$

(D) $\left(\underline{\bar{X}} - \underline{\bar{Y}}\right)' D \left(\underline{\bar{X}} + \underline{\bar{Y}}\right)$

157. Based on the random sample of size n = 16 taken by using SRSWOR it is observed that sample mean = 1000 and standard error in sample mean is 10 then 95% confidence interval for population mean is

(A) (974.2, 1025.8)

(B) (993.55, 1006.45)

(C) (980.40, 1019.60)

(D) (995.10, 1004.90)

158. Which of the following are *not* criteria for the selection of stratification variables in stratified sampling ?

(A) The strata should be mutually exclusive and collectively exhaustive so that every population element is assigned to one and only one stratum

(B) Across the strata, the elements should be as heterogeneous as possible

(C) Stratification variable should not be closely related to the characteristic of interest

(D) Elements within a stratum should be as homogeneous as possible

159. Suppose under the systematic sampling, when N = nK
(N : population size, n : sample size)
Let
$V(\bar{y}_{sy})$ = variance among means of all systematic samples.
S^2_{wsy} = variance among units belonging to the same systematic sample.
S^2 = population variance for study variable.
Then $V(\bar{y}_{sy})$ =

(A) $(N-1)S^2 - K(n-1)S^2_{wsy}$

(B) $\frac{1}{N}\left[(N-1)S^2 - K(n-1)S^2_{wsy}\right]$

(C) $(N-1)S^2 - n(K-1)S^2_{wsy}$

(D) $\frac{1}{N}\left[(N-1)S^2 - n(K-1)S^2_{wsy}\right]$

160. In a college library there are 10,000 books with numbers 1, 2, ..., 10,000. Suppose we want to get a random sample of size 50 using systematic sampling. Then what is the probability of selecting book number 9292 in the sample ?

(A) $1/200$

(B) $1/50$

(C) $1/100$

(D) $1/10,000$

161. In a class of 100 students, 52 passed in Mathematics, 60 passed in Statistics, then the number of passed in both the subjects will lie in the interval

(A) (0, 11)

(B) (12, 52)

(C) (53, 100)

(D) (15, 50)

162. The range, arithmetic mean and standard deviation of 10 items are 20, 75, 10 respectively, if each observation is increased by 5, then

(A) Range and coefficient of variation both will increase

(B) Range and coefficient of variation both will remain same

(C) Range will not change and coefficient of variation will increase

(D) Range will not change and coefficient of variation will decrease

163. Suppose weighted mean of a and b is computed taking weights 0.4 and 0.6 respectively. If a is increased by 20%, then the weighted mean will

(A) Increase by 8% of a

(B) Decrease by 8% of a

(C) Not change

(D) Increase by 18% of a

164. Let X ~ Poisson (λ), with $\lambda > 0$, what is $E\left(\frac{1}{X+1}\right)$?

(A) $\frac{1}{\lambda+1}$

(B) $\frac{1-e^{-\lambda}}{\lambda}$

(C) $\frac{e^{-\lambda}}{\lambda}$

(D) $\frac{1}{\lambda}$

165. A condition has 10% prevalence. A test has 90% sensitivity (chance of positive if diseased) and 90% specificity (chance of negative if healthy). Given a negative test result, what is the probability that the person is truly disease free ?

(A) 90%

(B) 98.8%

(C) 50%

(D) 10%

166. For the LPP

Max : x

Subject to $y \leq x + 1$, $x + y + 1 \leq 0$ and $x \geq 0$

(A) there is an unbounded solution

(B) there is a unique optimal solution

(C) there are infinitely many optimal solutions

(D) there is no feasible solution

167. For the LPP

Max : $9x + 2y$

Subject to $x - y \leq 1$, $y - x \leq 1$ and $x + y = 3$

(A) the optimal solution is $(x, y) = \left(\frac{3}{2}, \frac{3}{2}\right)$

(B) the optimal solution is (x, y) = (2, 1)

(C) the optimal solution is (x, y) = (1, 2)

(D) there are infinitely many optimal solutions

168. In a M/M/1 queueing system, assume that the arrival and service rates are 4 and 5 per minute respectively. Then the long run probability of finding at least three customers in the system is

(A) half

(B) more than half

(C) less than half but positive

(D) zero

169. In a M/M/1 queueing system, assume that the service rate is m times of the arrival rate for some m > 1. Then the long run average number of customers in the system is

(A) $\frac{1}{m-1}$

(B) $\frac{1}{m}$

(C) $\frac{1}{m+1}$

(D) $\frac{1}{m^2}$

170. In a M/M/1 queueing system, assume that the service and arrival rates are 8 and 2 per hour respectively. Then the long run probability of a customer's waiting time being more than 10 minutes is not

(A) 0.64

(B) less than $\frac{1}{\sqrt{2}}$

(C) more than $\frac{1}{\sqrt{3}}$

(D) $\frac{1}{\sqrt{e}}$

171. For the linear model $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, $i = 1, 2, ..., k$; $j = 1, 2, ..., n_i$. Which of the following is *not* estimable ?

(A) $\mu + \alpha_i$, $i = 1, 2, ..., k$

(B) $\sum_{i=1}^{k} C_i\alpha_i$ with $\sum_{i=1}^{k} C_i = 0$

(C) $\frac{1}{k}\sum_{i=1}^{k} \alpha_i$

(D) $\mu + \frac{1}{k}\sum_{i=1}^{k} \alpha_i$

172. Let $\underline{b}'\underline{y}$ be a linear function of observations belonging to error space in a linear model $\underline{y} = X\underline{\beta} + \underline{\varepsilon}$, $\underline{\varepsilon} \sim N_n\left(\underline{0}, \sigma^2 I_n\right)$. Which of the following is *not* true ?

(A) $\underline{b}'\underline{y}$ is independently distributed of $X(X'X)^{-}X'\underline{y}$

(B) Variance of $\underline{b}'\underline{y}$ is unbiased estimator of error variance σ^2

(C) $\underline{b}$ is orthogonal to columns of x-matrix

(D) $\underline{b}'\underline{y}$ is independently distributed of BLUE $\underline{\lambda}'\hat{\underline{\beta}}$ of an estimable parametric function $\underline{\lambda}'\underline{\beta}$

173. In a linear model $\underline{y} = X\underline{\beta} + \underline{\varepsilon}$, $\underline{\varepsilon} \sim N_n\left(\underline{0}, \sigma^2 I_n\right)$ the two estimable parametric functions T_1 and T_2 have BLUE's such that $\hat{V}\left(\hat{T}_1\right) = 4\hat{V}\left(\hat{T}_2\right)$. Then the ratio of lengths of their confidence intervals will be

(A) 1 : 4

(B) 1 : 2

(C) 2 : 1

(D) 4 : 1

174. Let $SSR\left(\underline{\beta}\right)$ be the regression sum of squares for testing the hypothesis $H_0 : X\underline{\beta} = \underline{0}$ in a model $\underline{y} = X\underline{\beta} + \underline{\varepsilon}$ with $\underline{\varepsilon} \sim N_n\left(\underline{0}, \sigma^2 I_n\right)$ and rank(X) = r < p, the number of parameters. Then distribution of $\frac{1}{\sigma^2} SSR\left(\underline{\beta}\right)$ is

(A) $\chi^2_{(r, \Delta^2)}$ where $\Delta^2 = \underline{\beta}' X' \times \beta / \sigma^2$

(B) $\chi^2_{(n-r, \Delta^2)}$ where $\Delta^2 = \underline{\beta}' X' \times \underline{\beta}$

(C) $\chi^2_{(r)}$

(D) $\chi^2_{(r, \Delta^2)}$ with $\Delta^2 = \frac{1}{\sigma^2} \underline{\beta}' \underline{\beta}$

175. Consider an RBD with V treatments and b blocks. Let SST, SSB and SSE denote treatment, block and error sum of squares respectively. If a same constant C ≠ 0 is added to every observation in one particular block leaving all other observations unchanged, then which of the following is necessarily true ?

(A) SST remain unchanged while SSE and SSB both change

(B) SST and SSE remain unchanged while SSB changes

(C) SSE remain unchanged while SST and SSB will change

(D) SST, SSB and SSE will all change

176. Let X follow uniform distribution on [– 1, 1]. Define $A_n := \left\{X \geq -1/n\right\}$. Then find the correct option.

(A) $\sum_{n=1}^{\infty} P(A_n) < \infty$

(B) $P(\{A_n \text{ occurs infinitely often}\}) = 1$

(C) $P(\{A_n \text{ occurs infinitely often}\}) < 1$

(D) $\{A_n\}_{n \geq 1}$ are independent events

177. Let $\{X_n\}_{n \geq 1}$ be a sequence of independent random variables with identical Bernoulli distribution of parameter p. Find incorrect statements as n tends to infinity.

(A) for every

$$\varepsilon > 0 \; P\left\{\left|\frac{1}{n}\sum_{m=1}^{n} X_m - p\right| > \varepsilon\right\} \to 0$$

(B) for every

$$\varepsilon > 0 \; P\left(\bigcup_{m \geq n}\left\{\left|\frac{1}{m}\sum_{i=1}^{m} X_i - p\right| > \varepsilon\right\}\right) \to 0$$

(C) $E\left(\left|\frac{1}{n}\sum_{i=1}^{n} X_i - p\right|\right) \to 0$

(D) $\frac{\frac{1}{n}\sum_{i=1}^{n} X_i - p}{\sqrt{p(1-p)/n}} \to 0$ with probability 1

178. Suppose $\{X_n, n \geq 0\}$ is a Markov chain where f_{ii} denotes the probability of ever return to state i and μ_i denotes the mean recurrence time. Which of the following is correct ? A state i is a null persistent state if

(A) $f_{ii} < 1$ and $\mu_i < \infty$

(B) $f_{ii} = 1$ and $\mu_i = \infty$

(C) $f_{ii} < 1$ and $\mu_i = \infty$

(D) $f_{ii} = 1$ and $\mu_i < \infty$

179. Suppose $\{x_n, n \geq 0\}$ is a Markov chain with state space S = {1, 2, 3, 4} and one-step transition probability matrix P given by

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1/2 & 0 & 1/2 & 0 \\ 0 & 1/3 & 0 & 2/3 \\ 2/3 & 0 & 1/3 & 0 \\ 0 & 1/4 & 0 & 3/4 \end{bmatrix} \end{array}$$

Which of the following is correct ?

(A) The Markov chain is irreducible

(B) All states are non-null persistent

(C) All states are transient

(D) States 1 and 3 are transient and states 2 and 4 are non-null persistent

180. Suppose $\underline{\pi} = (\pi_1, \pi_2, ...)$ is a stationary distribution of a Markov chain with one-step transition probability matrix P. Following are two statements.

I. $\underline{\pi}$ is a stationary distribution of a Markov chain with transition probability matrix P^4.

II. A stationary distribution of a Markov chain with transition probability matrix P^4 is given by $(\pi_1^4, \pi_2^4, ...,)$.

Which of the following is a correct option ?

(A) Both I and II are false

(B) Both I and II are true

(C) I is true but II is false

(D) I is false but II is true

ROUGH WORK

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