Test Booklet Code & Serial No.
प्रश्नपत्रिका कोड व क्रमांक

**D**

# Paper-II

# MATHEMATICAL SCIENCE

**Signature and Name of Invigilator**

1. (Signature) ..........................................

(Name) ................................................

2. (Signature) ..........................................

(Name) ................................................

Seat No. | | | | | | | |

(In figures as in Admit Card)

Seat No. ..............................................................

(In words)

OMR Sheet No. | | | | | | | |

(To be filled by the Candidate)

## APR - 30224

**Time Allowed : 2 Hours]** **[Maximum Marks : 200**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **180**

**Instructions for the Candidates**

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **180** objective type questions. Each question will carry *two* marks. Candidates should attempt *all* questions either from sections I & II or from sections I & III only.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   **Example :** where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. Use only Blue/Black Ball point pen.
11. Use of any calculator or log table, etc., is prohibited.
12. There is no negative marking for incorrect answers.

**विद्यार्थ्यांसाठी महत्त्वाच्या सूचना**

1. परीक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 180 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. विद्यार्थ्यांनी खण्ड I व II किंवा खण्ड I व III मधील **सर्व** प्रश्न सोडविणे अनिवार्य आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनिटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्वीकारू नये.
   (*ii*) पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.
   (*iii*) वरीलप्रमाणे सर्व पडताळून पाहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   **उदा.** : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणाव्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापि, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.
11. कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.
12. चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.

# Mathematical Science
## Paper II

**Time Allowed : 120 Minutes]** **[Maximum Marks : 200**

**Note :** This Paper contains **One Hundred Eighty (180)** multiple choice questions in THREE (3) sections, each question carrying **TWO (2)** marks. Attempt ***all*** questions either from **Sections I & II** only **or from Sections I & III** only. The OMR sheets with questions attempted from both the Sections viz. **II & III, will not be assessed.**

Number of questions, sectionwise :

**Section I : Q. Nos. 1 to 20,** **Section II : Q. Nos. 21 to 100,** **Section III : Q. Nos. 101 to 180.**

### SECTION I

1. Let $P : \mathbf{R}^3 \to \mathbf{R}^3$ represent the projection of $\mathbf{R}^3$ onto $xy$-plane along $z$-axis. Then :

(A) $P^2 = I$

(B) $P^3 = P$

(C) P is invertible

(D) P is not diagonalizable

2. Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$. Then, singular values of A :

(A) are 7 and 3

(B) are $\sqrt{3}$, 2 and 1

(C) do not exist

(D) $\sqrt{7}$ and $\sqrt{3}$

3. Let A and B be similar matrices. Consider the following statements :

(*i*) $tr\,A = tr\,B$

(*ii*) det A = det B

(*iii*) Characteristic polynomial of A is equal to the characteristic polynomial of B

(*iv*) Minimal polynomial of A is equal to the minimal polynomial of B

(A) All the statements are true

(B) None of the statement is true

(C) Only (*i*), (*ii*), (*iii*) are true

(D) Only (*ii*), (*iii*), (*iv*) are true

4. Let A be an $n \times n$ nilpotent non-zero real matrix. Then :

(A) $I_n + A$ is invertible

(B) $I_n - A$ is invertible

(C) A is invertible

(D) A is diagonalizable

5. Let A be a $4 \times 4$ real orthogonal matrix with determinant –1. Then the determinant of $I_4 - A$ is :

(A) 0

(B) 1

(C) 2

(D) 5

6. The set given by

$$\{1 + (-1)^n \mid n \in \mathbf{N}\}$$

is :

(A) {1, –1}

(B) **N**

(C) {0, 1}

(D) {0, 2}

7. The number of non-empty subsets of X = {1, 2, 3, 4, 5, 6, 7} containing only even integers is :

(A) 7

(B) 8

(C) 1

(D) 3

8. Let $K \leq \mathbf{R}^n$ be with the property that any real valued continuous function on K is bounded. Then K is :

(A) Bounded

(B) Compact

(C) Derise

(D) Connected

9. Suppose a bounded function $f : [0, 1] \to \mathbf{R}$ is Riemann integrable. Then which of the following is *not* true ?

(A) $f$ is Lebesgue integrable

(B) Riemann integral of $f$ is same as Lebesgue integral of $f$

(C) $f$ is not Lebesgue integrable whenever $f$ is discontinuous

(D) $|f|$ is Lebesgue integrable

10. Let $f : \mathbf{R} \to \mathbf{R}$ be a function. Then which of the following is *not* true ?
    (A) If $f$ is continuous then so is $|f|$
    (B) If $f$ is integrable then so is $|f|$
    (C) If $f$ is differentiable then so is $|f|$
    (D) If $f$ is bounded then so is $|f|$

11. For $n \geq 1$, let $A_n$ be the set of all irrational numbers in the interval $[1-1/n, 1+1/n]$. Then, which of the following is true ?
    (A) $\limsup A_n = \{1\}$
    (B) $\liminf A_n = \{1\}$
    (C) $\lim A_n = \phi$
    (D) $\lim A_n = \{0\}$

12. Suppose that $f$ is a function defined by :

$$f(x) = \begin{cases} x & \text{if} \quad 0 < x \leq ½ \\ x - ½ & \text{if} \quad ½ < x < 1 \end{cases}$$

If $M = \left(\frac{1}{4}, \frac{1}{2}\right)$, $N = \left(\frac{3}{4}, 1\right)$ and $Q = \left\{x \mid f(x) \in \left(\frac{1}{4}, \frac{1}{2}\right)\right\}$, then which of the following is true ?
    (A) Q = M
    (B) $Q \subseteq N$
    (C) $Q = M \cup N$
    (D) $Q = M \cap N$

13. Suppose that $\{a_n, n \geq 1\}$ is a sequence of real numbers such that each term of the sequence is either 0, 1, 2, 3. Let S be the set of all such sequences. Then, which of the following is true ?
    (A) S is finite
    (B) S is countable infinite
    (C) S is equivalent to the interval (0, 1)
    (D) S is equivalent to the set of all positive integers

14. Suppose that $f_1$, $f_2$ and $f_3$ are functions defined by :

$$f_1(x) = x^2, \; 0 \leq x < \infty$$

$$f_2(x) = \cos(x), \; 0 \leq x \leq \pi / 2$$

$$f_3(x) = \begin{cases} \sin(x)/x & \text{if} \quad x \neq 0 \\ 1 & \text{if} \quad x = 0 \end{cases}$$

Which of the above functions is /are uniformly continuous on their respective domains ?
    (A) None of the three functions
    (B) Only $f_1$ and $f_2$
    (C) Only $f_1$ and $f_3$
    (D) Only $f_2$ and $f_3$

15. Consider the function $f : \mathbf{R}^2 \to \mathbf{R}$ defined by :

$$f(x, y) = \begin{cases} x^2 + y^2, & \text{if } \quad x = 0, y = 0 \\ 1, & \text{otherwise} \end{cases}.$$

Which of the following statements is/are true ?

$S_1$ : $f$ is discontinuous at (0, 0)

$S_2$ : $\lim\limits_{y \to 0} \lim\limits_{x \to 0} f(x, y) = \lim\limits_{x \to 0} \lim\limits_{y \to 0} f(x, y)$

$S_3$ : $\lim\limits_{x \to 0} f(x, y)$ is continuous at $y = 0$

(A) Only $S_1$ and $S_2$

(B) Only $S_1$

(C) Only $S_3$

(D) Only $S_2$ and $S_3$

16. The dimension of the vector space over **R** of all 5 × 5 upper triangular matrices with complex entries is :

(A) 30

(B) 20

(C) 25

(D) 15

17. Let A be a 4 × 3 matrix and B a 3 × 4 matrix. Which of the following is always true ?

(A) det (AB) = 0, det (BA) need not be zero

(B) det (AB) and det (BA) need not be zero

(C) det (AB) = det (BA) = 0

(D) det (AB) = 0 only if either AB = 0 or BA = 0

18. The system of linear equations is consistent :

$$\begin{pmatrix} 2 & 0 & 6 \\ 2 & 2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}.$$

(A) Only if $b_1 = 4$ and $b_2 = -12$ and has a unique solution

(B) For all values of $b_1$ and $b_2$ and has a unique solution

(C) Only if $b_1 = 4$ and $b_2 = -12$ and has infinitely many solutions

(D) For all values of $b_1$ and $b_2$ and has infinitely many solutions

19. Let A be a 6 × 7 matrix of rank 6 and B be a 7 × 5 matrix of rank 4, then the rank of AB is :

(A) 5

(B) 6

(C) 4

(D) Cannot be determined exactly, but less than or equal to 4

20. Let T be a linear transformation from $R^3$ to R given by

$$T[x, y, z]^T = 2x - 5y + 9z,$$

where $[x, y, z]^T$ donote the transpose of the vector $[x, y, z]$. Then, the dimension of the null space of T is :

(A) 0

(B) 1

(C) 2

(D) 3

## SECTION II

21. If $c_1 + c_2 \ln x$ is the general solution of differential equation :

$$x^2 \frac{d^2y}{dx^2} + Kx\frac{dy}{dx} + y = 0, \; x > 0$$

then K equals :

(A) 2

(B) 3

(C) –1

(D) –3

22. The initial value problem :

$$\frac{dy}{dx} = \sqrt{|y|}, \; y(0) = 0,$$

(A) has no solution

(B) has unique solution

(C) has non-zero solution

(D) has more than one solution

23. The differential equation :

$$y'' + e^x y = 0$$

has a series solution $y(x) = \sum_{k=0}^{\infty} a_k x^k$ which satisfies $x(0) = 1$ and $x'(0) = 0$. Then the values of the coefficients $a_0$, $a_1$, $a_2$ are :

(A) $a_0 = 1, a_1 = 0, a_2 = -½!$

(B) $a_0 = 1, a_1 = 1, a_2 = 2!$

(C) $a_0 = 0, a_1 = 1, a_2 = 2!$

(D) $a_0 = 1!, a_1 = 2!, a_2 = 3!$

24. Which of the following is false :

(A) If $\phi_1, \phi_2$ are linearly independent functions on I, they are linearly independent on any interval J contained in I

(B) If $\phi_1, \phi_2$ are linearly dependent functions on I, they are linearly dependent on any interval J contained in I

(C) If $\phi_1, \phi_2$ are linearly independent solutions of a second order differential equation L(Y) = 0 on an interval I, they are linearly independent on any interval J contained in I

(D) If $\phi_1, \phi_2$ are linearly dependent solutions of a second order differential equation L(Y) = 0 on an interval I, they are linearly dependent on any interval J contained in I

25. Consider the following statements :

(I) $f(x, y) = xy^2$ satisfies Lipschitz condition on the rectangle $|x| \leq 1, |y| \leq 1$.

(II) $f(x, y) = xy^2$ satisfies Lipschitz condition on the strip $|x| \leq 1$, $|y| < \infty$.

Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

26. The Wronskian of the solutions of the differential equation

$$y'' - 6y'' + 12y' - 8y = 0$$

is :

(A) $2e^{6x}$

(B) $3e^{3x}$

(C) $4e^{2x}$

(D) $2e^{2x}$

27. The initial value problem

$$y' = 2\sqrt{y}\,,\quad y(0) = a$$

has :

(A) a unique solution if $a < 0$

(B) no solution if $a > 0$

(C) infinitely many solutions if $a = 0$

(D) a unique solution if $a \geq 0$

28. Which one of the following is not the general solution of the partial differential equation :

$$z(xp - yq) = y^2 - x^2$$

(A) $x^2 + y^2 + z^2 = f(xy)$

(B) $x^2 + y^2 + z^2 = f((x + y)^2 + z^2)$

(C) $x^2 + y^2 + z^2 = f(y / x)$

(D) $(x + y)^2 + z^2 = f(xy)$

29. A partial differential equation which represents a surface of revolution obtained by revolving a plane curve $f(x, y) = 0$ about $x$-axis is :

(A) $yp - xq = 0$

(B) $xp - yq = 0$

(C) $y + zq = 0$

(D) $x + zp = 0$

30. Consider the following two first order partial differential equations

$$xp - yq = x \text{ ................ } (1)$$

and $x^2p + q = xz$ ................ (2)

Then :

(A) they have common solutions

(B) every solution of (1) is also the solution of (2)

(C) every solution of (2) is also the solution of (1)

(D) they do not have any common solutions

31. The characteristic curve and the envelope of the one parameter family of surfaces

$$(x - a)^2 + (y - 2a)^2 + z^2 = 1$$

are :

(A) the great circle of the sphere and $(y - 2x)^2 + 5z^2 = 5$ respectively

(B) any circle on the sphere and $(y - 2x)^2 + 2z^2 = 2$ respectively

(C) exactly the same great circles

(D) a circle on the sphere and $x^2 + y^2 = 1,\ z = 2a$ respectively

32. The solution of the first order partial differential equation $f(x, y, z, p, q) = 0$ of the form $F(u, v) = 0$, where $u$ and $v$ are functions of $x$, $y$, $z$ is called :

(A) complete integral

(B) general integral

(C) particular integral

(D) singular integral

33. The region in which the partial differential equation

$$(1 - x^2)u_{xx} - u_{yy} = 0$$

is an ellipse if :

(A) $x > 1$

(B) $x < 1$

(C) $|x| > 1$

(D) $|x| < 1$

34. The partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial y^2} = 0, \quad y > 0$$

is :

(A) hyperbolic

(B) parabolic

(C) elliptic

(D) Laplacian

35. The partial differential equations

$f(x, y, p, q) = 0$ and $g(x, y, p, q) = 0$

are compatible if :

(A) $\frac{\partial(f, g)}{\partial(x, z)} + \frac{\partial(f, g)}{\partial(y, z)} = 0$

(B) $\frac{\partial(f, g)}{\partial(x, z)} - \frac{\partial(f, g)}{\partial(y, z)} = 0$

(C) $\frac{\partial(f, g)}{\partial(x, p)} + \frac{\partial(f, g)}{\partial(y, q)} = 0$

(D) $\frac{\partial(f, g)}{\partial(x, p)} - \frac{\partial(f, g)}{\partial(y, q)} = 0$

36. Let E be the shift operator and μ be the averaging operator. Then :

(A) $\mu = E^{1/2} - E^{-1/2}$

(B) $\mu = \frac{E^{1/2} + E^{-1/2}}{2}$

(C) $\mu = \frac{E^{1/2} - E^{-1/2}}{2}$

(D) $\mu = \frac{E^{1/2} + E^{-1/2}}{3}$

37. Let $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. Using Picard's method, the first approximation is :

(A) $y_1 = x + x^2$

(B) $y_1 = 2 + x + x^2$

(C) $y_1 = 20 + x + x^2$

(D) $y_1 = 2 + x$

38. If $f(x) = \frac{1}{x^2}$, then the first divided difference $[a, b]$ is :

(A) $\frac{-(a+b)}{a^2 b^2}$

(B) $\frac{+(a+b)}{a^2 b^2}$

(C) $\frac{a+b}{ab}$

(D) $\frac{a-b}{ab}$

39. Let $\frac{dy}{dx} = f(x, y)$ be an initial value problem with $y(x_0) = y_0$. Then in Runge-Kutta method of order 4, the value of $K_3$ to find first approximation is :

(A) $K_3 = h(f(x_0, y_0))$

(B) $K_3 = h(f(x_0 + h, y_0 + h))$

(C) $K_3 = h\left(f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)\right)$

(D) $K_3 = h\left(f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{k_2}\right)\right)$

40. A $n \times n$ matrix A is diagonally dominant of :

(A) The absolute value of each leading diagonal element is greater than or equal to the sum of the absolute values of the remaining elements in that row

(B) The value of the diagonal element is maximum in that row

(C) The value of the diagonal element is greater than or equal to the sum of the values of remaining elements in that row

(D) All diagonal elements are zero

41. The value of

$\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)]$

is :

(A) 24 × 10 !

(B) 24 × 8 !

(C) 12 × 10 !

(D) 12 × 8 !

42. The curve for which the area of surface of revolution is minimum when revolved about $x$-axis is the solution of the equation :

(A) $ay' = \sqrt{y^2 - a^2}$, $a$ is a constant

(B) $\sqrt{y^2 - a^2}\ y' = a$

(C) $\sqrt{x^2 - a^2}\ y' = a$

(D) $ay' = \sqrt{x^2 - a^2}$

43. The Euler-Lagrange's differential equation for extremization of the functional

$$I(y(x)) = \int_0^1 (y'^2 - y^2)\, dx$$

subject to the condition that

$$\int_0^{\pi} y dx = 1$$

is :

(A) $y'' - y = 0$

(B) $2y'' + 2y = \lambda$

(C) $y'' + y = 0$

(D) $y'' - y = \lambda$

44. The first integral of Euler-Lagrange's differential equation of the functional

$$I(y(x)) = \int_{x_1}^{x_2} f(x, y', y'')\, dx$$

is :

(A) $f - y'\dfrac{\partial f}{\partial y'} - y''\dfrac{\partial f}{\partial y''} = \text{constant}$

(B) $\dfrac{\partial f}{\partial y'} + \dfrac{d}{dx}\left(\dfrac{\partial f}{\partial y''}\right) = \text{constant}$

(C) $\dfrac{\partial f}{\partial y'} - \dfrac{d}{dx}\left(\dfrac{\partial f}{\partial y''}\right) = \text{constant}$

(D) $\dfrac{\partial f}{\partial y} - \dfrac{d}{dx}\left(\dfrac{\partial f}{\partial y'}\right) = \text{constant}$

45. The extremal of the functional

$I(y(x)) = \int_0^1 \dfrac{\dot{y}^2}{x^3}\, dx$, when $x(0) = 0$, $x(1) = 1$ is :

(A) $y = x$

(B) $y = c$

(C) $y = x^4$

(D) $y = 0$

46. The solution of the functional

$$I(y(x)) = \int_{x_1}^{x_2} f(x, y, y'')\, dx$$ is a :

(A) one parameter family of curves

(B) two parameter family of curves

(C) three parameter family of curves

(D) four parameter family of curves

47. Let **B** be a Banach space. Then $f : \mathbf{B} \to \mathbf{R}$ is said to be coercive if :

(A) $f(x) \to +\infty$ as $\| x \| \to 0$

(B) $f(x) \to 0$ as $\| x \| \to \infty$

(C) $f(x) \to +\infty$ as $\| x \| \to \infty$

(D) $f(x) \to -\infty$ as $\| x \| \to \infty$

48. The following integral equation

$$x(t) = \sin t + \lambda \int_0^{2\pi} \sin(t+s)\, x(s)\, ds$$

is :

(A) Fredholm integral equation of second kind

(B) Fredholm integral equation of first kind

(C) Volterra integral equation of second kind

(D) Volterra integral equation of first kind

49. The nontrivial solution of the integral equation

$$x(t) = \lambda \int_0^1 e^{t+s}\, x(s)\, ds$$

is :

(A) $t$

(B) $t^2$

(C) $e^{t^2}$

(D) $e^t$

50. The Volterra integral equation

$$x(t) = t + \int_0^t (s-t)\, x(s)\, ds$$

is equivalent to the initial value problem :

(A) $x''(t) + x(t) = 0$, $x(0) = 0$, $x'(0) = 1$

(B) $x''(t) - x(t) = 0$, $x(0) = 1$, $x'(0) = 0$

(C) $x''(t) + x'(t) = 0$, $x(0) = 1$, $x'(0) = 0$

(D) $x''(t) - x'(t) = 0$, $x(0) = 0$, $x'(0) = 1$

51. For a homogeneous Fredholm integral equation with separable kernel :

$$x(t) = \lambda \int_a^b k(t, s)\, x(s)\, ds,$$

consider the following statements :

(I) If Fredholm determinant $D(\lambda) \neq 0$, then integral equation has only trivial solution i.e. $x(t) = 0$.

(II) If Fredholm determinant $D(\lambda) = 0$, then integral equation has infinitely many solutions.

Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

52. The Neumann series solution of the Volterra integral equation

$$x(t) = 1 + t + \int_0^t (t - s)\, x(s)\, ds$$

is :

(A) $x(t) = \log t$

(B) $x(t) = \cos t$

(C) $x(t) = \sin t$

(D) $x(t) = e^t$

53. Eigen values of the homogeneous Fredholm integral equation

$$x(t) = \lambda \int_0^{2\pi} \sin(t + s)\, x(s)\, ds$$

are :

(A) 1, –1

(B) $\pi, -\pi$

(C) 1, $\pi$

(D) $1/\pi, -1/\pi$

54. A particle is thrown horizontally from the top of a building of height $h$ with initial velocity $u$. Neglecting all other forces except the gravity and the air resistance which is proportional to the velocity of the particle, then the equations of motion are given by :

(A) $\ddot{x} = 0, \; \ddot{y} + g = 0$

(B) $m\ddot{x} - k\dot{x} = 0, \; m\ddot{y} - k\dot{y} - mg = 0$

(C) $m\ddot{x} + k\dot{x} + mg = 0, \; m\ddot{y} + k\dot{y} = 0$

(D) $m\ddot{x} + k\dot{x} = 0, \; m\ddot{y} + k\dot{y} + mg = 0$

55. The Lagrangian of a particle of mass $m$ moving on the $xy$-plane which is rotating about $z$-axis with angular velocity $\omega$ is given by :

$$L = \frac{1}{2} m[(\dot{x} - \omega y)^2 + (\dot{y} + \omega x)^2] - V(x, y)$$

Then :

(A) Linear momenta $p_x$ and $p_y$ are conserved

(B) Hamiltonian H is conserved

(C) The total energy E is conserved

(D) Hamiltonian H represents the total energy

56. Let L be a Lagrangian of a particle and $p_j$ and $\dot{q}_j$ are respectively the conjugate momentum and generalised velocity. Then which one of the following represents the Lagrange's equation of motion ?

(A) $p_j = \frac{\partial L}{\partial \dot{q}_j}$

(B) $\dot{q}_j = \frac{\partial H}{\partial q_j}$

(C) $\dot{p}_j = \frac{\partial L}{\partial q_j}$

(D) $\dot{q}_j = -\frac{\partial L}{\partial p_j}$

57. Let $p_j$ and $p_j^{'}$ are the components of the canonical momenta corresponding to the Lagrangians L and L' respectively, where

$$L' = L + \frac{dF}{dt} \text{ and } F = F(q_j, 1).$$

If H and H' are the Hamiltonians corresponding to L and L', then :

(A) H' = H

(B) $H' = H + \frac{\partial F}{\partial t}$

(C) $H' = H + \frac{\partial F}{\partial q_j}$

(D) $H' = H - \frac{\partial F}{\partial t}$

58. A particle of mass $m$ is attached to one end of the string and the other end at a distance $r$ is fixed in space. If the particle starts with velocity $u$ from its lowest position, then the velocity of the particle at any angular distance $\theta$ is given by :

(A) $v^2 = u^2 + 2gr(1 - \sin\theta)$

(B) $v^2 = u^2 - 2gr(1 - \cos\theta)$

(C) $v^2 = u^2 + 2gr(1 + \cos\theta)$

(D) $v^2 = (r\dot{\theta})^2$

59. The first integral of equation of motion of the simple pendulum represents :

(A) Only the total energy E

(B) Only the Hamiltonian H

(C) Both the Hamiltonian H and the total energy E

(D) Neither the Hamiltonian H nor the total energy E

60. If $A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is the matrix of orthogonal transformation of a rigid body with one fixed, then $A^{-1}$ is given by :

(A) $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$

61. The function $f(z) = \frac{1}{z}$ has primitive in the domain :

(A) $\{z \in \mathbf{C} \mid 1 < |z| < 2\}$

(B) $\{z \in \mathbf{C} \mid |z| < 2\}$

(C) $\{z \in \mathbf{C} \mid 1 < |z|\}$

(D) $\{z = x + iy \in \mathbf{C} \mid x \geq 2\}$

62. Which of the following is false ?

(A) $\log z = \ln|z| + i \arg z, \quad z \neq 0$ and $-\pi < \arg z \leq \pi$

(B) $\cos^2 z = 1 - \sin^2 z$ for all $z \in \mathbf{C}$

(C) $\cosh^2 z - \sinh^2 z = 1$ for all $z \in \mathbf{C}$

(D) $\log(z_1 z_2) = \log z_1 + \log z_2$ for all $z_1, z_2 \in \mathbf{C} \setminus \{0\}$

63. Suppose $z^{10} = 1$ and $z \neq 1$. Which of the following statements is false ?

(A) $1 + z + z^2 + ...... + z^9 = 0$

(B) $1 + z^2 + z^4 + ...... + z^{18} = 0$

(C) $1 . z . z^2 . z^3 . ..... . z^9 = 1$

(D) $1 . z^2 . z^4 . z^6 . ..... . z^{18} = 1$

64. Consider the following two statements :

($i$) Every harmonic function $u$ on $\mathbf{C}$ has a harmonic conjugate.

($ii$) If $u$ and $v$ are harmonic conjugates of each other on a disk D, then $u$ and $v$ are constant on D.

Then :

(A) Only ($i$) is true

(B) Only ($ii$) is true

(C) Both ($i$) and ($ii$) are true

(D) Both ($i$) and ($ii$) are false

65. Which of the following Möbius transformations maps the unit disk onto itself ?

(A) $f(z) = \dfrac{2z - i}{2 + zi}$

(B) $f(z) = \dfrac{i - z}{i + z}$

(C) $f(z) = \dfrac{2z - 1}{2 + z}$

(D) $f(z) = \dfrac{iz + 1}{z + i}$

66. Let $f$ be analytic on $D = \{z \in \mathbf{C} / |z| < 3\}$ such that :

$$f\left(1 + \frac{1}{n}\right) = \frac{2n+1}{n+1}, \quad n = 1, 2, \ldots\ldots$$

Then $f(z)$ =

(A) $\dfrac{2z+1}{z}$

(B) $\dfrac{z}{z-1}$

(C) $\dfrac{z+1}{z}$

(D) $\dfrac{z}{z+1}$

67. Which of the following statements is false for an entire function $f$ ?

(A) If $f(x) = 0 \ \forall \ x \in [-1, 1]$, then $f \equiv 0$

(B) If A is an uncountable set in $\mathbf{C}$ and $f(z) = 0 \ \forall \ z \in A$, then $f \equiv 0$

(C) If $\{a_n\}$ is a sequence of distinct complex numbers and $f(a_n) = 0$ for all $n$, then $f \equiv 0$

(D) $f(1/n) = 0 \ \forall \ n \geq 1$, then $f \equiv 0$

68. Suppose $f(z) = u + iv$ is an entire function. Which of the following is true ?

(A) If $u \geq 0$, then $f$ is unbounded on $\mathbf{C}$

(B) If $|f(z)| \leq |z|$ for all $z$, then $f$ is constant

(C) If $u$ is bounded, then $v$ is bounded

(D) If $u \equiv 0$, then $v \equiv 0$

69. Suppose $f : \mathbf{C} \to \mathbf{C}$ is an analytic one-one function. Then which of the following statements is false ?

(A) $f$ is a polynomial of degree one

(B) $f$ is an open map

(C) $f$ is an onto map

(D) $f'(z) = 0$ for some $z \in \mathbf{C}$

70. Let $f(z) = e^{\frac{1}{z}}$ and $g(z) = e^z$ and

$$A = \left\{ z \,/\, 0 < |z| < \frac{1}{2} \right\}.$$

Then, on A :

(A) $f$ is one-one but $g$ is not one-one

(B) $f$ is not one-one but $g$ is one-one

(C) Both $f$ and $g$ are one-one

(D) Neither $f$ nor $g$ is one-one

71. If all roots of the polynomial $1 + z + z^2 + ..... + z^{10}$ are in the disk B(0, R), then :

$$\int_{|z|=R} \frac{1 + 2z + 3z^2 + ..... + 10z^9}{1 + z + z^2 + ..... + z^{10}} dz =$$

(A) 0

(B) 10

(C) $10 \pi i$

(D) $20 \pi i$

72. The value of the integral $\int_{\gamma} \frac{z^3}{(z-3)^3} dz$, where $\gamma(t) = 2e^{it}$, $t \in [0, 2\pi]$ is :

(A) $18 \pi$

(B) $18 \pi i$

(C) $6 \pi i$

(D) $-2 \pi i$

73. The residue of $f(z) = \frac{e^z - 1}{z^2}$ at $z = 0$ is :

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

74. The image the circle

$$c = \{z \in \mathbf{C} : |z-1| = 1\}$$

under the map $f(z) = \frac{1}{z}$ is :

(A) a circle

(B) a line

(C) a parabola

(D) an ellipse

75. Which of the following polynomial is reducible over the rationals ?

(A) $x^{11} - 11x - 11$

(B) $x^{10} - 7$

(C) $x^5 - 5$

(D) $x^9 + 5x^5 + x^4 + 5$

76. Which is correct ?

(A) The polynomial ring K[$x$] over a field K is local

(B) The additive group of rationals has a maximal subgroup

(C) Any ring (may not have unity) has a maximal ideal

(D) Any finitely generated ideal of a Boolean ring is principal

77. Consider the following statements :

(I) A p-sylow subgroup of the underlying additive group of a finite commutative ring R with unity is an ideal of R.

(II) If M is a maximal ideal of a commutative ring R and R* be the group of units of R. Such that R is a disjoint union of M and R*, then R has a maximal ideal other than M.

Which is true ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

78. Consider the following statements :

(I) 3 is prime in the integral domain $\mathbf{Z}[i\sqrt{5}]$

(II) $\overline{2}$ is a prime element in $\mathbf{Z}/10\,\mathbf{Z}$ but not irreducible in $\mathbf{Z}/10\,\mathbf{Z}$

Which is correct ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

79. Which of the following is false ?

(A) In a commutative ring with unity, every prime ideal is maximal

(B) In a commutative ring with unity, every maximal ideal is prime

(C) In a Boolean ring with unity, every prime ideal is maximal

(D) In a Boolean ring with unity, every maximal ideal is prime

80. Let G be a finite abelian group. If $n \in \mathbf{N}$ and $|G|$ are relatively prime, then the function $Q : G \to G$ defined by $Q(a) = a^n$ is :

(A) homomorphism but not $1-1$

(B) homomorphism but not onto

(C) isomorphism

(D) not a homomorphism

81. Let G be a group and a map $Q : G \to G$ defined by $Q(a) = a^{-1}$. Then which of the following is false ?

(A) Q is bijective

(B) Q is a homomorphism if G is abelian

(C) Q is a homomorphism only if G is abelian

(D) Q is a homomorphism, if G is any permutation group

82. Consider the following statements :

(I) If G is a finite group that has only 2 conjugate classes, then $|G| = 2$.

(II) If G is a group and $a \in Z(G)$, the center of G, then the conjugacy class of $a$ is $\{e, a\}$, where $e$ is the identity of G

Which of the following is true ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

83. Consider the following statements :

(I) If $n \geq 5$, then all cycles of length 3 are conjugates in $A_n$.

(II) Two elements of $A_n$ that are conjugates in $S_n$ are always conjugates in $A_n$ also.

Which is *correct* ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

84. Consider the following statements :

(I) Every finite group is isomorphic to a subgroup of the alternating group $A_n$ for some $n > 1$.

(II) The symmetric group $S_n$ is nilpotent for $n \geq 3$.

Which is correct ?

(A) Only (I) is correct

(B) Only (II) is correct

(C) Both (I) and (II) are correct

(D) Neither (I) nor (II) is true

85. The number of words of three distinct letters formed from the letters of the word "PNTU" is :

(A) 24

(B) $2^4$

(C) $4^2$

(D) $4^6$

86. Suppose that for any group of $n$ people has the property that at least two of them have birthdays that occur on the same day of the week. Then which one of the following must be the value of $n$ ?

(A) 10

(B) 7

(C) 6

(D) 5

87. Let $\mathbf{F}_q$ be a finite field with $q$ odd and $\alpha = \sum_{a \in \mathbf{F}_q} a$ and $\beta = \prod_{a \in \mathbf{F}_q \setminus \{0\}} a$.

Then :

(A) $\alpha = 0$ and $\beta = -1$

(B) $\alpha = 1$ and $\beta = -1$

(C) $\alpha = 0$ and $\beta = 1$

(D) $\alpha = 1$ and $\beta = 1$

88. If F is a finite field of characteristic $p$, a prime, then for some integer $n \geq 1$ :

(A) $F \approx \mathbf{Z}_p n$

(B) $F \approx \mathbf{F}_p \times \mathbf{F}_p \times ..... \times \mathbf{F}_p$ ($n$ times) where $\mathbf{F}_p$ is the field of $p$ elements

(C) F is the splitting field of $x^{p^n} - x \in \mathbf{F}_p[x]$

(D) F is the splitting field of $x^{p^n} - 1 \in \mathbf{F}_p[x]$

89. The closed unit ball centered at origin is compact in :

(A) Finite dimensional normed linear space

(B) $(C[0, 1], \| \|_\infty)$

(C) $(L^1[0, 1], \| \|_1)$

(D) $(L^2[0, 1], \| \|_2)$

90. Let $S^1 = \{(x, y) \in \mathbf{R}^2 / x^2 + y^2 = 1\}$ be the unit circle and

$$D = \{(x, y) \in \mathbf{R}^2 / |x| \leq 1, |y| \leq 1\}$$

Then :

(A) $S^1$ is homeomorphic to $\mathbf{R}$

(B) $S^1$ is not homeomorphic to D

(C) $S^1$ is homeomorphic to an interval in $\mathbf{R}$

(D) D is homeomorphic to an interval in $\mathbf{R}$

91. Let $X = A \cup B$ where A and B are connected sets such that $A \cap B = \phi$. Let $f : X \to f(x) \subset \mathbf{R}$ be a homeomorphism. Then :

(A) $f$(X) is always a finite subset of $\mathbf{R}$

(B) $f$(X) is a singleton set

(C) $f$(X) is an interval in $\mathbf{R}$

(D) $f$(X) is not connected

92. Which of the following sets are homeomorphic ?

(A) Square and Parabola

(B) Square and Ellipse

(C) Ellipse and Parabola

(D) Parabola and Hyperbola

93. A topological space X is regular if :

(A) Singleton sets are closed sets

(B) For each pair $x \in X$ and an open set B in X with $x \notin B$ there exists an open set U such that $x \in U$ and $u \cap B = \phi$

(C) For each pair $x \in X$ and a closed set B in X with $x \notin B$ there exists an open set U such that $x \in U$ and $u \cap B = \phi$

(D) For each pair $x \in X$ and a closed set B in X with $x \notin B$ there exist an open sets U, V such that $x \in U$, $B \subset V$ and $U \cap V = \phi$

94. Which of the following is true ?

(A) A metric space is always compact

(B) A metric space is always Hausdorff

(C) A metric space is always connected

(D) A metric space is always complete

95. Let A and B be connected sets. Then :

(A) $A \cap B$ is connected if $A \cap B \neq \phi$

(B) $A \cup B$ is connected if $A \cap B \neq \phi$

(C) $A \setminus B$ is connected if $B \subset A$

(D) $A \setminus B$ is connected if $B \not\subset A$

96. Let C[0, 1] denote the space of continuous functions on [0, 1] with sup norm $\| \|_\infty$ and G denotes the set of all constant functions on [0, 1]. Then :

(A) G is bounded subset of C[0, 1]

(B) G is dense in C[0, 1]

(C) G is closed subset of C[0, 1]

(D) G is compact subset of C[0, 1]

97. Let X be a topological space with a subspace Y. Which of the following statements is false ?

(A) A set F is closed in Y iff $F = F_1 \cap Y$ where $F_1$ is closed in X

(B) A set B is open in Y iff $B = B_1 \cap Y$ where $B_1$ is open in X

(C) A set C is connected in Y iff $C = C_1 \cap Y$ where $C_1$ is connected in X

(D) A set K is compact in Y iff $K = K_1 \cap Y$ where $K_1$ is compact in X

98. Consider the set :

$$A = \{(x, y) / xy = 0\}$$

Which of the following is false ?

(A) A is closed in $\mathbf{R}^2$

(B) A is path connected

(C) A is connected

(D) A is not connected

99. Let $A = \{(x, y) / y_2 \le x^2 + y^2 \le 1\}$ and $f : A \to \mathbf{R}$ be a continuous map. Which of the following is false ?

(A) $f(A)$ is an interval

(B) $f(A)$ is a bounded set

(C) $f(A)$ is a closed set

(D) $f(A)$ is a union of two disjoint intervals

100. Let $X = [0, 1) \times [0, 1)$, $Y = [0, 1] \times (0, 1)$ and $Z = [0, 1) \times (0, 1]$ be subspaces of the Euclidean space $\mathbf{R}^2$. Then :

(A) X is homeomorphic to both Y and Z

(B) X is homeomorphic to Y but not homeomorphic to Z

(C) X is homeomorphic to Z but not homeomorphic to Y

(D) X is not homeomorphic to Y and not homeomorphic to Z

## SECTION III

101. In the multiple linear regression setup under the assumption that the random errors $\varepsilon_1, \varepsilon_2, \ldots\ldots. \varepsilon_n$ are uncorrelated and homoscedastic, the residual variables $e_1, e_2, \ldots\ldots. e_n$ are :

(A) Uncorrelated and homoscedastic

(B) Correlated and homoscedastic

(C) Uncorrelated and heteroscedastic

(D) Correlated and heteroscedastic

102. In a multiple linear regression model $y = X\beta + \varepsilon$, the covariance between $y$ and $\hat{y}$, the predicted value of $y$ using least square estimator of $\beta$ is :

(A) $\sum X(X'X)^{-1}X'$

(B) $\sum (I - X(X'X)^{-1}X')$

(C) $\sum$

(D) None of the above

103. If $e_1, e_2, \ldots\ldots, e_n$ are the residuals obtained on fitting a simple linear regression $y = \beta_0 + \beta_1 X + \varepsilon$ to $(Y_i, X_i)$ $i = 1 \ldots n$ under the standard assumption, then :

(A) $\sum_{i=1}^{n} X_i e_i$ is always positive

(B) $\sum_{i=1}^{n} X_i e_i$ is always negative

(C) $\sum_{i=1}^{n} X_i e_i$ is always zero

(D) Nothing can be said about $\sum_{i=1}^{n} X_i e_i$

104. Consider a logistic regression model consisting of only one explanatory variable X which takes values in $\{-1, 1\}$. Then the odds ratio on changing X from –1 to 1 is :

(A) $\exp(2\beta)$

(B) $\exp(\beta)$

(C) $\exp(1 + \beta)$

(D) $\exp(1 - \beta)$

105. Suppose

$$\underline{X}_{3\times 1} \sim \text{MNormal}_3\left(\underline{0}, \begin{bmatrix} 2 & 0.5 & 2 \\ 0.5 & 1 & 0 \\ 2 & 0 & 4 \end{bmatrix}\right).$$

Then the correlation coefficient between $X_1 - X_3$ and $2X_2$ is :

(A) $\frac{1}{2\sqrt{2}}$

(B) 0

(C) $\frac{1}{\sqrt{2}}$

(D) 1

106. Suppose $\begin{bmatrix} X \\ Y \end{bmatrix} \sim$ Bivariate Normal $(\underline{\mu}_{2\times 1}, \Sigma_{2m})$. Then :

(A) $E(Y \mid X = x) = x\,E(Y)$

(B) $\text{Var}(Y \mid X = x) = x^2\,\text{Var}(Y)$

(C) $E(Y \mid X = x) = E(Y)$ + *a* linear function of $x$

(D) $\text{Var}(Y \mid X = x) = \text{Var}(Y)$ + *a* quadrate function of $x$

107. Suppose $\underset{\sim}{X}_{3\times1} \sim \text{MNormal}_3\ (\underline{\mu}, \Sigma_{3+3})$ and let $Y = \underline{X}'A\underline{X}$ where the matrix A has all eigenvalue equal to 1. Hence the probability distribution of Y

(A) cannot be determined based on the given information

(B) is normal $(\underline{\mu}'A\underline{\mu}, \Sigma'A\varepsilon)$

(C) is $\chi^2$ with 1 degree of freedom

(D) is $\chi^2$ with 3 degrees of freedom

108. Corresponding to the three variables $X_1$, $X_2$, $X_3$ the quadrate form $X_1^2 + 2X_2^2 + 4X_3^2 - X_1X_2 + X_1X_3$ is :

(A) Positive definite

(B) Positive semidefinite

(C) Negative definite

(D) Negative semidefinite

109. Suppose $\underset{\sim}{X}_{2\times1} \sim \text{Normal}_2(\underline{\mu}, \varepsilon)$ and $Y = 0.5X_1^2 + 0.5X_2^2 + X_1X_2$. Then the probability distribution of Y is :

(A) $\chi^2$ with 2 degrees of freedom

(B) $\chi^2$ with 1 degree of freedom

(C) F with 1, 2 degrees of freedom

(D) *t* with 1 degree of freedom

110. For the three random variables $X_1$, $X_2$, $X_3$ let $\hat{X}_1$ and $\hat{X}_2$ be the fitted values of $X_1$ and $X_2$ respectively after regressing on the linear function of $X_3$. The partial correlation coefficient between $X_1$ and $X_2$ after removing the effect of $X_3$ is the simple correlation coefficient between :

(A) $X_1$ and $\hat{X}_2$

(B) $\hat{X}_1$ and $X_2$

(C) $(X_1 - X_3)$ and $(X_2 - X_3)$

(D) $(X_1 - \hat{X}_1)$ and $(X_2 - \hat{X}_2)$

111. The Euclidean distance between the point $\underline{X}_{3=1} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$ and a cluster center $\bar{X}_{3=1} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ is equal to :

(A) 5

(B) 4

(C) 2

(D) 0

112. Principal component analysis is used for :

(A) reducing the number of observations in the data

(B) reducing the variability present in the data

(C) getting information on variable as a function of other variables

(D) reducing the number of variables in the data

113. Suppose $S_Y^2 = \frac{1}{n-1} \sum_{r=1}^{n} (Y_r - \bar{Y})^2$

$$s_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \bar{y})^2$$

$\bar{Y}$ is sample mean under SRSWR

$\bar{y}$ is population mean

$\bar{Y}_{sy}$ is sample mean under systematic sampling

Which of the following statements is *not* true ?

(A) Probability of selecting $i$th element of the population on the $r$th draw is $\frac{1}{N}$ under SRSWOR design for $i$ = 1, 2, 3, ......, N and $r$ = 1, 2, 3, ......., $n$

(B) $ES_Y^2 = s_y^2$ under SRSWOR

(C) $E(\bar{Y}_{sy}) = \bar{y}$ under systematic sampling

(D) $V(\bar{Y}) = \frac{N-1}{n} s_y^2$ under SRSWR

114. Consider a population of 50 units, {1, 2, 3, ....., 50} and suppose that 50 possible samples are listed as {1}, {1, 2}, {1, 2, 3}, ....., {1, 2, 3, ....., 50}. One of these samples is choosen at random. Let $\pi_i$ be the probability that unit $i$ of the population is in the selected sample. Then which of the following is necessarily true ?

(A) The expected sample size is 25.5

(B) $\sum_{i=1}^{50} \pi_i = 1$

(C) $\sum_{i=1}^{50} \pi_i = 25$

(D) The expected sample size is 25

115. Consider the following hypothetical population of 6 units. What is the estimated value of population total of $y$-variable based on SRSWOR sample of 2 units selected as unit 4 and unit 6 from the population using ratio method of estimation :

| Unit No. | $x$ | $y$ |
|---|---|---|
| 1 | 0 | 1 |
| 2 | 1 | 3 |
| 3 | 3 | 11 |
| 4 | 6 | 18 |
| 5 | 8 | 29 |
| 6 | 10 | 46 |

(A) 27

(B) 112

(C) 198

(D) 432

116. In stratified random sampling with a linear cost function

$$C = C_0 + \sum_{h=1}^{L} C_h n_h,$$

the variance of estimated mean $(\bar{Y}_{st})$ is minimum for a specified cost C and the cost is minimum for a specified variance of $\bar{Y}_{st}$ when :

(A) $n_h \propto \left\{\frac{w_h s_h}{\sqrt{C_h}}\right\}$

(B) $n_h \propto \left\{\frac{\sum_{h=1}^{L} w_h s_h}{C_h}\right\}$

(C) $n_h \propto \left\{\frac{N_h s_h}{s_h}\right\}$

(D) $n_h \propto \left\{\frac{n\, s_h}{\sqrt{C_h}}\right\}$

117. Under systematic sampling, if $s^2_{wsy}$ denotes the variance among the units belonging to the systematic sample then the variance of the sample mean $\bar{Y}_{sy}$ is :

(A) $[(N-1)s^2 - K(n-1)s^2_{wsy}]$

(B) $[(N-1)s^2 - K(n-1)s^2_{wsy}]$

(C) $\frac{1}{N^2}[(N-1)s^2 - K(n-1)s^2_{wsy}]$

(D) $\frac{1}{N}[(N-1)s^2 - K(n-1)s^2_{wsy}]$

118. In a BIBD with 7 treatments and block size 5, what will be the number of replications required ?

(A) 56

(B) 21

(C) 8

(D) 3

119. In a completely randomized design with unequal group numbers, that is, $n_1 = 5$, $n_2 = 7$, $n_3 = 6$, what is the degree of freedom for the error term ?

(A) 17

(B) 15

(C) 120

(D) 18

120. Suppose the analysis of variance calculations have been performed for a problem where there is a single factor with two levels. This would produce results equivalent to :

(A) a $t$-test with alternate hypothesis greater than

(B) a F-test with alternate hypothesis less than

(C) a $\chi^2$-test with alternate hypothesis not equal to

(D) a $t$-test with alternate hypothesis not equal to

121. The 'C'-matrix of a block design having $r$ treatments is :

(A) Positive definite

(B) Positive semidefinite

(C) Asymmetric

(D) Having rank $r$

122. In a completely randomized design suppose $\bar{y}_{i}.$ is computed from $n_i$ observations ($n_i$ not all equal with $\sum_{i=1}^{k} n_i = N$), then what will be $\text{Var}(\bar{y}_{i}. - \bar{y}..)$ ?

(A) $\sigma^2 / Nn_i$

(B) $\sigma^2 \left( \frac{N - n_i}{Nn_i} \right)$

(C) $\sigma^2 \frac{n_i}{(N - 1)}$

(D) $\frac{\sigma^2 (n_i - 1)}{Nn_i}$

123. Suppose the lifetimes of two systems, system 1 and system 2, follow exponential distributions with failure rates 2 and ½ respectively. Let the probability that system '*i*' fails prior to its mean time to failure be denoted by $p_i$, $i = 1, 2$. Which of the following is true ?

(A) $p_1 > p_2$

(B) $p_1 = p_2$

(C) $p_1 < p_2$

(D) $p_1 . p_2 = 1$

124. A system consists of 4 components, 3 of which form a parallel subsystem connected in series with the 4th component. The components work independently of each other. If the reliability of each component is 0.8, then the system reliability is :

(A) 0.8381

(B) 0.7132

(C) 0.4096

(D) 0.7936

125. Suppose only the first 6 failure times of 18 items kept on test are observed. Suppose the failure times are independent and follow an exponential distribution with mean θ. If the data are :

1.5, 2, 2.5, 3, 4, 5

the maximum likelihood estimate of θ :

(A) 9

(B) cannot be obtained

(C) is 3

(D) is 13

126. The final tableau of the simplex method of the LPP

$$\text{Max : } Z = 3x_1 + 2x_2$$

is given below :

| | | $x_1$ | $x_2$ | $x_3$ | $x_4$ | |
|---|---|---|---|---|---|---|
| | | 3 | 2 | 0 | 0 | |
| $x_1$ | 3 | 1 | 0.75 | 0.25 | 0 | 30 |
| $x_4$ | 0 | 0 | 2.25 | –0.25 | 0 | 60 |
| $z_j - c_j$ : | | 0 | 0.25 | 0.75 | 0 | 90 |

If the coefficient of $x_1$ in the objective function is changed from 3 to 1, then which of the following statements is/are true ?

(I) $x_2$ enters as a new basic variable in place of $x_4$.

(II) The optimal value decreases by 33.33.

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

127. In graphical solution method of LPP, the redundant constraint is the one :

(A) Which forms the boundary of the feasible region

(B) Which does not form the boundary of the feasible region

(C) Which does not optimize the objective function

(D) Which optimizes the objective function

128. Four workers are required to be assigned to 4 tasks. The number of hours required to complete the tasks by each of the workers is given in the following table :

| | Workers | | | |
|---|---|---|---|---|
| **Tasks** | 20 | 30 | 40 | 50 |
| | 40 | 50 | 60 | 70 |
| | 70 | 80 | 90 | 80 |
| | 30 | 50 | 80 | 40 |

Then, which of the following is true ?

(A) The minimum time required to complete all the tasks is 200 hours

(B) The minimum time required to complete all the tasks is 210 hours

(C) The basic feasible solution is non-optimum

(D) The basic feasible solution is non-degenerarate

129. In inventory control, which costs can vary with order quantity ?

(A) unit cost only

(B) reorder cost only

(C) holding cost only

(D) all of the above

130. Customer arrivals at a service facility follows a Poisson distribution with mean 10 per hour and the service times have an exponential distribution with average service rate of 4 minutes per customer. Then, which of the following statements is false ?

(A) The probability of two or more customers waiting in the queue is 4/9

(B) The average length of the queue is 4/3 customers

(C) The average waiting time of a customer in the queue is 8 minutes

(D) The probability that the server is idle is 2/3

131. Which of the following is not true for an exponential distribution ?

(A) It satisfies the lack of memory property

(B) The hazard rate is constant

(C) The distribution of $n$th order statistics is exponential

(D) The distribution of the first order statistics is exponential

132. Consider the following two statements :

(1) The correlation coefficient between two variables X and Y is the geometric mean of two regression coefficients $\beta_{YX}$ and $\beta_{XY}$.

(2) The arithmetic mean of the two regression coefficients $\beta_{YX}$ and $\beta_{XY}$ is greater than or equal to the correlation coefficient between X and Y.

Which of the above statement(s) is/are true ?

(A) Only (1)

(B) Only (2)

(C) Both (1) and (2)

(D) None of (1) and (2)

133. The ogive more than and ogive less than of the following grouped data

| Class | Frequency |
|---|---|
| 0—8 | 8 |
| 8—20 | 12 |
| 20—30 | 18 |
| 30—35 | 10 |
| 35—40 | 4 |

intersect at which of the following points ?

(A) 23.00

(B) 23.33

(C) 23.67

(D) 24.00

134. Suppose a class contains 44 students. On March 10, 25 students were present in the class and on March 16, 30 students were present in the class. Then the minimum number of students who were present in the class on March 15 as well as March 16 is equal to :

(A) 5

(B) 9

(C) 11

(D) 14

135. Which of the following is the smallest field containing a non-empty subset A of $\Omega$ ?

(A) Trivial field

(B) $\{A, A^C, \phi, \Omega\}$

(C) Power set of $\Omega$

(D) None of the above

136. Which of the following statements cannot be always true ?

(A) Let $X : \Omega \to \Omega^*$ and if $\tau_j$ be the $\sigma$-field of subsets of $\Omega^*$, then $X^{-1}(\tau_j)$ is also $\sigma$-field

(B) $\sigma\{X^{-1}(\tau) = X^{-1}\{\sigma(\tau)\}$ where, $\tau$ denote the class of subsets of $\Omega^*$ and $X : \Omega \to \Omega^*$

(C) If $B \cap C = \phi$, then $X(B) \cap X(C) = \phi$ where $X(A) = \{X(w) \mid w \in A\}$

(D) $X \geq 0$ and $E(X) = 0$, then $X = 0$ a.s.

137. Let $\Omega = \{1, 2, 3, ...., 10\}$, $A = \{1, 2\}$ and $\mathbf{F} = \{\phi, A, A^C, \Omega\}$, consider the following functions :

$$X(w) = 100 \;\; \forall w \in \Omega$$

$$Y(w) = \begin{cases} 100 & \text{if} \quad w \in A \\ 400 & \text{if} \quad w \notin A \end{cases}$$

$$Z(w) = w \;\; \forall w \in \Omega$$

Then which of the following is true ?

(A) Only X and Y are **F**-measurable r.v.s

(B) All the functions X, Y and Z are **F**-measurable r.v.s.

(C) Only X is a **F**-measurable r.v.

(D) All the functions X, Y, Z are not **F**-measurable

138. Suppose that probabilities are assigned to the subsets of $\Omega = \{a, b, c, d, e, f\}$ according to the classical definition of probability. Which of the following statements is/are true for the sets $A = \{a, b\}$, $B = \{b, d\}$ and $C = \{a, c, d\}$ ?

(I) C and A are independent

(II) C and B are independent

(III) C and $A \cup B$ are independent

(A) Only (I) and (II) are true

(B) Only (I) and (III) are true

(C) Only (II) and (III) are true

(D) All the three are false

139. The joint probability mass function $P_{X,Y}$ of the random variables X and Y is as given in the following table :

| $P_{X,Y}(x, y)$ | | Y | | | | $P_X(x)$ |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | |
| X | 1 | 0.02 | 0.04 | 0.06 | 0.08 | 0.20 |
| | 2 | 0.03 | 0.06 | 0.09 | 0.12 | 0.30 |
| | 3 | 0.05 | 0.10 | 0.15 | 0.20 | 0.50 |
| | $P_Y(y)$ | 0.10 | 0.20 | 0.30 | 0.40 | |

Then, which of the following statements is true ?

(A) $E(Y) < E(X)$

(B) $P(X > 2, Y > 2) = 1 - P(X \leq 2, Y \leq 3)$

(C) X and Y are uncorrelated

(D) X and Y are positively correlated

140. In order to detect an error, a computer program is being tested independently by three different testers. The probabilities with which the testers can detect the error are 0.2, 0.4 and 0.5 respectively. Then, which of the following statements is false ?

(A) The probability that the error will not be detected is 0.24

(B) The probability that exactly one tester will find the error is 0.46

(C) The probability that at least two testers will find the error is 0.30

(D) All the above are false

141. If E and F are events with $P(E) < P(F)$ and $P(E \cap F) > 0$, then :

(A) Occurrence of E $\Rightarrow$ occurrence of F

(B) Occurrence of F $\Rightarrow$ occurrence of E

(C) Non-occurrence of E $\Rightarrow$ non-occurrence of F

(D) None of the above implications hold

142. Suppose X is an arbitrary random variable. Which of the following statements is true ?

(A) $P[|X| \le 3] \le \frac{E(X^4)}{81}$

(B) $P[|X| \le 3] \ge \frac{E(X^4)}{81}$

(C) $P[|X| \ge 3] \le \frac{E(X^4)}{81}$

(D) $P[|X| \ge 3] \ge \frac{E(X^4)}{81}$

143. Suppose $\phi(t) = \frac{1}{(1+t^2)}$ is a characteristic function of a random variable X. Then E(X) and Var(X) are respectively given by :

(A) $\frac{1}{2}, \frac{1}{8}$

(B) 0, 2

(C) 0, 1

(D) $\frac{1}{2}, \frac{1}{16}$

144. Suppose the probability space $(\Omega, A, P)$ is defined as follows : $\Omega = [0, 1]$, A is a sigma field of subsets of $\Omega$ and P is a Lebesgue measure. Suppose a sequence $\{X_n, n \geq 1\}$ of random variables is defined as $X_n(w) = e^w + w^n, \quad n \geq 1$ and a random variable X is defined as $X(w) = e^w$ on $(\Omega, A, P)$. Which of the following is not true as $n \to \infty$ ?

(A) $X_n \to X$ almost surely

(B) $X_n \to X$ in probability

(C) $X_n \to X$ in distribution

(D) $X_n \to X$ pointwise

145. Suppose $\{X_n, n \geq 1\}$ is a sequence of random variables such that :

$$P[X_n = 0] = 1 - \frac{1}{n^3} \text{ and}$$

$$P[X_n = n] = \frac{1}{n^3}, \quad n \geq 1$$

Which of the following is not true ?

(A) $X_n$ converges to 0 in probability

(B) $X_n$ converges to 0 in quadratic mean

(C) $X_n$ does not converge to 0 almost surely

(D) $X_n$ converges to 0 in law

146. Suppose $\{X_n, n \geq 1\}$ is a sequence of random variables defined as $X_n = \left(1 + \frac{1}{n}\right) X$, where X is a discrete random variable with support {0, 1} and $P[X = 0] = \frac{2}{3}$. Following are three statements :

(I) $X_n \to 0$ in probability

(II) $X_n \to 0$ in law

(III) $X_n \to 0$ in quadratic mean

Which of the following options is true ?

(A) Only (I) and (II) are true

(B) Only (II) and (III) are true

(C) Only (III) is true

(D) All three are true

147. Let $\{X_n, n \geq 1\}$ and X be random variables defined on the same probability space. Which of the following statements is *not* always true as $n \to \infty$ ?

(A) If $X_n$ converges to X in probability and in distribution then $e^{X_n}$ converges to $e^X$ almost surely

(B) If $X_n$ converges to X almost surely then $e^{X_n}$ converges to $e^X$ in probability

(C) If $X_n$ converges to X in distribution then $e^{X_n}$ converges to $e^X$ in distribution

(D) If $X_n$ converges to X in probability then $e^{X_n}$ converges to $e^X$ in distribution

148. Let $\{X_n, n \geq 1\}$ be a sequence of independent identically distributed random variables with mean '0' and variance '2'. Let :

$$S_n = \sum_{i=}^{n} X_i^2 .$$

Which of the following statements is true ?

(A) $\frac{S_n}{n}$ converges to 0 almost surely as $n \to \infty$

(B) $\frac{S_n}{n}$ converges to 2 almost surely as $n \to \infty$

(C) $\frac{S_n}{\sqrt{n}}$ converges to 0 in probability as $n \to \infty$

(D) $\sqrt{n}\frac{S_n}{2}$ converges in distribution to a standard normal random variable as $n \to \infty$

149. Following are three statements. In a Markov chain, a persistent state $i$ is a null persistent if :

(I) $\lim\sup\ p_{ii}^{(n)} = 0$

(II) mean recurrence time is $\infty$

(III) $\lim\sup\ p_{ii}^{(n)} > 0$

Which of the following is true ?

(A) Only (I) is true

(B) Both (I) and (II) are true

(C) Only (III) is true

(D) Both (II) and (III) are true

150. Suppose $\{X_n, n \geq 0\}$ is a Markov chain with state space $S = \{1, 2, 3\}$ and the transition probability matrix P given by :

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.3 & \alpha & \beta \\ 0.4 & \gamma & 0.1 \\ \delta & 0.2 & \varepsilon \end{bmatrix} \end{array}$$

If P is a doubly stochastic matrix, the value of $\beta$ :

(A) Cannot be computed from the given information

(B) is < 0.4

(C) is > 0.4

(D) is 0.4

151. Suppose $\{X_n, n \geq 0\}$ is a Markov chain with state space $S = \{1, 2, 3\}$ and the transition probability matrix P given by :

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 0 \\ \frac{3}{8} & \frac{5}{8} & 0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix} \end{array}$$

Suppose $X_0 = 3$. Then realized values of $X_1$ and $X_2$ corresponding to random numbers 0.36 and 0.64 respectively, from U(0, 1) distribution are :

(A) $X_1 = 2$ and $X_2 = 2$

(B) $X_1 = 1$ and $X_2 = 1$

(C) $X_1 = 1$ and $X_2 = 2$

(D) $X_1 = 2$ and $X_2 = 1$

152. Suppose $\{X_n, n \geq 0\}$ is a Markov chain with state space $S = \{1, 2, 3, 4\}$ and the transition probability matrix P given by :

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} \end{bmatrix} \end{array}$$

Which of the following options is true ?

(A) States 1 and 3 are essential states

(B) States 1 and 3 are inessential states

(C) States 1 and 3 are transient states

(D) State 1 and 3 are null persistent states

153. Which of the following options is true ?

Messages arrive on a mobile phone according to the Poisson process with rate 5 messages per hour. The probability that no message arrives during 10 : 00 am to 12 : 00 noon is :

(A) $2e^{-5}$

(B) $e^{-10}$

(C) $1 - e^{-10}$

(D) $e^{-\frac{2}{5}}$

154. Suppose $\{X(t), t \geq 0\}$ is a pure birth process with birth rate $\lambda_i = \lambda$ for all $i \in S$ and $X(0) = 0$. The following are four statements.

(I) $\{X(t), t \geq 0\}$ is a process with stationary and independent increments.

(II) $E(X(t)) = e^{\lambda t}$

(III) $X(t)$ follows Poisson $P(\lambda t)$ distribution for every fixed $t$.

(IV) X(0) cannot be 0 in a pure birth process.

Which of the following is true ?

(A) Only (I) is true

(B) Only (I) and (II) are true

(C) Only (I) and (III) are true

(D) Only (IV) is true

155. Assume that $X \sim B(n, p)$ for some $n \geq 1$ and $0 < p < 1$ and $Y \sim$ Poisson $(\lambda)$ for some $\lambda > 0$. Suppose $E(X) = E(Y)$. Then :

(A) V(X) = V(Y)

(B) V(X) < V(Y)

(C) V(X) > V(Y)

(D) V(X) may be larger or smaller than V(Y)

156. Suppose X and Y are independently and uniformly distributed random variables on the interval (0, 1). Then the distribution of max (X, Y) is :

(A) Beta (1, 1)

(B) Beta (2, 2)

(C) Beta (1, 2)

(D) Beta (2, 1)

157. Suppose that X and Y are i.i.d. random variables with c.d.f. F which is binomial $(n, p)$. Then,

$$P(X = j \mid X + Y = k)$$

is equal to :

(A) $j/k$, for $j = 0, 1, 2, \ldots, k$

(B) $\binom{k}{j} p^j q^{k-j}$, for $j = 0, 1, 2, \ldots, k$

(C) $\binom{k}{j} \left(\frac{1}{2}\right)^k$, for $j = 0, 1, 2, \ldots, k$

(D) $\binom{n}{j} \binom{n}{k-j} / \binom{2n}{n}$, for $j = 0, 1, 2, \ldots, k$

158. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. random variables such that $X_1$ has uniform distribution over $(0, \theta)$, $\theta > 0$. Then, which of the following statements is true ?

(I) The conditional density $f_y(\cdot)$ of $X_{(1)} \mid X_{(n)} = y$ is given by :

$$f_y(x) = \begin{cases} (n-1).\frac{1}{y}\left(1 - \frac{x}{y}\right)^{n-2}, & \text{if } 0 < x < y \\ 0, & \text{otherwise} \end{cases}$$

(II) The conditional distribution of $X_{(1)} \mid X_{(n)} = y$ does not depend on $\theta$.

(A) Neither (I) nor (II) is true

(B) Both (I) and (II) are true

(C) Only (I) is true

(D) Only (II) is true

159. Suppose that F is a distribution function with mean support $[0, \infty)$ and mean $\mu$. Then, what is the value of the integral,

$$\int_0^\infty [F(x + a) - F(x)]\, dx \ ?$$

(A) $\mu$

(B) 0

(C) $a + \mu$

(D) $a$

160. A geometric distribution with support {1, 2, ....} has variance 6. Then, the height of the jump at 2 is equal to :

(A) cannot be determined

(B) 8/27

(C) 2/27

(D) 6/27

161. A random sample $X_1, X_2, ....., X_n$ is drawn from a Poisson distribution with parameter $\lambda$. Which one of the following is an unbiased estimator of $\lambda^2$ ?

(A) $\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^2$

(B) $\frac{1}{n}\sum_{i=1}^{n} X_i^2$

(C) $\frac{1}{n}\sum_{i=1}^{n} X_i(X_i - 1)$

(D) $\frac{1}{n}\sum_{i=1}^{n} X_i(X_i + 1)$

162. Cramer-Rao lower bound of variance for estimating the parameter $\theta$ of the distribution with pdf :

$$f(x, \theta) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2}$$

where $-\infty < x < \infty$ is :

(A) $\frac{2}{n}$

(B) $\frac{1}{n}$

(C) $\frac{2}{n^2}$

(D) $\frac{1}{2n}$

163. Based on a sample of size one from Binomial (6, P), it is decided to test the hypothesis $H_o : P = 1/2$ *Vs* $H_i : P = 1/4$ using the critical region $X \leq 3$. Then, the size of the critical region would be :

(A) 15/64

(B) 35/64

(C) 22/64

(D) 42/64

164. The following are some statements :

(I) If T is a boundedly complete sufficient statistic for θ and S is an ancillary statistic for θ, then T and S are independent.

(II) An ancillary statistic may or may not be independent of a sufficient statistic.

(III) It is possible to have variance smaller than that of the Cramer-Rao lower bound at some points of the parameter space.

(IV) The parametric function need not be differentiable for the Cramer-Rao lower bound to hold.

Which of the following options is true ?

(A) (I), (II) and (III) only

(B) (II), (III) and (IV) only

(C) (I), (III) and (IV) only

(D) (I), (II) and (IV) only

165. In the context of testing of statistical hypotheses, which one of the following statements is true ?

(A) When testing a simple hypothesis $H_0$ against an alternative simple hypothesis $H_1$, the likelihood ratio principle leads to the most powerful test

(B) When testing a simple hypothesis $H_0$ against an alternative simple hypothesis $H_1$

$P\{\text{rejecting } H_0 | H_0 \text{ is true}\} + P\{\text{accepting } H_0 | H_1 \text{ is true}\} = 1$

(C) For testing a simple hypothesis $H_0$ against an alternative simple hypothesis $H_1$, randomized test is used to achieve the desired level of the power of the test

(D) UMP tests for testing a simple hypothesis $H_0$, against an alternative composite $H_1$, always exist

166. Let $X_1, X_2, ......, X_n$ be independently and normally distributed with mean 0 and variance $\sigma^2$. What is the minimum variance unbiased estimator of $\sigma^2$ ?

(A) $\frac{1}{n-1} \sum_{i=1}^{n} X_i^2$

(B) $\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$

(C) $\frac{1}{n} \sum_{i=1}^{n} X_i^2$

(D) $\frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2$

167. A Statistics professor wants to see if more than 80% of her students enjoyed taking her class. At the end of the term, she takes a random sample of students from her large class and asks in an anonymous survey, if the students enjoyed taking her class.

Which of the following set of hypotheses should she set ?

(A) $H_0 : p = 0.80$ against $H_1 : p < 0.80$

(B) $H_0 : p > 0.80$ against $H_1 : p = 0.80$

(C) $H_0 : p = 0.80$ against $H_1 : p > 0.80$

(D) $H_0 : p = 0.80$ against $H_1 : p \neq 0.80$

168. Which of the following probability laws is *not* a member of an exponential family ?

(A) $f(x, \theta) = \theta(1-x)^{\theta-1}, \quad 0 < x < 1, \theta > 0$

(B) $f(x, \theta) = \theta^x (1-\theta)^{1-x}, \ x = 0, 1, \ 0 < \theta < 1$

(C) $N(\theta, \theta^2), \ \theta \in R$

(D) $N(\theta, \theta), \ \theta > 0$

169. Suppose $\{X_1, X_2, ......., X_n\}$ are independent random variables, where $X_i$ follows uniform $U(0, i\theta)$ distribution for $i = 1, 2, ......., n$. Which of the following statements is true ?

(A) $\frac{2}{n} \sum_{i=1}^{n} \frac{X_i}{i}$ is not consistent for $\theta$

(B) Sample mean is consistent for $\theta$

(C) $\max\left\{X_1, \frac{X_2}{2}, \frac{X_3}{3}, ......, \frac{X_n}{n}\right\}$ is consistent for $\theta$

(D) $X_{(n)}$ is consistent for $\theta$

170. Suppose $\{X_1, X_2, \ldots\ldots, X_n\}$ is a random sample from $N(\mu, \sigma^2)$ distribution. Then the approximate dispersion matrix of

$$\sqrt{n}\left((\bar{X}_n, S_n^2)' - (\mu, \sigma^2)'\right)$$

is :

(A) $\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^4 \end{pmatrix}$

(B) $\begin{pmatrix} \sigma^2 & 0 \\ 0 & 2\sigma^4 \end{pmatrix}$

(C) $\begin{pmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{\sigma^4} \end{pmatrix}$

(D) $\begin{pmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{2\sigma^4} \end{pmatrix}$

171. Suppose $X \sim B(1, p)$ distribution, $0 < p < 1$. On the basis of a random sample of size $n$ from the distribution of X, we want to test $H_0 : p = p_0$ against the alternative $H_1 : p \neq p_0$ , where $p_0$ is a specified constant. Then asymptotic null distribution of which of the following test statistics is N(0, 1) ?

(A) $\dfrac{n(\bar{X}_n - p_0)}{\sqrt{\bar{X}_n(1-\bar{X}_n)}}$

(B) $\dfrac{\bar{X}_n - p_0}{\sqrt{\bar{X}_n(1-\bar{X}_n)}}$

(C) $\dfrac{\sqrt{n}\,(\bar{X}_n - p_0)}{\sqrt{p_0(1-p_0)}}$

(D) $\dfrac{\bar{X}_n - p_0}{\sqrt{p_0(1-p_0)}}$

172. If T is MLE of $\theta$ and $\Psi(\theta)$ is one-to-one function of $\theta$, then $\Psi(T)$ is MLE of $\Psi(\theta)$. This follows from :

(A) Invariance property of MLE

(B) Asymptotic property of MLE

(C) Consistency property of MLE

(D) Regularity condition of MLE

173. In an $r \times s$ contingency table for testing $H_0$ : Two attributes A and B are independent against the alternative $H_1$ : A and B are not independent, the expected frequency $e_{ij}$ of $(i-j)$th cell is given by :

(A) $\frac{ni_j}{n}$

(B) $ni.n.j$

(C) $\frac{ni.n.j}{n}$

(D) $\frac{ni.n.j}{n^2}$

174. A manufacturer claims that the median lifetime of its product is more than 500 hours. To verify the claim based on 20 randomly observed lifetimes, which of the following tests is most appropriate ?

(A) Sign test

(B) Mann-Whitney test

(C) Kolmogorov-Smirnov test

(D) Chi-square test

175. Suppose

$$P(Y = y/\theta) = \binom{n}{y} \theta^y (1-\theta)^{n-y},$$

$y \in \{0, 1, 2, ....., n\}$. If $\theta \sim$ uniform (0, 1), then the posterior distribution will be :

(A) Uniform (0, $y$)

(B) Beta ($y$, $n - y$)

(C) Beta ($y + 1$, $n - y + 1$)

(D) Uniform (0, 1)

176. Let X ~ Poisson ($\lambda$) and a single realization of X resulted in 3. If $\lambda$ has a prior distribution

$$P(\lambda) = q\, \lambda\, e^{-3\lambda}, \ \lambda > 0,$$

then the posterior mean is given by :

(A) 4/5

(B) 3/5

(C) 5/4

(D) 2/5

177. Consider the linear model $\underline{Y}_{n-1} = X\underline{\theta}_{p-1} + \xi$ with $E(\xi) = \underline{0}$ $Cov(\xi) = \sigma^2 I_n$. Then which of the following statements is always true ?

(A) $\underline{\theta}$ is estimable

(B) the best linear unbiased estimate of $\underline{\theta}$ exists

(C) $\underline{l}'\underline{\theta}$ is estimable for any $p-1$ vector $\underline{l}$

(D) $\underline{l}'\underline{\theta}$ is estimable for only that $\underline{l}$ which belong to the column space of X'

178. $Y_1$, $Y_2$, $Y_3$ are three random variables with $E(Y_1) = Q_1 - Q_2 + Q_3$, $E(Y_2) = Q_1$ and $E(Y_3) = Q_3 - Q_2$. Then which of the following statements is *not* true ?

(A) $Q_2 - Q_3$ is estimable

(B) all three parameters $Q_1$, $Q_2$, $Q_3$ are estimable

(C) $Q_1$ is estimable, $Q_2$ $Q_3$ are not estimable

(D) $2Q_1 - Q_2 + Q_3$ is estimable

179. The values of $a$, $b$ and $c$ in the following ANOVA table

| **Source** | **d.f.** | **Ss** | **Mss** |
|---|---|---|---|
| Treat | 4 | $b$ | — |
| Error | $a$ | 0.35 | $c$ |
| Total | 29 | 1.25 | — |

are :

(A) $a = 15$, $b = 0.35$ and $c = 0.15$

(B) $a = 25$, $b = 0.35$ and $c = 0.015$

(C) $a = 15$, $b = 0.90$ and $c = 0.14$

(D) $a = 25$, $b = 0.90$ and $c = 0.014$

180. In a one-way ANOVA model $y_{ij} = \mu + d_i + e_{ij}$, $j = 1, ..., n_i$, $i = 1, ..., 4$ with $E(e_{ij}) = 0$. For all $j$ and $i$, which of the following statements is true ?

(A) $\mu$ and $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$ are all estimable

(B) $\mu$ is not estimable but $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$ are all estimable

(C) any linear function of $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$ is estimable

(D) $\alpha_i - \alpha_k$ $i = k = 1, 2, 3, 4$ are estimable

**ROUGH WORK**

ROUGH WORK