Test Booklet Code & Serial No.
प्रश्नपत्रिका कोड व क्रमांक

C

Paper-II
MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....
(Name) ....
2. (Signature) ....
(Name) ....

Seat No. | | | | | | | |
(In figures as in Admit Card)

Seat No. ....
(In words)

OMR Sheet No. | | | | | | | |
(To be filled by the Candidate)

MAR - 30223

Time Allowed : 2 Hours] **[Maximum Marks : 200**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **180**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **180** objective type questions. Each question will carry *two* marks. Candidates should attempt *all* questions either from sections I & II or from sections I & III only.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. Use only Blue/Black Ball point pen.
11. Use of any calculator or log table, etc., is prohibited.
12. There is no negative marking for incorrect answers.

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परीक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 180 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. विद्यार्थ्यांनी खण्ड I व II किंवा खण्ड I व III मधील **सर्व** प्रश्न सोडविणे अनिवार्य आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनिटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्वीकारू नये.
   (*ii*) पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.
   (*iii*) वरीलप्रमाणे सर्व पडताळून पाहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणाव्यतिरिक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापि, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.
11. कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.
12. चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.

Mathematical Science
Paper II

Time Allowed : 120 Minutes] **[Maximum Marks : 200**

Note : This Paper contains **One Hundred Eighty (180)** multiple choice questions in THREE (3) sections, each question carrying **TWO (2)** marks. Attempt ***all*** questions either from **Sections I & II** only **or from Sections I & III** only. The OMR sheets with questions attempted from both the Sections viz. **II & III, will not be assessed.**

Number of questions, sectionwise :

Section I : Q. Nos. 1 to 20, **Section II : Q. Nos. 21 to 100,** **Section III : Q. Nos. 101 to 180.**

SECTION I

1. Which of the following sets of vectors are linearly independent ?

I. $\left\{\begin{bmatrix}2\\0\\1\end{bmatrix}, \begin{bmatrix}3\\2\\1\end{bmatrix}, \begin{bmatrix}-1\\2\\0\end{bmatrix}\right\}$

II. $\left\{\begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}4\\1\\8\end{bmatrix}\right\}$

III. $\left\{\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}2\\1\\1\end{bmatrix}\right\}$

(A) I and II only

(B) II and III only

(C) I and III only

(D) None of the sets are linearly independent

2. The dimension of the vector space consisting of all 10×10 real symmetric matrices with determinant zero is :

(A) 55

(B) 54

(C) 99

(D) 45

3. Let T be a linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$ such that :

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix}, \ T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}2\\1\end{bmatrix}.$$

Then, $T\left(\begin{bmatrix}3\\4\end{bmatrix}\right)$:

(A) is $\begin{bmatrix}3\\2\end{bmatrix}$

(B) is $\begin{bmatrix}4\\5\end{bmatrix}$

(C) is $\begin{bmatrix}5\\3\end{bmatrix}$

(D) Cannot be determined from the given information

4. If A and B are two $m \times n$ non-null matrices such that AB = 0, then, which of the following is always *true* ?

   (A) The column space of B is contained in the null space of A

   (B) The column space of A is contained in the null space of B

   (C) Either A = 0 or B = 0

   (D) Either B or A can be non-singular

5. Let A be a square matrix and let $B = A - 0.2\,I$, where I is the identity matrix. Which of the following is always *true* ?

   (A) A and B have the same rank

   (B) B^{-1} exists

   (C) A and B have the same eigen vectors

   (D) A and B have the same eigen values

6. Suppose the eigen values of a 3×3 matrix A are 0, 1 and 2. Then, the determinant of $(I + A^2)^{-1}$.

   (A) Cannot be found from the given information

   (B) is 1/10

   (C) 0

   (D) 1/8

7. Suppose the two matrices A and B have same characteristic polynomial. Then :

   (A) A and B have same minimal polynomial

   (B) $A = B^t$

   (C) A and B are similar

   (D) A is invertible if and only if B is invertible

8. Which of the following is *not* a nilpotent operator ?

(A) An injective operator on $\mathbf{R}^4$

(B) The identity operator on $\mathbf{R}^4$

(C) The differential operator on the vector space of polynomials of degree upto 3 over $\mathbf{R}$

(D) $T : \mathbf{R}^4 \to \mathbf{R}^4$ defined by :

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ 2x_2 \\ 3x_3 \\ 4x_4 \end{bmatrix}$$

9. Let V be an inner product space, W be a subspace of V and

$$W^{\perp} = \{v \in V / < v, w > = 0 \text{ for all } w \in W\}$$

Then :

(A) $\dim W^{\perp} = \dim W$

(B) $W^{\perp\perp} = W$

(C) $W \subseteq W^{\perp\perp}$

(D) $V = W \oplus W^{\perp}$

10. The Jordan canonical form of a real symmetric matrix with characteristic polynomial $(x - 1)^4$ is :

(A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

11. Consider the sets given by

$X = \{2m + 1 / m \in \mathbf{Z}\}$ and

$Y = \{2n + 3 / n \in \mathbf{Z}\}$.

Then :

(A) X is a proper subset of Y

(B) Y is a proper subset of X

(C) X = Y

(D) Either X is finite or Y is finite

12. The series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\dots\dots\dots$$

(A) is divergent

(B) converges to zero

(C) converges to $\log_e{}^2$

(D) converges to $\log_e{}^1$

13. The improper integral $\int_0^\infty \frac{x^{s-1}}{1-x}$ is convergent if and only if :

(A) $0 \le s \le 1$

(B) $0 < s \le 1$

(C) $0 \le s < 1$

(D) $0 < s < 1$

14. Let X and Y be metric spaces and $f, g : X \to Y$ be continuous. Then $\{x \mid f(x) = g(x)\}$ is :

(A) Open

(B) Closed

(C) Connected

(D) Compact

15. Which of the following pair X, Y of sets do not have same cardinality, i.e. $|X| = |Y|$?

(A) $X = [0, 1]$, $Y = [0, 1] \times [0, 1]$

(B) X = the set of irrationals, Y = the set of algebraic numbers

(C) $X = \mathbf{R}$, $Y = \mathbf{C}$

(D) X = a bounded interval $[x, y]$ with $x < y$ in $\mathbf{R}$, $Y = (-\infty, \infty) = \mathbf{R}$

16. Suppose that $\{A_n\}$ is a sequence of sets such that :

$$A_n = \begin{cases} \{n, n+1, n+2\} & \text{if } n \text{ is even} \\ \{n, n+1, n+2, \dots\dots\} & \text{if } n \text{ is odd} \end{cases}$$

Then, which of the following is/are true ?

(A) $\lim_{n\to\infty} A_n$ does not exist

(B) $\lim_{n\to\infty} A_n = \{1, 2, 3, \dots\dots\}$

(C) $\lim_{n\to\infty} A_n = \{3, 5, 7, \dots\dots\}$

(D) $\lim_{n\to\infty} A_n = \phi$

17. Let **F** be the class of subsets of **R** defined by **F** = {A | either A is countable or A^c is countable}. Then, which of the following is *not* true ?

(A) $\mathbf{R} \in \mathbf{F}$

(B) $\bigcup_{n=1}^{\infty} B_n \in \mathbf{F}$, if $B_n \in \mathbf{F}$, for all n

(C) $[0, \infty) \in \mathbf{F}$

(D) $Q^c \in \mathbf{F}$, where Q is the set of all rational numbers

18. A function f is defined on **R** by :

$$f(x) = \begin{cases} x^2 & \text{if } x \le 3 \\ ax + b & \text{if } x > 3 \end{cases}$$

Then, for which values of a and b, $f'(3)$ exists ?

(A) $a = 6,\ b = -9$

(B) $a = 0,\ b = 9$

(C) $a = 3,\ b = 0$

(D) $a = 18,\ b = -9$

19. Consider the function g defined by :

$$g(x) = \begin{cases} \dfrac{1 - \cos(2x)}{x^2} & \text{if } x \ne 0 \\ u & \text{if } x = 0 \end{cases}$$

The function g is continuous at 0,

(A) if $u = 2$

(B) if $u = 1$

(C) if $u = 0$

(D) such a value u does not exist

20. Let f be a function from $\mathbf{R}^2$ to **R** given by :

$$f(x, y) = \begin{cases} x & \text{if } x = y \\ 1 & \text{if } x \ne y \end{cases}$$

Then, which of the following statements is *true* ?

S_1 : Both partial derivatives of f at (0, 0) exist and are equal

S_2 : f is differentiable at (0, 0)

(A) Both S_1 and S_2

(B) Only S_2

(C) Neither S_1 nor S_2

(D) Only S_1

SECTION II

21. $\mathbf{R}$ is not homeomorphic to $\mathbf{R}^2$ because :

(A) (0, 1) is open in $\mathbf{R}$ but not open as a subset of $\mathbf{R}^2$

(B) (0, 1) is open in $\mathbf{R}$ but closed as a subset of $\mathbf{R}^2$

(C) $\mathbf{R}\backslash(0, 1)$ is not connected in $\mathbf{R}$ and is also not connected in $\mathbf{R}^2$

(D) [0, 1] is closed in $\mathbf{R}$ but not closed as a subset of $\mathbf{R}^2$

22. Let A and B be proper dense subsets of a metric space. Then :

(A) $A \cap B$ is dense

(B) $A \cup B$ is dense

(C) $A \backslash B$ is dense

(D) Either A or B is a closed set

23. Which of the following statements is *true* ?

(A) Every connected subset of $\mathbf{R}^n$ is convex

(B) Every convex set in $\mathbf{R}^n$ is compact

(C) Every convex subset of $\mathbf{R}^n$ is connected

(D) The set :

$\{(x, y) \in \mathbf{R}^2 / x \geq 0, y \geq 0\} \cup \{(x, y) \in \mathbf{R}^2 / x \leq 0, y \leq 0\}$

is convex

24. Let $f : [0, 1] \to S^1$ be defined as $f(x) = e^{2\pi i x}$.

If Q denotes the set of rational numbers and S^1 is unit circle, then :

(A) $f([0, 1] \cap Q)$ is closed subset of S^1

(B) $f([0, 1] \cap Q)$ is an open subset of S^1

(C) $f([0, 1] \cap Q)$ is dense in S^1

(D) $f([0, 1] \cap Q)$ is connected in S^1

25. Which of the following collection is not a base for $\mathbf{Z} \times \mathbf{R}$, where $\mathbf{Z}$ denotes the set of integers.

(A) $\{\{k\} \times \mathrm{I} / \mathrm{I}$ is an open interval in $\mathbf{R}$, $k \in \mathbf{Z}\}$

(B) $\{\mathbf{Z} \times \mathrm{I} / \mathrm{I}$ is an open interval in $\mathbf{R}\}$

(C) $\{\{k, -k\} \times \mathrm{I} / \mathrm{I}$ is closed interval in $\mathbf{R}$, $k \in \mathbf{N}$, the set of natural numbers$\}$

(D) $\{\{k\} \times (k - 1, k + 1) / k \in \mathbf{Z}\}$

26. Consider the space $\mathbf{R}$ with the topology induced by the open base :

$\{u / u = (a, b)$ or $(a, b) \setminus \mathrm{F}\}$

where $\mathrm{F} = \{y_n / n \in \mathbf{N}\}$.

The space $\mathbf{R}$ with this topology is :

(A) not T_1

(B) regular

(C) both T_2 and regular

(D) T_2

27. Let $1 \le p < q < \infty$. Then :

(A) l^p is a closed subspace of l^q and l^q is a closed subspace of l^p

(B) l^p is a closed subspace of l^q and l^q is not a closed subspace of l^p

(C) l^p is not a closed subspace of l^q and l^q is a closed subspace of l^p

(D) neither l^p is a closed subspace of l^q nor l^q is a closed subspace of l^p

28. For $x, y \in \mathbf{R}$, define

$$d_1(x, y) = ||x| - |y||$$

and $d_2(x, y) = |x^2 - y^2|$. Then :

(A) d_1 and d_2 are metrics on $\mathbf{R}$

(B) d_1 is a metric on $\mathbf{R}$ and d_2 is not a metric on $\mathbf{R}$

(C) d_2 is a metric on $\mathbf{R}$ and d_1 is not a metric on $\mathbf{R}$

(D) neither d_1 nor d_2 is a metric on $\mathbf{R}$

29. Let $A = \{(x, y) \in \mathbf{R}^2 : x, y$ are integers$\}$ and $B = \{(x, y) \in \mathbf{R}^2 : y = x^2\}$. Then :

(A) $\mathbf{R}^2 - A$ is closed

(B) $\mathbf{R}^2 - A$ is connected

(C) $\mathbf{R}^2 - B$ is closed

(D) $\mathbf{R}^2 - B$ is connected

30. An equivalence relation ~ defined on X = [0, 1] by $s \sim t$ iff either $s = t$ or $s, t \in \{0, 1\}$. Then quotient space X/~ (if [0, 1] is given the metric topology induced by modulus) is homeomorphic to :

(A) [0, 1]

(B) [0, 1)

(C) (0, 1)

(D) $s^1 = \{(x, y) \in \mathbf{R}^2 : x^2 + y^2 = 1\}$

31. The initial value problem :

$$y^1 = f(x, y), \; y(x_0) = y_0$$

where (x, y) belongs to a domain **D**, has a unique solution :

(A) If $f(x, y)$ is continuous in **D**

(B) If $f(x, y)$ is continuous and bounded in **D**

(C) If $f(x, y)$ is continuous and satisfies Lipschitz condition for all points in **D**

(D) For all functions $f(x, y)$

32. Let $y_1(x)$ and $y_2(x)$ be two solutions of :

$$(r(x)y')' + p(x)y = 0$$

such that $y_1(x)$ and $y_2(x)$ have a common zero in $[a, b]$. Then :

(A) y_1, y_2 are linearly independent on $[a, b]$

(B) $W(y_1, y_2) = 0$

(C) $W(y_1, y_2) \neq 0$

(D) None of the above

33. The differential equation of family of circles of radius r whose center lie on x-axis is :

(A) $y\left(\frac{dy}{dx}\right) + y^2 = r^2$

(B) $y\left[\left(\frac{dy}{dx}\right)^2 + 1\right] = r^2$

(C) $y^2\left[\frac{dy}{dx} + 1\right] = r^2$

(D) $y^2\left[\left(\frac{dy}{dx}\right)^2 + 1\right] = r^2$

34. Let $f(x) = x^2$ and $g(x) = x^2 \log x$ on $|x| > 0$. Then :

(A) f, g are linearly dependent

(B) f, g are linearly independent but cannot be solutions of an ordinary differential equation

(C) f, g can be solutions of an ordinary differential equation

(D) Wronskian of f and g is x^2

35. For the initial value problem

$$y^1 = y^2 + \cos^2 x, \quad y(0) = 0$$

the interval of existence of solution in rectangle

$$R : \{(x, y) \mid 0 \le x \le a, |y| \le b, \quad a > ½, b > 0\}$$

is :

(A) [0, 1]

(B) $[0, 1/\sqrt{2}]$

(C) [0, ½]

(D) [0, 2]

36. Let y be solution of $y^1 + y = |x|$, $x \in \mathbf{R}$ and $y(-1) = 0$, then $y(1)$ equals :

(A) $2/e$

(B) $2e$

(C) $2/e^2$

(D) $2e^2$

37. Let Γ be a curve which passes through (0, 1) and intersects each of the curve of the family $y = cx^2$ orthogonally. Then Γ also passes through :

(A) $(0, \sqrt{2})$

(B) (1, 1)

(C) $(\sqrt{2}, 0)$

(D) $(-1, 1)$

38. The locus of points of intersection of two surfaces represented by the partial differential equations $xdx + ydy + zdz = 0$ and $yp - xq = 0$ is a :

(A) parabola

(B) circle

(C) straight line

(D) helix

39. A given partial differential equation can have :

(I) no integral surface passing through a given curve

(II) unique integral surface passing through a given curve

(III) infinitely many integral surfaces passing through a given curve

Then :

(A) (I) and (II) are true

(B) (II) and (III) are true

(C) (I) and (III) are true

(D) (I), (II), (III) are true

40. The partial differential equation $yp - xq = 0$, has :

(A) unique integral surface passing through the curve $x^2 = z, y = 0$

(B) finitely many integral surfaces passing through the curve $x^2 = z, y = 0$

(C) infinitely many integral surfaces passing through the curve $x^2 = z, y = 0$

(D) no integral surface passing through the curve $x^2 = z, y = 0$

41. The partial differential equation :

$f(x, y, z, p, q) = 0$ and

$g(x, y, z, p, q) = 0$

are compatible on a domain D if :

(A) $\frac{\partial(f,g)}{\partial(x,y)} \neq 0$ and the equation $dz = pdx + qdy$ is integrable

(B) $\frac{\partial(f,g)}{\partial(p,q)} \neq 0$ and the equation $dz = pdx + qdy$ is integrable

(C) $\frac{\partial(f,g)}{\partial(p,x)} \neq 0$ and the equation $dz = pdx + qdy$ is integrable

(D) $\frac{\partial(f,g)}{\partial(q,y)} \neq 0$ and the equation $dz = pdx + qdy$ is integrable

42. Consider the following two statements :

    (I) Every surface generated by a one parameter family of characteristics is an integral surface

    (II) Every integral surface is generated by the characteristic curves

    Then :

    (A) (I) is true and not (II)

    (B) (II) is true and not (I)

    (C) (I) and (II) both are true

    (D) (I) and (II) both are false

43. The two partial differential equations $f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$ are said to be compatible if :

    (A) every solution of $f(x, y, z, p, q)=0$ is also a solution of $g(x, y, z, p, q)=0$

    (B) every solution of $g(x, y, z, p, q)=0$ is also a solution of $f(x, y, z, p, q)=0$

    (C) they have common solution

    (D) they do not have common solution

44. The normals to surfaces represented by the Pfaffian differential equation Pdx + Qdy + Rdz = 0 and the Lagrange's equation Pp + Qq = R :

    (A) are collinear

    (B) are orthogonal

    (C) are parallel

    (D) intersect

45. The partial differential equation

$$x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = xyz$$

    is :

    (A) hyperbolic

    (B) parabolic

    (C) elliptic

    (D) Laplacian

46. Let

$$x^{(n)} = x(x-h)(x-2h)...(x-(n-2)h)(x-(n-1)h),$$

where h is the step size. Then which of the following is *correct* ?

(A) $\Delta^n x^{(n)} = n!h^n$ and $\Delta^{n+1} x^{(n)} = 0$

(B) $\Delta^n x^{(n)} = n!$ and $\Delta^{n+1} x^{(n)} \neq 0$

(C) $\Delta^n x^{(n)} = n!h$ and $\Delta^{n+1} x^{(n)} = (n+1)!$

(D) $\Delta^n x^{(n)} = n!$ and $\Delta^{n+1} x^{(n)} = (n+1)!$

47. Let $f(x) = e^x$, then $\Delta^n e^x$ = ...

(A) $e^x(e^h - 1)^n$

(B) $e^x(e^h + 1)^n$

(C) e^x

(D) e^{-x}

48. The relation to solve $x = e^{-x}$ using Newton-Raphson method is :

(A) $x_{n+1} = \dfrac{(1+x_n)\, e^{-x_n}}{1+e^{-x_n}}$

(B) $x_{n+1} = \dfrac{(1-x_n)\, e^{-x_n}}{1-e^{-x_n}}$

(C) $x_{n+1} = \dfrac{(1+x_n)\, e^{-x_n}}{1-e^{-x_n}}$

(D) $x_{n+1} = \dfrac{x_n(e^{x_n})}{1+e^{-x_n}}$

49. Everett's formula truncated after second differences is equivalent to .... truncated after third differences.

(A) String's formula

(B) Bessel's formula

(C) Stirling's formula

(D) Lagrange's formula

50. The value of

$$\Delta^5 [(1-x)(1-x^2)(1+x^2)]$$

is :

(A) 5!

(B) 0

(C) 1

(D) 5

51. The missing term in the following table

x	***f*(*x*)**
0	1
1	3
2	9
3	–
4	81

using difference operator method is :

(A) 28

(B) 29

(C) 30

(D) 31

52. A curve of quickest descent in the vertical plane, where y is vertical and x is horizontal, is the solution of the equation :

(A) $y(1+y'^2)=c$

(B) $\sqrt{y^2-c^2-y'}=c$

(C) $y'\sqrt{a^2-x^2}=\sqrt{x}$

(D) $y'\sqrt{a^2-x^2}=a$

53. The shape of a curve in the plane with fixed perimeter that encloses maximum area is :

(A) a square

(B) an ellipse

(C) a rectangle

(D) a circle

54. The extremal of the functional

$$\mathrm{I}\left(y(x)\right)=\int_0^4 \left(xy'-(y')^2\right)dx$$

determined by the boundary conditions $y(0) = 0$ and $y(4) = 3$ is :

(A) $y=\frac{x^2}{4}$

(B) $y=\frac{x^2}{2}+3x$

(C) $y=\frac{x^2}{4}-x+1$

(D) $4y=x(x-1)$

55. The Euler's equation for the extremization of the functional :

$$\mathrm{I}\left(y(x)\right)=\int_0^{\pi/2} \left(2xy+x'^2+y'^2\right)dt,$$

$$x'=dx/dt$$

is :

(A) $x''-y=0$

(B) $y''-x=0$

(C) $\frac{d^4y}{dt^4}-y=0$

(D) $x''+y=0$

56. The Euler-Lagrange's differential equation for the extremization of the functional :

$$I(y(x)) = \int_{x_1}^{x_2} f(x, y, y')\, dx$$

is :

(A) $f_{y'y'} \frac{d^2y}{dx^2} + (f_{y'x} - f_y) = 0$

(B) $f_{y'y'} \frac{d^2y}{dx^2} + f_{y'y} \frac{dy}{dx} + (f_{y'x} - f_y) = 0$

(C) $f_{y'x} \frac{d^2y}{dx^2} + f_{y'y} \frac{dy}{dx} + (f_{y'y} - f_y) = 0$

(D) $f_{y'x} \frac{d^2y}{dx^2} + (f_{y'x} - f_y) = 0$

57. The Euler Lagrange equation associated with the functional

$$I(w) = \frac{1}{2} \int_u |Dw|^2\, dx + \int_u f\, w\, dx$$

where u is an open set is :

(A) $-\Delta w = w$ in u

(B) $-\Delta w = f$ in u

(C) $\Delta w = w$ in u

(D) $\Delta w = f$ in u

58. The solution of the homogeneous integral equation

$$x(t) = \lambda \int_0^1 t\, s\, x(s)\, ds,$$

for $\lambda \pm 0$, is :

(A) t^2

(B) $1 + t^2$

(C) t

(D) $1 + t$

59. The kernel of the linear integral equation

$$x(t) = \lambda \int_0^1 (t^2 - s^2)\, x(s) ds$$

is :

(A) Separable

(B) Symmetric

(C) Convolution

(D) Singular

60. The initial value problem
$x''(t) - x(t) = 0,\ x(0) = 1,\ x'(0) = 1$
is equivalent to the integral equation :

(A) $x(t) = 1 + \int_0^t (t+s)^2 x(s)\, ds$

(B) $x(t) = t + \int_0^t (t+s)\, x(s)\, ds$

(C) $x(t) = 1 + \int_0^t (t-s)^2\, x(s)\, ds$

(D) $x(t) = 1 + t + \int_0^t (t-s)\, x(s)\, ds$

61. For a homogeneous Fredholm integral equation with symmetric kernel :

$$x(t) = \lambda \int_a^b k(t, s)\, x(s)\, ds$$

consider the following statements.

(I) The eigen values are not real.

(II) The eigen functions corresponding to distinct eigen values are orthogonal on $[a, b]$. Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

62. The solution of the integral equation

$$x(t) = t + \int_0^t (s-t)\, x(s)\, ds$$

with the help of resolvent kernel is :

(A) $x(t) = \cos t$

(B) $x(t) = \sin t$

(C) $x(t) = e^t$

(D) $x(t) = 1$

63. Eigen values of the homogeneous Fredholm integral equation

$$x(t) = \lambda \int_0^1 (1 - 3ts)\, x(s)\, ds$$

are :

(A) 1, 2

(B) 2, –2

(C) 2, –1

(D) 1, –2

64. Let a particle of mass m be projected from the ground with initial velocity u making an angle θ with the horizontal. Then the total energy of the particle is :

(A) $\frac{1}{2m}\left(p_x^2 + p_y^2\right)$

(B) $\frac{1}{2}m\left(p_x^2 + p_y^2\right) + mgy$

(C) $\frac{1}{2m}\left(p_x^2 + p_y^2\right) + mgy$

(D) $\frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - mgy$

65. For a particle the kinetic energy and the potential energy is given by

$$T = \frac{1}{2}m\dot{r}^2, \quad U = \frac{1}{r}\left(1 + \frac{\dot{r}^2}{c^2}\right),$$

where c is a constant. Then the Hamiltonian H of the particle is :

(A) H = T + U

(B) $H = P_r\dot{r}^2 - \frac{1}{2}m\dot{r}^2 + \frac{1}{r}\left(1 + \frac{\dot{r}^2}{c^2}\right)$

(C) $H = \frac{1}{2}\left(\frac{p_r^2 r c^2}{(mrc^2 - 2)} - \frac{1}{r}\right)$

(D) $H = \frac{1}{2}\frac{p_r^2 r c^2}{(mrc^2 - 2)} + \frac{1}{r}$

66. A Lagrangian of a particle of mass m is given by

$$L = \frac{1}{2}m\dot{r}^2 - \frac{1}{r}\left(1 + \frac{\dot{r}^2}{c^2}\right),$$

c is a constant. Then :

(A) the Hamiltonian H represents the total energy E

(B) H is conserved

(C) E is conserved

(D) Both H and E are conserved

67. Let a particle of mass m be projected with initial velocity u making an angle θ with the x-axis. Then its velocity after time t is given by :

(A) $v^2 = u^2 - g^2t^2$

(B) $v^2 = u^2 + g^2t^2 - 2gut\cos\theta$

(C) $v^2 = u^2 + g^2t^2 - 2ugt\sin\theta$

(D) $v^2 = u^2 - 2ugt\sin\theta$

68. A particle has Lagrangian

$$L = \frac{1}{2}\left[f(\theta)\dot{\theta}^2 + 2g(\theta)w\dot{\theta} + w^2 h(\theta)\right] - V$$

Then $\frac{1}{2}\left[f(\theta)\dot{\theta}^2 - w^2 h(\theta)\right] + V$ represents :

(A) The Hamiltonian H

(B) The total energy E

(C) Both Hamiltonian H and the total energy E

(D) Neither the Hamiltonian H nor the total energy E

69. A particle of mass m falling vertically under gravity. Then the Lagrangian L of the particle is given by :

(A) $L = \frac{1}{2}m\dot{z}^2 - mgz$

(B) $L = \frac{1}{2}m\dot{z}^2 + mgz$

(C) $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy$

(D) $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mgy$

70. A rigid body with one point fixed has :

(A) Three degrees of freedom

(B) Six degrees of freedom

(C) Two degrees of freedom

(D) One degree of freedom

71. One of the values of $i^{(1+i)}$ is :

(A) $ie^{\frac{\pi}{2}}$

(B) $e^{-\frac{\pi i}{2}}$

(C) $e^{\frac{\pi i}{2}}$

(D) $ie^{-\pi/2}$

72. If $f(z) = u + iv$ is analytic on a domain and $v = y + 2xy$, then u =

(A) $x + 2xy + c$

(B) $x + x^2 - y^2 + c$

(C) $y^2 - x^2 + y + c$

(D) $x^2 - y^2 + c$

73. The radius of convergence of the series

$1 - 4z^2 + 16z^4 - 64z^6 + 256z^8 - .....$

is :

(A) ∞

(B) 1

(C) 2

(D) $\frac{1}{2}$

74. Let $D = \{z \in \mathbf{C} : |z| < 1\}$ and let $f : D \to D$ be a bijective analytic function such that $f\left(\frac{1}{2}\right) = 0$. Then :

(A) $f(z) = \frac{2z-1}{2-z}$

(B) $f(z) = e^{i\theta}\left(\frac{2z-1}{2-z}\right)$ for some real number θ

(C) $f(z) = \frac{2z-1}{2+z}$

(D) $f(z) = \frac{z - \frac{1}{2}}{1+z}$

75. Suppose f is a Möbius transformation on $\mathbf{C}_\infty$ that has ∞ as its only fixed point. Then :

(A) f is a rotation

(B) f is a dilation

(C) f is a translation

(D) f is an inversion

76. A function f analytic on a disk D is necessarily constant if f(D) is contained in :

(A) a disk

(B) a half plane

(C) a circle

(D) **C**

77. Let A denote the set of all complex numbers lying on and within the square having vertices 0, 1, πi and $\pi i + 1$. Let $f(z) = e^z$, $z \in \mathbf{C}$. Then f(A) =

(A) $\{w / 1 \le |w| \le e\}$

(B) $\{w / |w| \le e\}$

(C) $\{w / 1 \le |w| \le e, 0 \le \arg w \le \pi\}$

(D) $\{w / 1 \le |w| \le e, 0 \le \arg w \le \frac{\pi}{2}\}$

78. $\int_{|z|=1} z^4 \sin\frac{1}{z}\, dz =$

(A) $12\pi i$

(B) $\frac{\pi i}{60}$

(C) $60\pi i$

(D) $\frac{\pi i}{30}$

79. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n! z^n}{n^n}$ is :

(A) 1

(B) e

(C) ∞

(D) $\frac{1}{e}$

80. Let G be a region and suppose $f_n : G \to \mathbf{C}$ is analytic for each $n \geq 1$. Suppose $\{f_n\}$ converges uniformly to a function $f : G \to \mathbf{C}$. Then :

(A) f is continuous but not differentiable

(B) f is differentiable but not analytic

(C) f' is analytic

(D) f is analytic but f' is not analytic

81. Let $G = \{z \in \mathbf{C} : |z - 1| < 1\}$ and let $f : G \to \mathbf{C}$ be an analytic function such that :

$$f(1 - 1/n) = \frac{2n - 1}{n + 1}$$

for n = 2, 3, ...

Then :

(A) $f(z) = \frac{2z - 1}{z + 1}$

(B) $f(z) = \frac{z + 1}{2z - 1}$

(C) $f(z) = \frac{1 + z}{2 - z}$

(D) $f(z) = \frac{2 - z}{1 + z}$

82. Consider the following two statements :

    (*i*) An entire function f with Re $f \leq$ Im f is constant.

    (*ii*) There exists an analytic function f from the unit disk Δ into Δ such that $f(0) = \frac{1}{2}$ and $f'(0) = \frac{3}{4}$.

    Then :

    (A) Only (*i*) is true

    (B) Only (*ii*) is true

    (C) Both (*i*) and (*ii*) are true

    (D) Both (*i*) and (*ii*) are false

83. The residue of $f(z) = \cot z$ at $z = \pi$ is :

    (A) 0

    (B) π

    (C) 1

    (D) 2π

84. Let $G = \{z : 0 < |z| < 0.1\}$ and $f(z) = e^{1/z}$, $z \in G$. Then :

    (A) $f(G) = G$

    (B) $f(G) = \mathbf{C}$

    (C) $f(G) = \{z : 0 < |z| < e^{10}\}$

    (D) $f(G)$ is dense in $\mathbf{C}$

85. What is the degree of the splitting field $x^3 - 2$ over $\mathbf{Q}$?

    (A) 6

    (B) 4

    (C) 3

    (D) 2

86. Let F be a finite extension of $\mathbf{R}$. Then the degree $[F : \mathbf{R}]$ is :

    (A) a power of 2

    (B) even

    (C) a power of a prime

    (D) prime

87. Which of the following statements is *not* true ?

(A) There is a field with 4 elements

(B) There is a Boolean ring with 8 elements

(C) Every group of order 12 is cyclic

(D) Every group of order 15 is abelian

88. The number of non-constant irreducible polynomials of degree ≤ 3 in $\mathbf{Z}_2[x]$ is :

(A) 5

(B) 4

(C) 6

(D) 3

89. Suppose the ideal $A = (p(x))$ in $F[x]$ F-field is a maximal ideal. Then the polynomial $p(x)$:

(A) has no root in F

(B) is monic

(C) is irreducible

(D) has all roots in F

90. Which is FALSE ?

(A) The polynomial ring $K[x]$ over a field K is a Euclidean domain

(B) Every principal ideal domain is a Euclidean domain

(C) The polynomial ring $K[x]$, where K is a field, is a Euclidean domain if and only if it is a principal ideal domain

(D) The polynomial ring $K[x, y]$ is not a principal ideal domain, where K is a field

91. Consider the following statements :

(I) The polynomial ring $\mathbf{Z}[x]$ is a principal ideal domain.

(II) The polynomial ring $\mathbf{Z}[x]$ is a Euclidean domain.

Which is *correct* ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

92. Consider the following statements :

(I) If R is a commutative ring with unity and $f : R \to R$ be a ring homomorphism defined by $f(a) = a^2$, then $1 + 1 = 0$.

(II) If R is a commutative ring with unity and $f : R \to R$ be a ring homomorphism defined by $f(a) = a^3$, then $1 + 1 + 1 = 0$.

Which of the following is *true* ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

93. Consider the following statements :

(I) Let G be a finite group. Then every element is of finite order.

(II) Let G be a group such that every element is of finite order. Then G is a finite group.

Which is *true* ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

94. Which of the following is *true* ?

(A) Let G be a finite group such that $|G| = p^k$, where p is prime and $k \in \mathbf{N}$. Then G has a non-trivial center

(B) The group of order p^2 (p-prime) is simple

(C) The group of order pq, where p and q are primes is simple

(D) The group of order 45 is simple

95. Consider the following statements :

(I) Let H and K be subgroups of an abelian group G. Suppose $|H| = m$ and $|K| = n$. Let $d = \text{lcm}(m, n)$. Then G has a subgroup of order d.

(II) The group of order 15 is abelian.

Which of the following is *true* ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

96. Let S_3 be the symmetric group, $I_n(S_3)$ be the group of inner automorphisms of S_3 and $Z(S_3)$ be the center of S_3. Then which is FALSE ?

(A) $Z(S_3) = \{e\}$, where e is the identity of S_3

(B) $I_n(S_3) = S_3$

(C) $I_n(S_3) = \{e\}$, where e is the identity of S_3

(D) $I_n(S_3) \cong S_3 / Z(S_3)$

97. How many distinct terms are there in the expansion of $(w+x+y+z)^{10}$?

(A) 228

(B) 224

(C) 281

(D) 286

98. What is the last digit in 3^{55} ?

(A) 3

(B) 5

(C) 1

(D) 7

99. Let $f : \mathbf{R}^2 \to \mathbf{R}^3$ be a linear map and

$A = \{(x, y) \in \mathbf{R}^2 / |x| \leq 1, |y| \leq 1\}$.

Then :

(A) $f(A)$ is a connected subset of $\mathbf{R}^3$

(B) $f(A)$ is dense in $\mathbf{R}^3$

(C) $f(A)$ is an open subset of $\mathbf{R}^3$

(D) $f(A)$ is an unbounded subset of $\mathbf{R}^3$

100. If

$A = \{(x, y) / -1 \leq x < 1, -1 \leq |y| \leq 1\}$

and

$B = \{(x, y) / (x - 1)^2 + y^2 \leq 1/2\}$

then :

(A) $A \setminus B$ is compact in $\mathbf{R}^2$

(B) $B \setminus A$ is compact in $\mathbf{R}^2$

(C) $A \cap B$ is compact in $\mathbf{R}^2$

(D) $A \cup B$ is compact in $\mathbf{R}^2$

SECTION III

101. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample from Cauchy $C(\theta, 1)$ distribution. A test procedure for testing $H_0 : \theta = 0$ against the alternative $H_1 : \theta \neq 0$ is *not* consistent if the test statistic is based on :
(A) Sample median
(B) Sample mean
(C) Sample first quartile
(D) Sample third quartile

102. The Mann-Whitney U statistic is defined as :
(A) The minimum absolute difference between the two empirical distribution functions
(B) The maximum absolute difference between the two empirical distribution functions
(C) The number of times a Y precedes X in the combined ordered arrangement of two independent random samples on X's and Y's
(D) The sum of ranks of Y's in the combined ordered arrangement of two independent random samples on X's and Y's

103. Consider the following statements :
(I) The Fisher information $I(\theta)$ associated with a logistic distribution with location parameter θ and scale parameter 1 is 1/3.
(II) The Fisher information $I(\theta)$ associated with a Cauchy distribution with location parameter θ and scale parameter 1 is 1/2.
(A) Only (I) is true
(B) Only (II) is true
(C) Both (I) and (II) are true
(D) Both (I) and (II) are not true

104. Suppose n randomly chosen persons were enrolled to examine whether different skin creams A and B have different effects on the human body. Cream A was applied to one of the randomly chosen arm of each person, cream B to the other arm. Response is measured as a continuous variable. Which of the following tests is most appropriate to examine the difference ?
(A) Two-sample Kolmogorov-Smirnov test
(B) Two-sample t-test if normality can be assumed
(C) Paired t-test if normality can be assumed
(D) Mann-Whitney test

105. Suppose given θ, the random variable X follows a normal distribution with mean θ and variance 1. Let the prior density of θ be Bernoulli with parameter ½. Then the posterior distribution of θ is :

(A) $2e^{-1/2\,(x-\theta)^2}$

(B) $\frac{1}{2}e^{-1/2\,(x-\theta)^2}$

(C) $\frac{e^{x\theta - \theta^2/2}}{1 + e^{x - 1/2}}$

(D) $\frac{e^{x^2 + x\theta - \theta^2/2}}{1 + e^{x - 1/2}}$

106. Suppose we have 10 observations $\underset{\sim}{y} = (y_1, y_2, \ldots, y_{10})$ from Poisson (λ) and we assume a Gamma (a, b) prior distribution for λ.

$$\left(p(\lambda) = \left\{\Gamma(a)\right\}^{-1} b^a e^{-b\lambda} \lambda^{a-1,\, a>0,\, b>0}\right)$$

Then the posterior predictive distribution $p(y_{\text{new}} / \underset{\sim}{y})$ of a new observation y_{new} is given by :

(A) Gamma $\left(a + \sum_{i=1}^{10} y_i,\ b + 10\right)$

(B) Negative Binomial $\left(a + \sum_{i=1}^{10} y_i,\ b + 10\right)$

(C) Gamma $\left(b + 10,\ a + \sum_{i=1}^{10} y_i\right)$

(D) Poisson $\left(a + \sum_{i=1}^{10} y_i\right)$

107. Consider the linear model $\underline{y}_{n-1} = X\underline{\theta}_{p-1} + \xi_{n-1}$ where $E(\xi) = 0$, $\text{cov}(\xi) = \sigma^2 I_n$. The assumption that ξ is multivariate Normal is needed for :

(A) estimating the parameter $\underline{\theta}$

(B) estimating the parameter σ^2

(C) splitting the total sum of squares into two orthogonal parts

(D) using F test to test the model fit

108. In a linear model with 6 observations, $E(Y_1) = Q_1 + Q_2$, $E(Y_2) = E(Y_4) - Q_1 + Q_3$, $E(Y_3) = Q_2 - Q_3$, $E(Y_5) = 2E(Y_1)$ and $E(Y_6) = 3E(Y_2)$. Then which of the following statements is *true* ?

(A) Q_1, Q_2, Q_3 are all estimable

(B) $Q_1 + Q_2 + Q_3$ is estimable

(C) $Q_1 + Q_2 + 2Q_3$ is estimable

(D) $Q_1 + 2Q_2 - Q_3$ is estimable

109. For a given data, the two-way ANOVA table is :

Source	df	Ss	Mss	F_{ratio}
Treat	5	6	—	*c*
Block	3	4.5	—	—
Error	*a*	2.0	—	
Total	23	12.5		

Hence the values of *a*, *b* and *c* are :

(A) $a = 15 \quad b = 6.5 \quad c = 1$

(B) $a = 15 \quad b = 4.5 \quad c = 2.25$

(C) $a = 15 \quad b = 6.0 \quad c = 3$

(D) $a = 15 \quad b = 6.0 \quad c = 9$

110. In case of a one-way ANOVA, the data strongly supports the null hypothesis that all the treatments have the same effect. Hence the *p*-value corresponding to the F ratio is :

(A) much larger than 0.05

(B) between 0.01 and 0.05

(C) much smaller than 0.01

(D) nothing can be said about the *p*-value

111. In simple linear regression with regression X and response variate Y the least squares regression line :

(A) may be used to predict a value of Y given the value of X

(B) implies a cause-effect relationship between X and Y

(C) can be determined only if a linear relationship exists between Y and X

(D) all the above statements are true

112. In case of a multiple linear regression set up $\underline{Y} = r\underline{B} + \underline{\varepsilon}$ with covariance matrix of $\underline{\varepsilon}$ being $\sigma^2 t_n$, $\sigma^2 > 0$, which of the following statements is *true* ?

(A) mean regression sum of squares (ss) and mean error ss are unbiased estimators of σ^2, mean total ss is a biased estimator of σ^2

(B) all three mean ss are unbiased estimators of σ^2

(C) mean error ss is an unbiased estimator of σ^2, mean regression ss and mean total ss are biased estimators of σ^2

(D) all three mean ss are biased estimators of σ^2

113. In case of regression of Y_i on X_i with fitted values $\hat{y}_i$, $i = 1, \ldots, n$ we get $\sum_{i-1}^{n} y_i = \sum_{i-1}^{n} \hat{y}_i$.

(A) always

(B) only if the regression equation is $y = \beta_0 + \beta_1 X + \varepsilon$

(C) If the regression equation is $y = \beta_0 + \beta_1 X + \varepsilon$

(D) if and only if the regression equation is $y = \beta_0 + \beta_1 X + \varepsilon$

114. For the logistic regression model, the response variables u is :

(A) binary

(B) discrete with finite sample space consisting of more than two values

(C) discrete with countably infinite sample space

(D) continuous

115. Suppose $\underline{X}_{3\times 1}$ ~ MNormal_3

$$\left(\underline{0} \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right).$$

Hence var $(2X_2 | X_1 = x)$ is equal to :

(A) 16

(B) 3.75

(C) 8

(D) 15

116. Suppose $\underline{X}_{2\times 1}$ ~ Bivariate Normal

$$\left(\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ -1 & 5 \end{bmatrix}\right).$$

Then the distribution of $3Y = 3X_1 - 4X_2$ is :

(A) Normal (–1, 26)

(B) Normal (8, 62)

(C) Normal (8, 98)

(D) Normal (8, 122)

117. Based on n *iid* observations from multivariate normal distribution with mean $\underline{\mu}$ and covariance matrix Σ. The likelihood ratio test statistics to test whether $\underline{\mu} = \underline{\mu}_0$ follows :

(A) F distribution with one-sided critical region

(B) χ^2 distribution with one-sided critical region

(C) t distribution with one-sided critical region

(D) t distribution with two-sided critical region

118. The quadrate form

$$9X_1^2 + 4X_2^2 + X_3^2 - 4X_1X_3 - 2X_2X_3$$

is :

(A) Positive semidefinite

(B) Positive definite

(C) Negative semidefinite

(D) Negative definite

119. Suppose $\underline{X}_{3\times1}$ ~ multivariable Normal$_3$ (μ, Σ) and let

$$Y = 2X_1^2 - X_2^2 + X_3^2.$$

Then the probability distribution of Y is :

(A) χ^2 with 2 degrees of freedom

(B) χ^2 with 3 degrees of freedom

(C) χ^2 with 6 degrees of freedom

(D) not χ^2

120. Given three random variables X_1, X_2, X_3 with $\rho_{12} = 0.6$, $\rho_{13} = 0.8$ and $\rho_{23} = 0.24$. Hence the partial correlation coefficient between X_2 and X_3 after removing the effect of X_1 is :

(A) –0.24

(B) +0.24

(C) –0.5

(D) +0.5

121. The Mahalanobis distance between the two Bivariate Normal Population

$$N_2\left(\begin{bmatrix}-2\\3\end{bmatrix},\begin{bmatrix}2&2\\2&4\end{bmatrix}\right) \text{ and}$$

$$N_2\left(\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}2&2\\2&4\end{bmatrix}\right)$$

is :

(A) 10.5

(B) 8.5

(C) 4.0

(D) 2.0

122. Which of the following techniques is used for the dimension reduction of the data ?

(A) Linear discriminant analysis

(B) Quadratic discriminant analysis

(C) Principal component analysis

(D) None of the above

123. Under systematic sampling :

(A) First unit in the sample is selected by probability proportionate to size (pps) sampling and remaining $(n-1)$ units are selected by simple random sampling

(B) Variance of sample mean is always less than that of sample mean from SRSWOR

(C) Each population unit has unequal chance of being selected in the sample

(D) Sample mean is unbiased estimator for population mean

124. Suppose there are five units, u_1, u_2, u_3, u_4, u_5 in a population. The probabilities of selecting a sample of size two with different units are :

$p(\{u, u_2\}) = \frac{1}{2}$, $p(\{u_3, u_4\}) = \frac{1}{6}$

$p(\{u_3, u_5\}) = \frac{1}{6}$, $p(\{u_4, u_5\}) = \frac{1}{6}$

If π_i denote the probability of including ith unit in the sample, then $(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$ =

(A) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

(B) $\left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$

(C) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}\right)$

(D) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}, \frac{1}{2}\right)$

125. Under Midzuno's sampling design the inclusion probability of ith element π_i is at least :

(A) $\frac{n-1}{N-1}$

(B) $\frac{n}{N}$

(C) $\frac{n(n-1)}{N-1}$

(D) $\frac{n(n-1)}{N}$

126. Post stratification is used when :

(A) Stratum sizes are equal

(B) Number of strata of the population is not known

(C) Population size and stratum sizes are not known

(D) Stratum to which a unit belongs is not known before sampling

127. Double sampling is useful :

(A) When complete information on auxiliary variable is not available

(B) When population is heterogeneous

(C) To improve the performance of ratio estimator for population mean

(D) To improve the performance of regression estimator for population mean

128. Consider the model $E(Y_1) = \beta_1 + \beta_2$, $E(Y_2) = \beta_1 - \beta_2$, $E(Y_3) = \beta_1 + 2\beta_2$ with the assumption that Y_i's are uncorrelated and $\text{var}(Y_i) = \sigma^2$ for all $i = 1, 2, 3$. Then the best linear unbiased estimator of β_2 is :

(A) $(Y_1 + Y_2 + Y_3)/3$

(B) $(4Y_1 + 8Y_2 + 2Y_3)/14$

(C) $(Y_1 - 5Y_2 + 4Y_3)/14$

(D) $(4Y_1 - 8Y_2 + 4Y_3)/3$

129. Which of the following properties is *not* true about the incidence matrix M of a BIBD (r, b, s, k, λ) ?

(A) Every column of M contains exactly k '1's

(B) Every row of M contains exactly s '1's

(C) M is a symmetric matrix

(D) Two distinct rows of M contains both 1's in exactly λ columns

130. Suppose we wish to have a confounding arrangement in 2^r blocks of a 2^n factorial experiment. Then, how many interactions have to be confounded ?

(A) 2^{r-1}

(B) $r - 1$

(C) $n - r + 1$

(D) $2^r - 1$

131. In a completely randomized design with unequal group numbers, that is, $n_1 = 5$, $n_2 = 7$, $n_3 = 6$, what is the degrees of freedom for the error term ?

(A) 17

(B) 18

(C) 15

(D) 120

132. When the conditional effect (also known as sample effect) for one of the factors in a 2^2 design differ only in sign, then the corresponding main effect estimate will be :

(A) zero

(B) one

(C) $(a + b)/2$

(D) $(a - b)/2$

133. Suppose the lifetimes of systems A and B are independent and follow exponential distributions with failure rates 3 and 2 respectively. Then the probability that system A fails before system B is :

(A) 0.4

(B) 0.5

(C) 0.6

(D) 1

134. If the failure rate of a random variable is $h(t) = 3t + 2$, $t \geq 0$, then its distribution function :

(A) is $1 - \exp\left(-\frac{3t^2}{2} - 2t\right)$, $t \geq 0$

(B) is $1 - \exp\left(-(3t+2)^2\right)$, $t \geq 0$

(C) is $\exp\left(-(3t+2)^2\right)$, $t \geq 0$

(D) cannot be obtained from the given information

135. Suppose the lifetime of a system follows the exponential distribution with failure rate 0.8. Then the probability that the system will survive longer than twice its mean time to failure is closest to :

(A) 0.331

(B) 0.261

(C) 0.135

(D) 0.101

136. The two LPPs P and Q are duals of each other, where

P : Max $ax + 2y$ s.t. $2x + y \leq 3$, $bx + 2y \leq c$, $x, y \geq 0$, and

Q : Min $3u + 4v$ s.t. $2u + 4v \geq 2$, $u + 2v \geq 5$, $u, v \geq 0$.

Then, (a, b, c) is equal to :

(A) $(a, b, c) = (5, 4, 4)$

(B) $(a, b, c) = (4, 5, 5)$

(C) $(a, b, c) = (5, 4, 4)$

(D) $(a, b, c) = (4, 5, 4)$

137. Consider the LPP :

Max $Z = 3x + 5y$

Subject to $x + 5y \leq 10$;

$2x + 2y \leq 5$;

$x \geq 0, y \geq 0$.

Which of the following statements is *true* ?

S_1 : There exists a unique optimal solution to the LPP.

S_2 : There exists a unique optimal solution to the dual problem.

(A) Only S_1 is true

(B) Only S_2 is true

(C) Both S_1 and S_2 are true

(D) Neither S_1 nor S_2 is true

138. Solution to the balanced assignment problem is binary due to :

(A) linear formulation of the problem

(B) its non-empty feasible region

(C) the approximate nature of the algorithm

(D) the uni-modular property

139. In terms of the usual notation, for an M|M|1 queueing system, which of the following equations does *not* hold ?

(A) $L_s = \rho/(1 - \rho)$

(B) $W_q = W_s$

(C) $W_s = W_q + 1/\mu$

(D) $L_q = \lambda W_q$

140. For a M|M|3 queueing system with arrival rate 18, which one of the statements is *true* ?

(A) The traffic density is 2

(B) The traffic density is 6

(C) The traffic density is 3

(D) The traffic density is 12

141. If the probability generating function of a random variable X is given by :

$$P_X(t) = (0.4 + 0.6t)^{10},$$

then the distribution of X is :

(A) binomial with mean 4

(B) binomial with mean 6

(C) negative binomial with mean $\frac{20}{3}$

(D) negative binomial with mean 15

142. Suppose the distribution of a random variable is normal (μ, σ^2). Then, which of the following statements is *true* ?

(A) The coefficient of skewness $\beta_1 = 3$

(B) The coefficient of kurtosis $\beta_2 = 0$

(C) 95% area under the normal curve lies in the range of $\mu \pm \sigma$

(D) The mean deviation from the mean is same as the mean deviation from the median

143. The waiting time of the customer (in min) is grouped as :

Waiting Time	**Frequency**
4—6	6
6—8	8
8—10	12
10—12	10
12—14	6

The mode of waiting time is :

(A) 9.33

(B) 9.67

(C) 10.33

(D) 10.67

144. In how many ways can a lady having 10 dresses, 5 pairs of shoes, and 2 hats be dressed ?

(A) 50

(B) 100

(C) 500

(D) 10 ! 5 ! 2 !

145. Which of the following statements is *correct* ?

(A) Probability measure is countably sub-additive

(B) Distribution function of Binomial (n, p) distribution has n discontinuity points

(C) Power set of Ω is always a finite field

(D) If X is simple random variable, then its set of continuity points is countable set

146. The distribution of a random variable X is given by :

$$F(x) = P[X \le x] = \begin{cases} 0 & \text{if} \quad x < 0 \\ \frac{1}{4} & \text{if} \quad 0 \le x < \frac{1}{4} \\ \frac{1}{2} & \text{if} \quad \frac{1}{4} \le x < \frac{1}{2} \\ \frac{3}{4} & \text{if} \quad \frac{1}{2} \le x < \frac{3}{4} \\ \frac{x+3}{5} & \text{if} \quad \frac{3}{4} \le x < 2 \\ 1 & \text{if} \quad x \ge 2 \end{cases}$$

Then $P\left[\frac{1}{2} \le X \le \frac{3}{4}\right]$ is :

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

147. Which of the following statements is *not* true ?

(A) E(X) may not exist

(B) $E(X+Y)^2 \ge \frac{1}{2}\left\{EX^2 + EY^2\right\}$

(C) If X is integrable, then $|X|$ is also integrable

(D) Moments of even order always exist

148. Suppose that Ω_1 = {1, 2, 3, 4, 5, 6} and that P is a probability measure such that P({*j*}) = 1/6, for *j* = 1, 2, 3, 4, 5, 6. Let I_A and I_B be the indicator functions of the subsets A = {1, 3, 5} and B = {2, 3, 4, 6} respectively. The following are two statements :

(I) I_A and I_B are identically distributed

(II) I_A and I_B are independent random variables

Which of the following is *true* ?

(A) Both (I) and (II) are true

(B) Only (I) is true

(C) Neither (I) nor (II) is true

(D) Only (II) is true

149. Suppose that 12 independent observations on a random variable X are given by 2, 2, 2, 2, 4, 4, 5, 5, 6, 9, 9, 10 and that Y is a random variable with c.d.f. F_{12}, where F_{12} is the empirical distribution function of the above data. Then :

(A) E(Y) = 4.5

(B) $P[Y \leq 6] = 0.75$

(C) P(Y = 7) = P(Y = 6)

(D) $P(Y = 2) \leq P(Y = 9)$

150. The cumulative distribution function F of a random variable X is given by :

$F(x) = 0.2\ \Delta_0(x) + 0.4\ \Delta_1(x) + 0.2\ \Delta_2(x) + 0.2\ \Delta_3(x)$,

where Δ_a is the degenerate distribution function degenerate at 'a'. The following are two statements :

(I) $P(X = 1 | X \leq 1.5) = 2/3$

(II) Variance (X) = 0

Which of the following is *true* ?

(A) Both (I) and (II) are true

(B) Only (I) is true

(C) Only (II) is true

(D) Neither (I) nor (II) is true

151. Suppose that the conditional distribution of X given that N = n is binomial with parameters n and 0.25 and that the marginal distribution of N is geometric with mean 2 and variance 2. Then, variance (X) is equal to :

(A) 1

(B) 10/16

(C) 8/16

(D) Cannot be determined

152. If X is a non-negative random variable, which of the following statements is *not* true ?

(A) $E(X^2) \geq (E(X))^2$

(B) $E(X^3) \geq (E(X))^3$

(C) $E(X^{1/4}) \geq (E(X))^{1/4}$

(D) $E(X^{-2}) \geq (E(X))^{-2}$

153. Suppose $\phi(t) = \exp(-|t|)$ is a characteristic function of a random variable X. Then the distribution of X is :

(A) Normal

(B) Laplace

(C) Student's t with 1 degree of freedom

(D) Gamma

154. Suppose $\{A_n, n \geq 1\}$ is a sequence of events on a probability space (Ω, A, P). Which of the following is always *true* ?

(A) $P(\limsup A_n) = 0 \Rightarrow \sum_{n\geq1} P(A_n) < \infty$

(B) $\sum_{n\geq1} P(A_n) < \infty \Rightarrow P(\limsup A_n) = 0$

(C) $P(\limsup A_n) = 1 \Rightarrow \sum_{n\geq1} P(A_n) = \infty$

(D) $\sum_{n\geq1} P(A_n) = \infty \Rightarrow P(\limsup A_n) = 1$

155. Suppose $\{X_n, n \geq 1\}$ is a sequence of independent and identically distributed random variables each following uniform U(2, 6) distribution. Which of the following is *not* true ?

(A) $X_{(n)}$ converges to 6 in probability

(B) $X_{(1)}$ converges to 2 in probability

(C) $X_{(n)}$ converges in law to exponential distribution

(D) M_n converges in probability to 4, where M_n is the median of $\{X_1, X_2, ..., X_n\}$

156. Suppose X is a real random variable on a probability space (Ω, A, P) and $\{A_n, n \geq 1\}$ is a sequence of events in A such that :

$$A_n \to \Omega \text{ as } n \to \infty.$$

Which of the following is *true* ?

(A) $XI_{A_n} \xrightarrow{P} 0$

(B) $XI_{A_n} \xrightarrow{L} 0$

(C) $XI_{A_n} \xrightarrow{P} X$

(D) $XI_{A_n} \xrightarrow{P} 1$

157. $\{X_n, n \geq 1\}$, $\{Y_n, n \geq 1\}$ and Y are random variables defined on the same probability space. Suppose as $n \to \infty$, X_n converges in probability to 3 and Y_n converges in distribution to Y. Then which of the following is *true* as $n \to \infty$?

(A) $X_n Y_n^2$ converges in probability and in distribution to $3Y^2$

(B) $X_n Y_n^2$ need not converge in probability nor in distribution

(C) $X_n Y_n^2$ converges in probability but not in distribution

(D) $X_n Y_n^2$ need not converge in probability but converges in distribution to $3Y^2$

158. Let $\{X_n, n \geq 1\}$ be independent and identically distributed random variables with mean 0 and variance 1. Which of the following is *true* as $n \to \infty$?

(A) $\sum_{i=1}^{n} X_i / \sqrt{n}$ converges in distribution to a degenerate random variable

(B) $\sum_{i=1}^{n} X_i / \sum_{i=1}^{n} X_i^2$ converges in probability to 0

(C) $\sqrt{n} \sum_{i=1}^{n} X_i$ converges in distribution to a non-zero random variable

(D) $\dfrac{\sum_{i=1}^{n} X_i^2}{\sqrt{n} \sum_{i=1}^{n} X_i}$ converges in probability to 1

159. The following are three statements related to a Markov Chain :

(I) For any state i, $\mu i \geq fii$

(II) For a persistent state i, $\mu i \geq 1$

(III) For a transient state i, $\mu i < 1$

where μi denote the mean recurrence time for state i and fii is the probability of first return to state i.

Which of the following is *true* ?

(A) Only (I) is true

(B) Both (I) and (II) are true

(C) Both (I) and (III) are true

(D) Only (III) is true

160. Suppose f_{ij} denotes the probability of the first visit to j from i in a Markov chain with finite state space. The following are two statements :

(I) If C is a closed communicating class of persistent states, then for any transient state i which leads to $j, k \in$ C, $f_{ij} = f_{ik}$.

(II) If C is a single closed communicating class and if $i \to j$ where $i \notin$ C and $j \in$ C, then $f_{ij} = 1$.

Which of the following is *true* ?

(A) Both (I) and (II) are false

(B) Both (I) and (II) are true

(C) (I) is true but (II) is false

(D) (I) is false but (II) is true

161. Suppose $f_{ii}^{(n)}$ denotes the probability of the first return to i in n steps in a Markov chain. The following are three statements :

(I) For any state i, $f_{ii}^{(n)} \leq p_{ii}^{(n)}$, $n \geq 1$.

(II) For any state i, $\sum_{n \geq 1} p_{ii}^{(n)} \geq 1$.

(III) For a persistent state i, $\Sigma p_{ii}^{(n)} \geq 1$.

Which of the following is *true* ?

(A) All three are true

(B) Both (I) and (II) are true

(C) Both (I) and (III) are true

(D) Only (I) is true

162. The following are four statements about a Markov chain. A unique stationary distribution exists if the Markov chain is irreducible and (I) ergodic (II) non-null persistent, (III) transient and (IV) null persistent.

Which of the following is *true* ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Either (III) or (IV) is true

163. Which of the following is *true* ?

The life time of a component of a machine is modeled by the exponential distribution with mean 2 per week. Failed component is immediately replaced by a new one. Then the probability that no component is replaced in two weeks is :

(A) e^{-2}

(B) e^{-4}

(C) $e^{-0.5}$

(D) e^{-1}

164. Which of the following is *true* ?

A linear death process is a continuous time Markov chain, where :

(A) all states are non-null persistent

(B) all states are transient

(C) 0 is a non-null persistent state and all other states are transient

(D) 0 is a non-null persistent state and all other states are null persistent

165. To find out the prevalence of a virus in a metrocity with population size 10,00,000 a blood test was carried out on 200 randomly selected citizens. Test found 8 positive cases. Then the distribution of number of affected persons in a random sample of size 500 from the population of the same metrocity can approximately be taken as :

(A) Poisson (40)

(B) Poisson (20)

(C) Poisson (8)

(D) Poisson (4)

166. If X and Y are independent N(0, 1) random variables, then E{max (X, Y)} equals :

(A) 1

(B) 3

(C) $\frac{1}{\sqrt{\pi}}$

(D) $\frac{1}{\sqrt{2\pi}}$

167. Suppose that X is a non-negative integer valued random variable having φ as its probability generating function. Then, variance (X) = 1, when :

(A) $\varphi(s) = s/(3-2s), 0 \le s \le 1$

(B) $\varphi(s) = 1/(2-s), 0 \le s \le 1$

(C) $\varphi(s) = (1+s)/2, 0 \le s \le 1$

(D) $\varphi(s) = (s+s^3)/2, 0 \le s \le 1$

168. Suppose that the random vector $(X_1, X_2, X_3)'$ has a multinomial distribution with parameters n, p_1, p_2 and p_3. Then, which of the following statements is FALSE ?

(A) X_1 and X_2 are independent random variables having binomial distributions with parameters n and p_1, and n and p_2 respectively

(B) Covariance $(X_1, X_3) = -n\,p_1p_3$

(C) Variance $(X_3) = n\,p_3(1 - p_3)$

(D) $X_1 + X_2$ has binomial distribution with parameters n and $(1 - p_3)$

169. Suppose that X_1 and X_2 are independent and identically distributed random variables such that X_1 has exponential distribution with scale parameter $1/\lambda$. Let $S_1 = X_1$, $S_2 = X_1 + X_2$. Then, which of the following statements is *true* ?

(A) $E(S_1 | S_2 = 4) = \lambda/2$

(B) $E(S_1 | S_2 = 4) = 2\lambda$

(C) $E(S_1 | S_2 = 4) = 2/\lambda$

(D) $E(S_1 | S_2 = 4) = 2$

170. If $(X, Y)'$ follows bivariate normal distribution with mean vector $(0, 0)'$ and variance-covariance matrix $\Sigma = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$, which of the following does *not* have a χ^2–distribution ?

(A) $X^2 + Y^2$

(B) X^2

(C) Y^2

(D) $4/3\ (X^2 - XY + Y^2)$

171. If one observation X from Bernoulli distribution with parameter $p \in \left[\frac{1}{3}, \frac{2}{3}\right]$ is obtained, then the MLE of p is :

(A) X

(B) $\frac{2X+1}{3}$

(C) $\frac{X+1}{3}$

(D) $\frac{X-1}{3}$

172. Let $X_1, X_2, ..., X_n$ be i.i.d. Bernoulli (p) random variables and $S = \sum_{i=1}^{n} X_i$. Then unbiased estimator of p^3 is :

(A) $\left(\frac{S}{n}\right)^3$

(B) $\frac{S(S-1)(S-2)}{n(n-1)(n-2)}$

(C) $\frac{(S-1)(S-2)(S-3)}{(n-1)(n-2)(n-3)}$

(D) None of the above

173. Let X be random sample of size 1 from U(0, θ). Consider the critical region $C = \{x \mid x > 1\}$ to test $H_0 : \theta = 1$ against $H_1 = \theta = 2$. The size of the test is :

(A) 0

(B) 0.01

(C) 0.05

(D) 0.10

174. Suppose $\{X_1, X_2, ..., X_n\}$, $n \geq 2$ is a random sample from Poisson distribution with mean θ. If $T = \sum_{i=1}^{n} X_i$, then the MVUE of $e^{-\theta}$ is :

(A) $\left(\frac{n}{n-1}\right)^T$

(B) $\left(\frac{n-1}{n}\right)^T$

(C) $\frac{T}{n-T}$

(D) $\frac{n-T}{T}$

175. Suppose that X_1 and X_2 are independent random variables having Bernoulli distribution with mean p and $2p$ respectively, where $0 < p < ½$. The following are two statements :

(I) (X_1, X_2) is minimal sufficient for p

(II) $2X_1 + 5X_2$ is minimal sufficient for p

Which of the following is *true* ?

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Neither (I) nor (II) is true

176. A pharmaceutical company claims that its weight loss drug allows women to lose 8.5 lb after one month of treatment. If we want to conduct an experiment to determine if the patients are losing less weight than advertised, which of the following hypothesis should be used ?

(A) $H_0 : \mu = 8.5, H_a : \mu > 8.5$

(B) $H_0 : \mu = 8.5, H_a : \mu < 8.5$

(C) $H_0 : \mu = 8.5, H_a : \mu \neq 8.5$

(D) $H_0 : \mu > 8.5, H_a : \mu \leq 8.5$

177. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample from $N(\theta, 1)$ distribution. Which of the following is an MVUE of θ^2 ?

(A) $\overline{X}_n^2 - 1$

(B) $\overline{X}_n^2$

(C) $\dfrac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}$

(D) $\overline{X}_n^2 - \dfrac{1}{n}$

178. Which of the following distributions does not belong to the Cramer family ?

(A) Exponential distribution with location parameter $\theta \neq 0$ and scale parameter 1

(B) Cauchy distribution with location parameter θ and scale parameter 1

(C) $N(\theta, \theta)$ $\theta > 0$

(D) Laplace distribution with location parameter 0 and scale parameter θ

179. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample from a Laplace distribution with p.d.f. $f(x, \theta)$ given by :

$$f(x,\theta) = \frac{1}{2\theta} \exp\left(-\frac{|x|}{\theta}\right),$$

$$x \in \mathrm{R}, \ \theta > 0 .$$

Which of the following estimators is consistent for θ ?

(A) $\sum_{i=1}^{n} \frac{|X_i|}{n}$

(B) $\left(\frac{1}{n}\sum_{i=1}^{n} X_i^2\right)^{1/2}$

(C) Sample mean

(D) Sample median

180. Suppose $\{X_1, X_2, ..., X_n\}$ is a random sample from normal $N(\mu, \sigma^2)$ distribution. Suppose M_n is the sample median. Which of the following is *true* ?

(A) $\sqrt{n}\,(M_n - \mu) \xrightarrow{d} Z$ where $Z \sim N(0, 2\sigma^2)$

(B) $\sqrt{n}\,(M_n - \mu) \xrightarrow{d} Z$ where $Z \sim N\left(0, \frac{\pi\sigma^2}{2}\right)$

(C) $\sqrt{n}\,(M_n - \mu) \xrightarrow{d} Z$ where $Z \sim N\left(0, \frac{\pi}{2\sigma^2}\right)$

(D) $\sqrt{n}\,(M_n - \mu) \xrightarrow{d} Z$ where $Z \sim N\left(0, \frac{\sigma^2}{2\pi}\right)$

ROUGH WORK

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