Test Booklet Code & Serial No.
प्रश्नपत्रिका कोड व क्रमांक

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# Paper-II
# MATHEMATICAL SCIENCE

**Signature and Name of Invigilator**

1. (Signature) ........................................

(Name) ...............................................

2. (Signature) ........................................

(Name) ...............................................

Seat No. | | | | | | | |

(In figures as in Admit Card)

Seat No. ............................................................

(In words)

OMR Sheet No. | | | | | | | |

(To be filled by the Candidate)

## SEP - 30221

**Time Allowed : 2 Hours]** **[Maximum Marks : 200**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **190**

**Instructions for the Candidates**

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. This paper consists of **190** objective type questions. Each question will carry *two* marks. Candidates should attempt *all* questions either from sections I & II or from sections I & III only.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   **Example :** where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. Use only Blue/Black Ball point pen.
11. Use of any calculator or log table, etc., is prohibited.
12. There is no negative marking for incorrect answers.

**विद्यार्थ्यांसाठी महत्त्वाच्या सूचना**

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. सदर प्रश्नपत्रिकेत 190 बहुपर्यायी प्रश्न आहेत. प्रत्येक प्रश्नास **दोन** गुण आहेत. विद्यार्थ्यांनी खण्ड I व II किंवा खण्ड I व III मधील **सर्व** प्रश्न सोडविणे अनिवार्य आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चुकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.
   (*iii*) वरीलप्रमाणे सर्व पडताळून पाहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   **उदा. :** जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापि, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.
11. कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.
12. चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.

# Mathematical Science
## Paper II

**Time Allowed : 120 Minutes]** **[Maximum Marks : 200**

**Note :** This Paper contains **One Hundred Ninety (190)** multiple choice questions in THREE (3) sections, each question carrying **TWO (2)** marks. Attempt ***all*** questions either from **Sections I & II** only **or from Sections I & III** only. The OMR sheets with questions attempted from both the Sections viz. **II & III, will not be assessed.**

Number of questions, sectionwise :

**Section I : Q. Nos. 1 to 10,** **Section II : Q. Nos. 11 to 100,** **Section III : Q. Nos. 101 to 190.**

### SECTION I

1. Suppose P($x$, $y$) denotes the joint probability mass function of two random variables X and Y where : P(0, 0) = 0.4, P(0, 1) = 0.2, P(1, 0) = 0.1, P(1, 1) = 0.3.

   What is the conditional probability of X = 0, given that Y = 1 ?

   (A) $\frac{1}{5}$

   (B) $\frac{3}{5}$

   (C) $\frac{2}{5}$

   (D) 0

2. If 3 balls are randomly drawn from a bowl containing 6 white and 5 black balls, what is the probability that one of the balls is white and other two black ?

   (A) $\frac{3}{11}$

   (B) $\frac{4}{11}$

   (C) $\frac{8}{11}$

   (D) $\frac{2}{11}$

3. Which of the following sets are convex sets ?

   (*i*) $\{(x, y) \mid y = x\}$

   (*ii*) $\{(x, y) \mid y \leq |x|\}$

   (*iii*) $\{(x, y) \mid y \geq x^2\}$

   (A) (*i*) only

   (B) (*i*) and (*ii*) only

   (C) (*i*) and (*iii*) only

   (D) (*ii*) and (*iii*) only

4. Which of the following statement(s) is/are true for an LP problem ?

   (I) The set of all feasible solutions to an LP problem is a convex set.

   (II) Every hyperplane is a convex set.

   (A) Both are true

   (B) Only (I) is true

   (C) Only (II) is true

   (D) Neither (I) nor (II) is true

5. Let $(x_n)\ n \in \mathbf{N}$ be a convergent sequence of positive real valued numbers. Which of the following statements is *true* ?

   (A) Limit of the sequence is positive

   (B) Limit of the sequence may be negative

   (C) There is a subsequence of the sequence which has positive limit

   (D) The limit of every subsequence of the sequence is non-negative

6. Let $f : \mathbf{R} \to \mathrm{Q}$ be a non-constant increasing function. Then :

   (A) $f$ is constant on some interval $[a, b]$

   (B) $f$ is one-one

   (C) $f$ is onto

   (D) $f$ is not constant on any interval

7. Which of the following is *true* for a complex number $z = x + iy$ ?

   (A) $|x| + |y| = |z|$

   (B) $|x| + |y| = \sqrt{2}\,|z|$

   (C) $|x| + |y| \geq |z|$

   (D) $|x| + |y| \leq |z|$

8. The function $f : \mathbf{C} \to \mathbf{C}$ defined by $f(z) = \cos z$ is :
   (A) one-one
   (B) onto
   (C) bijective
   (D) bounded

9. Let $R_n [x]$ denote the vector space of all polynomials over $\mathbf{R}$ having degree atmost $n$. Then the dimension of $\mathbf{R}_n [x]$ is :
   (A) $n^2$
   (B) $n + 1$
   (C) $n$
   (D) $\infty$

10. Let V and V' be finite dimensional vector spaces over K and let $\{\sqrt{1}, \ldots\ldots, \sqrt{n}\}$ be a basis of V. If $T \in L(V, V')$ and T is injective, then which of the following statements is FALSE ?
   (A) $T\sqrt{1}, \ldots\ldots, T\sqrt{n}$ are linearly independent
   (B) dim V $\leq$ dim V'
   (C) dim V = dim V'
   (D) Ker T = $\{0\}$

## SECTION II

11. Let $X = \{(x, y) \in \mathbf{R}^2 : (x - n)^2 + y^2 = 1, n \in \mathbf{Z}\}$ be the subspace of the Euclidean space $\mathbf{R}^2$. Then :
   (A) X is compact and connected
   (B) X is connected, but not compact
   (C) X is compact, but not connected
   (D) X is neither connected nor compact

12. Let X be a separable first countable space, then X × X in the product topology is :
   (A) first countable and separable
   (B) first countable, but may not always be separable
   (C) separable and may not always be first countable
   (D) neither always first countable nor always separable

13. The subgroup lattice of a finite cyclic group is :
   (A) finite direct product of chains
   (B) modular but not distributive
   (C) not modular
   (D) not distributive

14. Let G be a simple graph having no isolated vertex and having no vertex induced subgraph with exactly two edges. Then :
(A) G is planar
(B) G is self-complementary
(C) G is complete
(D) G is a tree

15. The coefficient of $x^8$ in $(x^2+x^3+x^4+x^5)^5$ is :
(A) 13
(B) 10
(C) 20
(D) 0

16. Let $u_1$ and $u_2$ be solutions of the equation :

$\Delta u = 0$ in D

$\dfrac{\partial u}{\partial n} = g(s)$ on $\partial$D

where $D = \{(x, y) \in \mathbf{R}^2 / \frac{x^2}{4} + \frac{y^2}{9} \le 1\}$ is an ellipse with boundary $\partial D = \{(x, y) \in \mathbf{R}^2 / \frac{x^2}{4} + \frac{y^2}{9} = 1\}$. If $u_1(0, ½) - u_2(0, ½) = 1$. Then :
(A) $u_1$ and $u_2$ are dependent solutions
(B) $u_1 \equiv u_2 + 1$ in D
(C) $u_1 - u_2$ changes sign in D
(D) $u_1 \equiv u_2$ in D

17. The equation

$$xu_{xx} - yu_{yy} + \frac{1}{2}(u_x - u_y) = 0$$

is hyperbolic in the maximal domain :
(A) $\{(x, y) \in \mathbf{R}^2 / x > 0,\ y > 0\}$
(B) $\{(x, y) \in \mathbf{R}^2 / x < 0,\ y > 0\}$
(C) $\{(x, y) \in \mathbf{R}^2 / xy > 0\}$
(D) $\{(x, y) \in \mathbf{R}^2 / x < 0,\ y < 0\}$

18. The partial differential equation

$(x^2 + z^2)\dfrac{\partial z}{\partial x} - xy\dfrac{\partial z}{\partial y} = z^3 x + y^2$ is :
(A) linear
(B) semi-linear
(C) quasi-linear
(D) non-linear

19. Every natural number is :
(A) a sum of distinct powers of 2
(B) a sum of distinct powers of 3
(C) a sum of distinct factorials
(D) a sum of distinct triangular numbers

20. Which of the following is a solution to the system of congruences :

$x \equiv 2 \pmod 5$
$x \equiv 3 \pmod 4$
$x \equiv 1 \pmod 3$

(A) 22
(B) 27
(C) 47
(D) 67

21. Modulo the prime 101, the number $(50\,!)^2$ is congruent to :
    (A) $-1$
    (B) 1
    (C) 0
    (D) 2

22. Which of the following statements is *correct* ?
    (A) A system of $n$ particles can have infinite degrees of freedom
    (B) A system of $n$ particles with $k$ holonomic constraints has $3n - k$ degrees of freedom
    (C) A system of $n$ particles with $k$ constraints has $n - k$ degrees of freedom
    (D) A system of $n$ particles has 6 degrees of freedom

23. The Lagrangian of a dynamical system is given by :

$$L(q_1, q_2, \dot{q}_1, \dot{q}_2) = \frac{m}{2}(\dot{q}_1^2 + \dot{q}_2^2) + a(q_1\dot{q}_2 - q_2\dot{q}_1)$$

where $m$ and $a$ are constants. Then canonical momenta of the system are given by :
    (A) $p_1 = m\dot{q}_1,\ p_2 = m\dot{q}_2$
    (B) $p_1 = m\dot{q}_1 + aq_2,\ \ p_2 = m\dot{q}_2 - aq_1$
    (C) $p_1 = a\dot{q}_2,\ p_2 = -a\dot{q}_1$
    (D) $p_1 = m\dot{q}_1 - aq_2,\ \ p_2 = m\dot{q}_2 + aq_1$

24. If the Hamiltonian of the dynamical system is

$$H(q_1, q_2, p_1, p_2) = p_1^2 + p_2^2 - k\,q_1^2,$$

then the Lagrangian is given by :
    (A) $L = \frac{\dot{q}_1^2}{2} + \frac{\dot{q}_2^2}{2} + kq_1^2$
    (B) $L = \dot{q}_1^2 + \dot{q}_2^2 - kq_1^2$
    (C) $L = \frac{\dot{q}_1^2}{4} + \frac{\dot{q}_2^2}{4} + kq_1^2$
    (D) $L = \frac{\dot{q}_1^2}{2} + \frac{\dot{q}_2^2}{2} - kq_1^2$

25. Which of the following statements is *correct* ?
    (A) Finite rotation can be represented by a single vector
    (B) Infinitesimal transformation can be represented by a single vector
    (C) The matrix representing infinitesimal transformation is idempotent matrix
    (D) Addition of finite rotations is commutative

26. Identify the velocity field $(u, v)$ which satisfies conservation of mass for incompressible plane flow :
    (A) $u = 3x^2 + y^2,\ \ v = x^3 - 4xy^2$
    (B) $u = x^2,\ \ v = -y^2$
    (C) $u = 2xy - x^2,\ \ v = 2xy - y^2$
    (D) $u = x^2y,\ \ v = xy^2$

27. $u = ax + by$, $v = cx + dy$ where $a$, $b$, $c$, $d$ are non-zero constants are velocity components of a possible incompressible fluid motion only, if :

(A) $a = d$

(B) $a + d = 0$

(C) $b = c$

(D) $b + c = 0$

28. If the velocity potential of the fluid is given by :

$$\phi = x^2 - y^2 + \frac{xy^3}{3} - \frac{x^3 y}{3} + 6,$$

then the velocity components of the fluid flow are :

(A) $u = x^2 y - 2x - \frac{y^3}{3}$,

$$v = 2y - xy^2 + \frac{x^3}{3}$$

(B) $u = 2y - xy^2 - \frac{x^3}{3}$,

$$v = 2x + \frac{y^3}{3} - x^2 y$$

(C) $u = -2y + xy^2 - \frac{x^3}{3}$,

$$v = -2x - \frac{y^3}{3} + x^2 y$$

(D) $u = 2x$, $v = -2y$

29. In two-dimensional fluid flow consider a source of strength $m$ at $z = f$, where $f$ is real, outside the circular cylinder of radius $a$ whose centre is at the origin. The image of the source in a cylinder is :

(A) a sink of strength $m$ at $z = \frac{a^2}{f}$ and a source of strength $m$ at the origin

(B) a source of strength $m$ at $z = \frac{a^2}{f}$ and a sink of strength $m$ at the origin

(C) a sink of strength $m$ at $z = \frac{a^2}{f}$ and a sink of strength $m$ at the origin

(D) a source of strength $m$ at $z = \frac{a^2}{f}$ and a source of strength $m$ at the origin

30. Let $Edu^2 + 2Fdudv + Gdv^2$ and $Ldu^2 + 2Mdudv + Ndv^2$ be the first and second fundamental forms respectively of a surface patch in $\mathbf{R}^3$. If $X = \begin{bmatrix} E & F \\ F & G \end{bmatrix}$ and $Y = \begin{bmatrix} L & M \\ M & N \end{bmatrix}$, then :

(A) $X^{-1}Y$ is non-singular

(B) $X^{-1}Y$ may not be non-singular, but symmetric

(C) $X^{-1}Y$ may not be symmetric, but have real eigen values

(D) $X^{-1}Y$ is non-singular, but not symmetric

31. For any unit speed curve in $\mathbf{R}^3$ with tangent $\overline{t}$, normal $\overline{n}$, binormal $\overline{b}$, curvature $k$ and torsion $\tau$, $\tau \overline{t}\,' + k\overline{b}\,'$ is in :

(A) the linear span of $\overline{t}$ and $\overline{b}$

(B) the linear span of $\overline{t}$ and $\overline{n}$

(C) the linear span of $\overline{b}$ and $\overline{n}$

(D) none of the linear spans of any two vectors in $\{\overline{t}, \overline{n}, \overline{b}\}$

32. The Gaussian curvature of the elliptic cylinder

$$\{(x, y, z) \in \mathbf{R}^3 : \frac{x^2}{2} + \frac{y^2}{3} = 1\}$$

is :

(A) always > 0

(B) always < 0

(C) always = 0

(D) > 0 at some points and < 0 at some points

33. The extremal problem

$$J[y(x)] = \int_0^{\pi} (y'^2 - y^2)dx,\ y(0) = 1,\ y(\pi) = \lambda$$

has :

(A) a unique extremal if $\lambda = 1$

(B) infinitely many extremals if $\lambda = 1$

(C) a unique extremal if $\lambda = -1$

(D) infinitely many extremals if $\lambda = -1$

34. If the integrand $f$ does not depend on $y$, the Euler Lagrange's equation of the functional

$$I[y(x)] = \int_{x_1}^{x_2} f(x, y', y'')\, dx$$

under the conditions that both $y$ and $y'$ are prescribed at the end points has the first integral as :

(A) $\frac{\partial f}{\partial y''} - \frac{d}{dx}\left(\frac{\partial f}{\partial y''}\right) = \text{constant}$

(B) $\frac{\partial f}{\partial y'} - \frac{d}{dx}\left(\frac{\partial f}{\partial y''}\right) = \text{constant}$

(C) $\frac{\partial f}{\partial y'} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = \text{constant}$

(D) $\frac{\partial f}{\partial y''} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = \text{constant}$

35. An extremum of the functional

$$\int_0^1 (y'^2 - 12xy)\, dx,\ y(0) = 0,\ y(1) = 1$$

can occur only along the curve :

(A) $y(x) = x^3$

(B) $y(x) = -x^3 + 2x$

(C) $y(x) = x^3 - 2x$

(D) $y(x) = x^2 - x^3$

36. The curve connecting two given points A and B that do not lie on a vertical line such that a particle sliding down this curve under gravity from the point A reaches point B in shortest time is :

(A) a straight line connecting A and B

(B) a parabola passing through A and B

(C) there is no curve that minimizes time

(D) the cycloid of which one arc contains both the points A and B

37. Let $\lambda_1 \neq \lambda_2$ be the characteristic numbers and $\phi_1, \phi_2$ be the corresponding eigen functions for the homogeneous Fredholm integral equation with separable kernel :

$$u(t) = \lambda \int_0^1 k(t, s)\, u(s)\, ds.$$

Then which of the following is *not* true ?

(A) Fredholm determinant $D(\lambda) = 0$

(B) $\int_0^1 \phi_1(t)\, \phi_2(t)\, dt = 0$

(C) $\phi_1, \phi_2$ are linearly dependent

(D) $\phi_1, \phi_2$ are linearly independent

38. The initial value problem

$$y'' + y = 0, \ y(0) = y'(0) = 0$$

is equivalent to the integral equation :

(A) $y(t) = \int_0^t (t-s)\, y(s)\, ds$

(B) $y(t) = \int_0^t t(t-s)\, y(s)\, ds$

(C) $y(t) = \int_0^t (s-t)\, y(s)\, ds$

(D) $y(t) = \int_0^t t(s-t)\, y(s)\, ds$

39. If $K(t, s)$ is symmetric kernel of the Fredholm integral equation. Define the operator K by :

$$K\phi = \int_a^b K(t, s)\, \phi(s)\, ds$$

and consider the following statements :

(I) K is self-adjoint operator.

(II) Inner product $< K\phi, \phi >$ is always real.

Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are false

40. The third divided difference of the function $f(x) = 1/x$ with arguments $p, q, r, s$ is :

(A) $\frac{1}{pq}$

(B) $\frac{1}{pqr}$

(C) $-\frac{1}{pqrs}$

(D) $\frac{-s}{pqr}$

41. Using Newton's forward interpolation formula, the cubic polynomial which takes the following value

| $x$ : | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f(x)$ : | 1 | 2 | 1 | 10 |

is :

(A) $2x^3 - 7x^2 + 6x + 1$

(B) $2x^3 + 7x^2 - 6x + 1$

(C) $2x^3 + x^2 - 2x + 1$

(D) $2x^3 + 2x^2 + 3x + 1$

42. For a given initial value problem

$$y' = y - x, \ y(0) = 2$$

the value of $y(0.1)$ by Runge-Kutta second order method is :

(A) 2.2100

(B) 2.0050

(C) 2.2050

(D) 2.1900

43. If

$$f(t) = \begin{cases} e^{-xt} g(t), & t > 0 \\ 0 \quad , & t < 0 \end{cases}.$$

Then which of the following is *true* ?

(A) $L\{f(t)\} = e^{s} F\{g(t)\}$

(B) $F\{f(t)\} = sL\{g(t)\}$

(C) $F\{f(t)\} = L\{g(t)\}$

(D) $L\{f(t)\} = F\{g(t)\}$

44. If $\tilde{f}(s)$ is the Fourier transform of $f(t)$, then Fourier transform of $f(t-a)$ is :

(A) $e^{isa} \tilde{f}(s+a)$

(B) $e^{sa} \tilde{f}(s+a)$

(C) $e^{sa} \tilde{f}(s)$

(D) $e^{isa} \tilde{f}(s)$

45. Solution of the Fredholm integral equation

$$\int_0^{\infty} \cos \lambda t \; u(t) \; dt = e^{-\lambda}$$

is :

(A) $\frac{2}{\pi} \frac{1}{(1+t^2)}$

(B) $\frac{2}{\pi} \frac{1}{(1-t^2)}$

(C) $\frac{2}{\pi} \frac{1}{(1+t)^2}$

(D) $\frac{2}{\pi} \frac{1}{(1-t)^2}$

46. Consider the following auxiliary LP problem for two-phase simplex method.

$$\text{Max } Z^* = \sum_{i=1}^{m} (-1) A_i ,$$

where $A_i$'s are artificial variables, subject to

$$\sum_{i=1}^{n} a_{ij} x_j + A_i = b_i , \quad i = 1, 2, ....., m$$

and $x_j, A_i \geq 0$.

If Max $Z^*$ = 0 and at least one artificial variable is present in the basis with positive values then the original L.P. problem has :

(A) no feasible solution

(B) a feasible solution

(C) degenerate feasible solution

(D) cannot say anything about the existence of solution

47. There exists a graph with vertex-connectivity $a$, edge-connectivity $b$ and the minimum degree $c$, if :

(A) $a = 4, b = 2, c = 3$

(B) $a = 4, b = 3, c = 4$

(C) $a = 2, b = 3, c = 4$

(D) $a = 2, b = 4, c = 3$

48. Which of the following statements is true for a graph G ?

(A) Every graph G has a perfect matching

(B) Every regular graph G with even degree $\geq 1$ has a 2-factor

(C) Every bipartite graph G having no pendant vertex has a 2-factor

(D) Line graph of a regular graph G has a 1-factor

49. Consider the signed measure $v$ on [0, 2] defined by $\nu(A) = \int_A f(x)\, dx$

where $f(x) = \begin{matrix} 1 & \text{if} & x \in [0,1) \\ -1 & \text{if} & x \in [1,2] \end{matrix}$.

Then $\nu\left(\frac{1}{2}, \frac{3}{2}\right)$ is :

(A) equal to the Lebesgue measure of (0, 1)

(B) 1

(C) 0

(D) $-\frac{1}{2}$

50. Let (X, M, μ) be a measurable space and $E \in M$ be a measurable set. Let $f$ be a non-negative measurable function with support contained in E. Define $\lambda(A) = \int_{A \cap E} f\, d\mu$.

Which of the following statements is *false* ?

(A) λ is a measure on M

(B) λ is a measure on

$$M(E) = \{A \cap E / A \in M\}$$

(C) λ is absolutely continuous with respect to μ on M(E)

(D) λ is singular with respect to μ on M(E)

51. Let $f : [0, 1] \to \mathbf{R}$ be a continuous function. Then $f$ is absolutely continuous, if :

(A) $f$ is uniformly continuous

(B) $f$ is of bounded variation

(C) $f$ is Riemann integrable

(D) $f'$ exists and is continuous

52. Consider the set X of all sequences $(x_n)$ of natural numbers.

Let $Y = \{(x_n) \in X : x_n = 0$ for infinitely many $n\}$,

$Z = \{(x_n) \in X : x_n = 0$ for infinitely many $n\}$,

$W = \{(x_n) \in X : x_n \neq 0$ for finitely many $n\}$.

Then which of the following sets is countable ?

(A) Y

(B) Z

(C) W

(D) Complement of Z in X

53. Which of the following real numbers $x$, the inequality $(1+x)^n \geq 1+nx$ holds for all $n \in \mathbf{N}$ ?

(A) $x \in \mathbf{R}$

(B) $x \in [-4, 4]$

(C) $x \in (-1, \infty)$

(D) $x \in (-\infty, -4] \cup [0, \infty)$

54. How many solutions are there for the equation $\cos x - e^{-x} = 0$ ?

(A) No solution

(B) Finitely many solutions

(C) Countably infinite solutions

(D) Uncountably many solutions

55. Which of the following statements holds for a function $f : [0, 1] \to \mathbf{R}$ ?

(A) If $f$ is of bounded variation, then $f$ is Riemann integrable

(B) If $f$ is of bounded variation, then it is uniformly continuous

(C) If $f$ is bounded, then it is of bounded variation

(D) If $f$ is uniformly continuous, then it is of bounded variation

56. Consider the following statements :

($a$) Every non-empty open subset of $\mathbf{R}$ is uncountable.

($b$) Every connected subset of $\mathbf{R}$ containing at least two elements is uncountable.

Then :

(A) Only ($a$) is true

(B) Only ($b$) is true

(C) Both ($a$) and ($b$) are true

(D) Both ($a$) and ($b$) are false

57. Which of the following Mobius transformations maps the unit disc onto the right half plane ?

(A) $f(z) = \dfrac{z-i}{z+i}$

(B) $f(z) = \dfrac{z-1}{z+1}$

(C) $f(z) = \dfrac{1+z}{1-z}$

(D) $f(z) = i\left(\dfrac{1+z}{1-z}\right)$

58. Let S be the set of zeros of a non-zero entire function. Then the set S is :

(A) non-empty open

(B) infinite and bounded

(C) closed

(D) uncountable

59. If $f = u + iv$ is analytic on $\mathbf{C}$, then which of the following is FALSE ?

(A) $u$, $v$ are harmonic

(B) $u^2$, $v^2$ are harmonic

(C) $u + v$ is harmonic

(D) $uv$ is harmonic

60. Let $f$ be analytic on $\mathbf{C} - \{0\}$ and let $\nu_1, \nu_2 : [0, 1] \to \mathbf{C} - \{0\}$ be defined by $\nu_1(t) = e^{2\pi i t}$ and $\nu_2(t) = 10\, e^{2\pi i t}$. Suppose $I_1 = \int_{\nu_1} f(z)\, dz$ and $I_2 = \int_{\nu_2} f(z)\, dz$.

Then :

(A) $I_1 = I_2$

(B) $I_1 = 10 I_2$

(C) $I_1 = \dfrac{1}{10} I_2$

(D) There is no relation between $I_1$ and $I_2$

61. Consider the function $f(z) = \dfrac{\cot z}{z}$.

Then :

(A) $f$ has a simple pole at $z = 0$

(B) $f$ has a removable singularity at $z = 0$

(C) $f$ has uncountably many singularities

(D) the residue of $f$ at $z = 0$ is 1

62. Let G be a finite group and H, H' be two subgroups of G. Then :

(A) $H = H' \Leftrightarrow \# H = \# H'$

(B) $H = H' \Leftrightarrow [G : H] = [G : H']$

(C) $H = H' \Leftrightarrow aH = bH'$ for some $a, b \in G$

(D) $H = H' \Leftrightarrow$ both H and H' are normal in G

63. Let I be the ideal in the polynomial ring $\mathbf{Z}_4[X]$ generated by the polynomial $1 + 2X$. Then :

(A) I is a prime ideal

(B) I is a maximal ideal

(C) I is a proper ideal which is not a prime ideal

(D) I is a unit ideal

64. In the group $S_3$, there are exactly :

(A) 2 elements satisfying $x^3 = e$ and 3 elements satisfying $x^2 = e$

(B) 3 elements satisfying $x^3 = e$ and 4 elements satisfying $x^2 = e$

(C) 2 elements satisfying $x^3 = e$ and 4 elements satisfying $x^2 = e$

(D) 3 elements satisfying $x^3 = e$ and 3 elements satisfying $x^2 = e$

65. Let G be a cyclic group of order 20. Then the number of elements of G of order 10 is :

(A) 10

(B) 4

(C) 6

(D) 2

66. Which of the following is an integral domain but not a field ?

(A) $\mathbf{Z}_{10}$

(B) $\mathbf{Z}_{31}$

(C) $\mathbf{R}[x]$

(D) $\mathbf{C}(x)$

67. $A^{35}$ if $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is :

(A) $\begin{pmatrix} 1 & 70 \\ 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 35 \\ 0 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 35 \\ 0 & 2 \end{pmatrix}$

(D) $\begin{pmatrix} 35 & 35 \\ 0 & 35 \end{pmatrix}$

68. Let W be the vector space of real skew symmetric $n \times n$ matrices. Then the dimension of W =

(A) $n^2$

(B) $n^2 - n$

(C) $\frac{n^2 - n}{2}$

(D) $\frac{n^2 + n}{2}$

69. If A is a real $3 \times 3$ matrix, which of the following statements can *not* be true about A ?

(A) Minimum polynomial = characteristic polynomial

(B) A has at least one complex eigen value

(C) Minimum polynomial of A = $x^2 - 1$

(D) Minimum polynomial of A = $x^2 + 1$

70. Matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$ is :

(A) Diagonalizable over **R**

(B) Triangulable over **R** but not diagonalizable over **R**

(C) Triangulable over **C** but not diagonalizable over **C**

(D) Diagonalizable over **C**

71. Let W be the subspace of $\mathbf{R}^4$ given by :

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \Big/ \begin{matrix} x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \\ x_1 - x_2 + x_3 = 0 \end{matrix} \right\}$$

Then $W^{\perp}$ is generated by :

(A) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

72. The partial differential equation obtained by eliminating arbitrary constants from

$$z = ae^{bx} \sin by$$

is :

(A) $\dfrac{\partial^2 z}{\partial y^2} + \dfrac{\partial z}{\partial x} = 0$

(B) $\dfrac{\partial^2 z}{\partial y^2} + \dfrac{\partial^2 z}{\partial x^2} = 0$

(C) $\dfrac{\partial^2 z}{\partial y^2} + x\dfrac{\partial^2 z}{\partial x^2} = 0$

(D) $\dfrac{\partial^2 z}{\partial y^2} + x\dfrac{\partial z}{\partial x} = 0$

73. The solution of the linear partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial y \partial x} + \frac{\partial^2 z}{\partial y^2} = 0$$

is :

(A) $z = \phi_1(x+y) + \phi_2(x-y)$

(B) $z = \phi_1(x+y) + x\phi_2(x-y)$

(C) $z = \phi_1(x+y) + x\phi_2(x+y)$

(D) $z = \phi_1(x+y) + x^2\phi_2(x+y)$

74. If $\phi_1$ and $\phi_2$ are two linearly independent solutions of $y'' + ay = 0$, where $a$ is a constant, on an interval I. Then which of the following statements about the Wronskin $W(\phi_1 \phi_2)(x)$ is true ?

(A) $W(\phi_1, \phi_2)(x)$ is a constant function

(B) $W(\phi_1 \phi_2)(x)$ is a monotonically increasing function

(C) $W(\phi_1 \phi_2)(x)$ is a monotonically decreasing function

(D) $W(\phi_1 \phi_2)(x) = 0$ for all $x \in I$

75. A solution of the differential equation

$$xy' = 2y$$

passing through (1, 2), also passes through :

(A) (2, 1)

(B) (2, 4)

(C) (2, 8)

(D) (2, 16)

76. The differential equation obtained by eliminating arbitrary constants from the expression $y = ae^{bx}$ is :

(A) $y\dfrac{d^2y}{dx^2} - \left(\dfrac{dy}{dx}\right)^2 = 0$

(B) $y\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = 0$

(C) $\dfrac{d^2y}{dx^2} + xy\left(\dfrac{dy}{dx}\right)^2 = 0$

(D) $\dfrac{d^2y}{dx^2} - xy\dfrac{dy}{dx} = 0$

77. Assign the programmers U, V, W to the programmes I, II, III in such way that the total computer time is minimum. The estimated computer time required by the programmers to execute the programmes is given in the table.

| | | **Programmers** | | |
|---|---|---|---|---|
| | | U | V | W |
| | I | 120 | 100 | 80 |
| **Programmes** | II | 80 | 90 | 110 |
| | III | 110 | 140 | 120 |

The optimal assignment is :

(A) $I \to W$, $II \to V$, $III \to U$

(B) $I \to V$, $II \to U$, $III \to W$

(C) $I \to U$, $II \to W$, $III \to V$

(D) Optimal assignment cannot be done

78. A company management and the labour unions are negotiating a new three year settlement. Each of these has 4 strategies mentioned below :
(I) : Hard and aggressive bargaining.
(II) : Reasoning and logical approach.
(III) : Legalistic strategy.
(IV) : Conciliatory approach.
The cost to the company are given for every pair of strategy choice.

| | | Company Strategies | | | |
|---|---|---|---|---|---|
| | | I | II | III | IV |
| **Labour** | I | 20 | 15 | 12 | 35 |
| **Union** | II | 25 | 14 | 8 | 10 |
| **Strategies** | III | 40 | 2 | 10 | 5 |
| | IV | –5 | 4 | 11 | 0 |

Then the value of the game is :
(A) 12
(B) –5
(C) 0
(D) 2

79. Consider the following LP problem.
Max $Z = 6x_1 - 4x_2$
Subject to the constraints :
$$2x_1 + 4x_2 \leq 4$$
$$4x_1 + 8x_2 \geq 16$$
$$x_1, x_2 \geq 0.$$
Then L.P. has :
(A) Infeasible solution
(B) Unbounded solution
(C) Basic feasible solution
(D) Feasible solution

80. Consider the LP problem :
Max $Z = x_1 - x_2 + 3x_3$
Subject to the constraints :
$$x_1 + x_2 + x_3 \leq 10$$
$$2x_1 - x_2 - x_3 \leq 2$$
$$2x_1 - 2x_2 - 3x_3 \leq 6 \text{ and}$$
$$x_1, x_2, x_3 \geq 0.$$
Then the dual of the above LP problem is :
Min W : $py_1 + qy_2 + ry_3$
Subject to the constraints :
$$ay_1 + by_2 + cy_3 \geq 1$$
$$y_1 - y_2 - 2y_3 \geq -1$$
$$y_1 - y_2 - 3y_3 \geq 3 \text{ and}$$
$$y_1, y_2, y_3 \geq 0$$
where the values of $p, q, r, a, b, c$ are :
(A) $p = 10, q = 2, r = -6, a = -1, b = 2, c = -2$
(B) $p = 10, q = 2, r = 6, a = 1, b = 2, c = 2$
(C) $p = 10, q = -2, r = -6, a = -1, b = -2, c = 2$
(D) $p = -10, q = 2, r = 6, a = -1, b = -2, c = 2$

81. The number of basic variables in an $(m \times n)$ transportation table are :
(A) $m + n - 1$
(B) $m + n + 1$
(C) $mn$
(D) $m + n$

82. Consider the function

$$f : [0, 1] \times [0, 1] \to \mathbf{R}$$

defined as :

$$f(x, y) = 1 \text{ if either } x \text{ or } y \text{ is irrational}$$
$$= 0 \text{ otherwise}$$

Then $\int_{[0,1] \times [0,1]} f$ is :

(A) 0

(B) 1

(C) Undefined

(D) 2

83. Consider the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by $f(x, y) = x^2 + y^2 - 4y$. Then in a neighbourhood of (0, 0) :

(A) $x$ variable can be expressed as a function of $y$

(B) $y$ variable can be expressed as a function of $x$

(C) none of $x$ and $y$ be expressed as a function of other variable

(D) both $x$ and $y$ can be expressed as a function of the other variable

84. Consider the function $f : \mathbf{R}^2 \to \mathbf{R}^2$ defined as $f(x, y) = (x^2 - y^2, 2xy)$. Then the image of the ball $B(P, \frac{1}{2})$, where P = (1, 0) is the centre and $\frac{1}{2}$ is radius, is :

(A) a closed set

(B) an open set

(C) neither open nor closed set

(D) an unbounded set

85. Let $f$ be a non-constant entire function. Then which of the following is *impossible* ?

(A) $f(\mathbf{C}) = \mathbf{C}$

(B) $f(\mathbf{C}) = \mathbf{C} - \{0\}$

(C) $\overline{f(\mathbf{C})} = \mathbf{C}$

(D) $f(\mathbf{C}) \subseteq \mathbf{R}$

86. Consider the following statements :

(*a*) $|e^{\sin z}|$ does not assume its maximum value in **C**.

(*b*) $|\sin(e^z)|$ does not assume its minimum value in **C**.

Then :

(A) Only (*a*) is true

(B) Only (*b*) is true

(C) Both (*a*) and (*b*) are true

(D) Both (*a*) and (*b*) are false

87. Let $f : \mathbf{C} \to \mathbf{C}$ be a non-constant analytic such that $|f(z)| \le |z|$ for $|z| \ge 2019$. Then which of the following is *false* ?
(A) $f$ maps a line onto a circle
(B) $f$ is one-one
(C) $f$ is onto
(D) $f'(z) \ne 0$ for all $z \in \mathbf{C}$

88. Let L|K be an algebraic field extension and $\sigma : L \to L$ be a K-algebra homomorphism. Then :
(A) $\sigma$ is bijective
(B) $\sigma$ is injective, but not surjective
(C) $\sigma$ is surjective, but not injective
(D) $\sigma$ is neither injective nor surjective

89. The number of linear factors of the polynomial $X^{16} + 1$ in $\mathbf{R}[X]$ is :
(A) 0
(B) 1
(C) 2
(D) 4

90. The number of irreducible quadratic factors of the polynomial $X^8 + 1$ in $\mathbf{R}[X]$ is :
(A) 4
(B) 3
(C) 2
(D) 1

91. Let X be a metric space and $Y, Z \subset X$ such that $Y \cap Z = \phi$. Then :
(A) $d(Y, Z) > 0$
(B) $d(Y, Z) > 0$ if Y and Z are closed
(C) $d(Y, Z) > 0$ if Y or Z is compact
(D) $d(Y, Z) > 0$ if one of Y or Z is compact and the other is closed in X

92. Let $f, g : \mathbf{R} \to \mathbf{R}$ be defined by
$$f(x) = \sin\frac{1}{x} \quad \text{and} \quad g(x) = x\sin\frac{1}{x}.$$
Then :
(A) $f$ and $g$ are of bounded variation
(B) $f$ is of bounded variation, but $g$ is not of bounded variation
(C) $g$ is of bounded variation, but $f$ is not of bounded variation
(D) neither $f$ nor $g$ is of bounded variation

93. Let X, Y be metric spaces and $f : X \to Y$ be a surjective continuous map. Then :
(A) $f$ maps open sets in X to open sets in Y
(B) $f$ maps closed sets in X to closed sets in Y
(C) $f$ maps dense subsets of X to dense subsets of Y
(D) $f$ maps nowhere dense subsets of X to nowhere dense subsets of Y

94. Let G be a group of order $n$, $p$ a prime such that $p \mid n$. Then :

(A) every Sylow $p$-subgroup of G is normal in G

(B) every Sylow $p$-subgroup of G is abelian

(C) the number of Sylow $p$-subgroups of G is equal to the index of N(P) in G, where N(P) is the normaliser of any Sylow $p$-subgroup P of G

(D) the number of Sylow $p$-subgroups of G is equal to the index of P in G, where P is any Sylow $p$-subgroup of G

95. The number of elements of order 3 in the direct product of groups $\mathbf{Z}_9 \times \mathbf{Z}_3$ is :

(A) 6

(B) 8

(C) 9

(D) 18

96. The field $\mathbf{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ is not equal to :

(A) $\mathbf{Q}(\sqrt{30})$

(B) $\mathbf{Q}(\sqrt{2} + \sqrt{3} + \sqrt{5})$

(C) $\mathbf{Q}(\sqrt{2} + \sqrt{3}, \sqrt{5})$

(D) $\mathbf{Q}(\sqrt{2} + \sqrt{3} - \sqrt{5})$

97. Let X and Y be Banach spaces with $\|\cdot\|_1$ and $\|\cdot\|_2$ respectively denoting their norms. Let $T : X \to Y$ be a bijective bounded linear map. If $\|x\|_3 = \|T_x\|_2$ and $\|y\|_4 = \|x\|_1$, where $T_x = y$, for all $x \in X$ and $y \in Y$. Then :

(A) $\|\cdot\|_3$ and $\|\cdot\|_4$ are norms and complete

(B) $\|\cdot\|_3$ is a complete norm and $\|\cdot\|_4$ is not a norm

(C) $\|\cdot\|_4$ is a complete norm and $\|\cdot\|_3$ is not a complete norm

(D) neither $\|\cdot\|_3$ is a complete norm nor $\|\cdot\|_4$ is a complete norm

98. Let $V_k = \{(a_n) \in l^2 : a_n = 0$ for all $n > k\}$. Then :

(A) $V_k$ is finite dimensional for all $k$ and $l^2$ is a direct sum of $\{V_k \mid k \in \mathbf{N}\}$

(B) $V_k$ is finite dimensional for all $k$ and $l^2$ is a sum of $\{V_k \mid k \in \mathbf{N}\}$ and is not a direct sum

(C) $V_k$ is infinite dimensional for all $k$ and $l^2$ is a sum of $\{V_k \mid k \in \mathbf{N}\}$

(D) $V_k$ is finite dimensional for all $k$ and $l^2$ is not a sum of $\{V_k \mid k \in \mathbf{N}\}$

99. Let X be the space of all continuous complex valued functions on [0, 1] with inner product

$$< f, g > = \int_0^1 f(t)\overline{g(t)}\, dt.$$

Let $T : X \to X$ be defined by $T(f)(t) = t\, e^{it} f(t)$. Then :

(A) T is self-adjoint, but not unitary

(B) T is unitary, but not self-adjoint

(C) T is normal, but neither self-adjoint nor unitary

(D) T is not normal

100. Let $X_i$ denote the set X with topology $\theta_i$ and $Y_i$ denote the set Y with topology $\tau_i$ for $i = 1, 2$. If $f : X_1 \to Y_1$ is continuous, then $f : X_2 \to Y_2$ is continuous provided :

(A) $\tau_1 \subset \tau_2$ and $\theta_1 \subset \theta_2$

(B) $\tau_2 \subset \tau_1$ and $\theta_1 \subset \theta_2$

(C) $\tau_1 \subset \tau_2$ and $\theta_2 \subset \theta_1$

(D) $\tau_2 \subset \tau_1$ and $\theta_2 \subset \theta_1$

## SECTION III

101. Let $(X, Y) \sim N(\underline{0}, \Sigma)$ where

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \quad \rho \neq 0 \quad \text{and}$$

$$W = \frac{X - Y}{X + Y} \sqrt{\frac{1+\rho}{1-\rho}}.$$

Then which of the following statements is *not* correct ?

(A) $W^2 \sim F_{1,1}$

(B) $1/W^2 \sim F_{1,1}$

(C) (X − Y) and (X + Y) are independently distributed

(D) $W \sim \chi_1^2$

102. Let $\underline{X} \sim N_n(\underline{\mu}, \sigma^2 I)$ and A be a $n \times n$ matrix. Then $\underline{Y} = AX$ are uncorrelated normally distributed if and only if which of the following conditions hold ?

(A) A is any non-singular matrix

(B) A is an idempotent matrix with rank < $n$

(C) A is an orthogonal matrix

(D) All elements of A are equal to 1

103. If $S_1$, $S_2$, ......, $S_5$ are the corrected sum of squares and sum of products matrices based on five independent samples of sizes 100 each coming from a four variate normal population. The distribution of $\sum_{i=1}^{5} Si$.
(A) is Wishart with $k = 496$
(B) is Wishart with $k = 495$
(C) is Wishart with $k = 499$
(D) cannot be decided on the basis of given information

104. Let $M \sim W_p(n, \Sigma)$ and $CC' = \Sigma$ where C is a non-singular matrix. Then $C^{-1}MC^{-1}$ is distributed as :
(A) $W_p(n, C)$
(B) $W_p(n, \Sigma^{-1})$
(C) $W_p(n, I_p)$
(D) $W_p(n, C^{-1})$

105. The characteristic function of a vector $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ is given by :
(A) $e^{\underline{\mu}'\underline{t} - \frac{1}{2}\underline{t}'\Sigma\underline{t}}$
(B) $e^{i\underline{\mu}'\underline{t} - \frac{1}{2}\underline{t}'\Sigma\underline{t}}$
(C) $e^{\underline{\mu}'\underline{t} - \frac{i}{2}\underline{t}'\Sigma\underline{t}}$
(D) $e^{i\underline{\mu}'\underline{t} - \frac{i}{2}\underline{t}'\Sigma\underline{t}}$

106. $y_1$, $y_2$, $y_3$, $y_4$ are four uncorrelated random variables with common variance $\sigma^2$ and $E(y_1) = E(y_2) = \theta_1 + 2\theta_2 + \theta_3$, $E(y_3) = E(y_4) = \theta_1 + \theta_3$. Hence :
(A) $\theta_2$ is not estimable
(B) $2\theta_2 + 3\theta_3$ is not estimable
(C) $\theta_1 + \theta_3$ is not estimable
(D) $\theta_1 + \theta_2 + \theta_3$ is not estimable

107. In a simple linear regression set up $y = \beta_a e \beta, X e \varepsilon$, the prediction of response variable $y$ :
(A) does not depend on regressor X
(B) will have same variation irrespective of X
(C) will have larger variation for larger values of X
(D) will have larger variation if the distance between X and mean of regressors increases

108. In a multiple linear regression set up $\underline{Y} = X\underline{\beta} e \underline{\varepsilon}$ under the assumption that the random errors $\varepsilon_1.....\varepsilon_n$ are uncorrelated and homoscedastic the residuals $e_1.....e_n$ are :
(A) correlated and homoscedastic
(B) correlated and heteroscedastic
(C) uncorrelated and homoscedastic
(D) uncorrelated and heteroscedastic

109. In a multiple linear regression model $\underline{Y} = X\underline{\beta}\, e\underline{\varepsilon}$ with $E(\underline{\varepsilon}) = 0$, $Var(\underline{\varepsilon}) = \sigma^2 I$ :
(A) the regression sum of squares and error sum of squares are independent
(B) the regression sum of squares and error sum of squares are positively correlated
(C) the regression sum of squares and error sum of squares are uncorrelated
(D) the regression sum of squares and error sum of squares are negatively correlated

110. Suppose $Y_i = \beta_1 X_{i_1} + \beta_2 X_{i_2} + \varepsilon_i$ where $E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2$ $i = 1, 2, 3$ and $X_{i_1} = i - 4$, $i = 1, 2, 3$ $X_{i_2} = i - 2$, $i = 1, 2$ and $X_{i_2} = i$ for $i = 3$. A least-squares fit is carried out and sum of squares are calculated then :
(A) the sum of squares due to $\beta_1$ and $\beta_2$ are uncorrelated with each other
(B) the sum of squares due to $\beta_1$ and $\beta_2$ are independent of each other
(C) the sum of squares due to $\beta_1$ and $\beta_2$ are correlated with each other
(D) the sum of squares due to $\beta_1$ and $\beta_2$ are uncorrelated with total sum of squares

111. Let N = 4. Two units are selected for inclusion in the sample based on the following design. The $i$-th unit is selected with probability $p_i$ where $p_1$ = 1/6, $p_2$ = 1/6, $p_3$ = 2/3. The second unit is selected randomly from among the remaining three units in the population. Then, the first unit is included in the sample with probability :
(A) 2/9
(B) 4/9
(C) 6/9
(D) 7/9

112. The regression estimator of the sample mean based on a SRSWOR :
(A) is always better than the sample mean
(B) and the sample mean have the same efficiency
(C) is worse than the sample mean
(D) is better than the sample mean if $n$, the sample size, is sufficiently large

113. Let $p$ be the observed proportion of units with the attribute A in a SRSWOR of size $n$ from a finite population size N. Let P be the population proportion and Q = 1 – P. Then, Var($p$) equals :

(A) $\frac{PQ}{n}\frac{N-n}{N-1}$

(B) $\frac{PQ}{n}$

(C) $\frac{PQ}{n}\frac{1}{N-1}$

(D) $\frac{PQ}{N}\frac{N-n}{N-1}$

114. A block design with $v$ treatments and $b$ blocks is connected. Hence the rank of $c$ matrix is :

(A) $v - 1$

(B) $v$

(C) $b - 1$

(D) $b$

115. For a BIBD with parameters $(v, b, r, k, \lambda)$ to exist, the condition $b \geq v$ is :

(A) neither necessary nor sufficient

(B) not necessary but sufficient

(C) necessary but not sufficient

(D) necessary and sufficient

116. A row-column design with 7 treatments arranged in 3 rows and 7 columns is given below :

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 1 |
| 4 | 5 | 6 | 7 | 1 | 2 | 3 |

Hence the design is :

(A) Latin Square Design

(B) Youden Square Design

(C) Quasi-Latin Square Design

(D) Split-Plot design

117. In a $2^3$ factorial design with three treatments A, B, C each at two levels, blocks of 4 plots are available. The design is replicated twice and the allocation of treatment combination is :

| **Replication 1** | | | **Replication 2** | | |
|---|---|---|---|---|---|
| Block | I | II | Block | I | II |
| | 1 | b | | 1 | a |
| | a | c | | b | c |
| | bc | ab | | ac | ab |
| | abc | ac | | abc | bc |

Hence the two confounded treatment combinations in replication 1 and 2 respectively are :

(A) AB and ABC
(B) AB and AC
(C) BC and ABC
(D) BC and AC

118. Let $\{X_t\}$ be a stationary time series with autocovariance function $r(h)$. Then, which of the following statements is *not* true about $r(h)$ ?

(A) $\gamma(0) = \text{Var}\,(X_t)$
(B) $\gamma(k) = \gamma(-k)$
(C) $|\gamma(k)| \geq \gamma(0)$
(D) $\sum_i \sum_j \alpha_i \alpha_j \ \gamma(|t_i - t_j|) \geq 0$ for every set of time points $\{t_1, t_2, ......, t_n\}$ and real members $\{\alpha_1, \alpha_2, ......, \alpha_n\}$

119. Given two time series,

$$X_t = 0.3\, X_{t-1} - 0.2\, X_{t-2} + Z_t$$

$$Y_t - 0.4\, Y_{t-1} = Z_t + 1.2\, Z_{t-1}$$

where $\{Z_t\}$ iid WN(0, $\sigma^2$), which of the following statements is *true* ?

(A) $\{Y_t\}$ is not causal, but invertible
(B) $\{X_t\}$ is not causal
(C) $\{Y_t\}$ is causal, but not invertible
(D) $\{X_t\}$ is not invertible, but causal

120. Let $\{X_t\}$ be a stationary AR(1) model with $X_t = \mu + \phi X_{t-1} + Z_t$, where $Z_t \sim WN(0, \sigma^2)$. Then, which of the following statements is *true* ?

(*i*) $E(X_t) = \mu$
(*ii*) $V(X_t) = \sigma^2 / 1 - \phi^2$
(*iii*) $\text{Corr}(X_t, X_{t+s}) = \phi^s$
(*iv*) $V(X_t) = 1 - \phi^2$

(A) (*ii*) and (*iii*)
(B) (*i*) and (*iii*)
(C) (*i*) and (*ii*)
(D) (*iii*) and (*iv*)

121. Consider a Markov chain on $S = \{1,2,3,4,5\}$ with its transition probability matrix given by :

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 3/4 & 0 & 1/4 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 2/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 2/3 \end{pmatrix} \end{array}.$$

Then :

(A) there exists a unique stationary distribution

(B) there does not exist any stationary distribution

(C) all states are transient

(D) there exist infinitely many stationary distributions

122. Let $\{X(t), t \geq 0\}$ be a time-homogeneous Poisson process with rate $\lambda$. Let T be a positive continuous r.v. distributed independently of $\{X(t), t \geq 0\}$. Then :

(A) $\text{Cov}(X(T), T) = \frac{1}{2}$

(B) $\text{Cov}(X(T), T) = \lambda / (\lambda + E(T))$

(C) $\text{Cov}(X(T), T) = \lambda \, \text{Var}(T)$

(D) $\text{Cov}(X(T), T) < 0$

123. Let the off-spring probability distributions of the three Branching processes be given by :

I. $p_0 = \frac{1}{4}$ $p_1 = \frac{1}{4}$ $p_2 = \frac{1}{2}$

II. $p_0 = \frac{1}{4}$ $p_1 = \frac{1}{2}$ $p_2 = \frac{1}{4}$

III. $p_1 = \frac{1}{4}$ $p_2 = \frac{3}{4}$

Then, the extinction probabilities of the three Branching processes are respectively given by :

(A) 0, 0, 1

(B) ½, 0, 1

(C) ½, 1, 0

(D) ½, ½, ½

124. If all states of a Markov chain are persistent non-null :

(A) it has a unique stationary distribution

(B) it does not have a stationary distribution

(C) $\lim_{n \to \infty} p_{ij}^{(x)} > 0$ for all $i$ and $j$

(D) it has at least one stationary distribution

125. The method of age-standardisation of fertility rates is used to :
   (A) exclude the effect of age distribution in measuring fertility
   (B) exclude effects of differences in fertility levels
   (C) compare fertility in two population groups
   (D) compare birth rates in two populations

126. A column $L_x$ in the life table denotes :
   (A) the number of years lived by the persons in the age interval $(x, x + 1)$
   (B) the number of individuals surviving to age $x$
   (C) the number of individuals surviving to age $x + 1$
   (D) the average number of years of life remaining after age $x$

127. The following sampling plan is to be applied for acceptance or rejection of large lots.

   *"Select two items from the lot and inspect both the items. If both are good, accept the lot, if both are defective, reject the lot; otherwise, select one more item from the lot and accept the lot if and only if this item is good."*

   If the incoming lot quality is 0.8, then the average outgoing quality (AOQ) is equal to :
   (A) 0.512
   (B) 0.896
   (C) 0.800
   (D) 0.640

128. Identify the correct match between the terms in Column-I below with the definitions given in Column-II.

**Column-I**

(*i*) AOQ

(*ii*) AQL

(*iii*) ATI

(*iv*) RQL

**Column-II**

($\alpha$) Number of items per lot that will be inspected on average

($\beta$) A lower bound on the incoming quality for which a lot is to be rejected

($\gamma$) Proportion of defective items remaining in the lot after inspection

($\delta$) An upper bound on the incoming quality for which a lot to be accepted

| | (*i*) | (*ii*) | (*iii*) | (*iv*) |
|---|---|---|---|---|
| (A) | ($\gamma$) | ($\delta$) | ($\alpha$) | ($\beta$) |
| (B) | ($\gamma$) | ($\beta$) | ($\alpha$) | ($\delta$) |
| (C) | ($\beta$) | ($\delta$) | ($\alpha$) | ($\gamma$) |
| (D) | ($\alpha$) | ($\delta$) | ($\gamma$) | ($\beta$) |

129. If the demand distribution is exponential with mean $1/\alpha$, then the optimal inventory level S* is :

(A) $\alpha \log \dfrac{h+p}{h+c}$

(B) $\dfrac{1}{\alpha} \log \dfrac{h+c}{h+p}$

(C) $\dfrac{1}{\alpha} \log \dfrac{h+p}{h+c}$

(D) $\alpha \log \dfrac{h+c}{h+p}$

130. As the variance of service time increases, the waiting time of a customer in M|G|1 queue :

(A) increases

(B) decreases

(C) remains the same

(D) cannot be determined

131. Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates servers 1 and 2 are respectively 8 and 10. A customer at server 1 is equally likely to join server 2 upon completion of service or leave the system. Similarly a customer at server 2, upon completion of the service will go to server 1 with probability 1/4. Then the expected number of customers in the queue waiting for service is :

(A) 7/9

(B) 1/4

(C) 1/5

(D) None of the above

132. The dynamic programming approach to an optimization problem gives the recurrence equation for optimal solution as :

$$f_k(b_k) = \min_{0<x\leq b} \{x^2 + f_{k-1}(b_{k-1})\}$$

for $k > 1$; and $f_1(b_1) = (b_2 - x)^2$.

Then, $f_3(15) =$

(A) 25

(B) 112.5

(C) 75

(D) None of the above

133. Which of the following characteristics of dynamic programming is *not true* ?

(A) The problem can be subdivided into stages

(B) Every stage consists of a number of states

(C) Linear programming problems cannot be solved using dynamic programming

(D) The optimal solution to the problem is obtained using optimal solutions to sub-problems

134. Consider the LPP :

Minimize $Z = 4x_1 + 6x_2 + 18x_3$

Subject to : $x_1 + 3x_2 \geq 3$,

$x_2 + 2x_3 \geq 5$

and $x_1, x_2, x_3 \geq 0$.

Then, which of the following is not a constraint in the corresponding dual problem ? (Assume that $w_1$ and $w_2$ are the dual variables.)

(A) $w_1 \leq 4$

(B) $w_2 \leq 9$

(C) $3w_1 + w_2 \geq 6$

(D) $w_1 \geq 0, w_2 \geq 0$

135. In critical path analysis, CPM is :

(A) event oriented

(B) deterministic in nature

(C) dynamic in nature

(D) none of the above

136. In a class of 100 students, 40 students got 60 marks and 60 students got 40 marks out of 100. Hence the mean, median and mode of the class marks are respectively.
(A) 48, 40 and 60
(B) 48, 40 and 40
(C) 24, 60 and 40
(D) 24, 40 and 60

137. The coefficient of variation of a random variable X is 1.2. If $Y = \frac{X}{4}$, the coefficient of variation of Y is :
(A) 0.3
(B) 0.6
(C) 2.4
(D) 1.2

138. The random variable X follows normal distribution with mean 5 and standard deviation 6. Hence :
(A) $P[X \geq 5] = P[X \leq 6]$
(B) $P[X \leq 5] > P[X \leq 6]$
(C) $P[X \geq 5] < P[X \geq 6]$
(D) $P[X \geq 5] > P[X \geq 6]$

139. Suppose P(A/B) = 0.4 and P(B) = 0.8. Which of the following is a possible value for P(A) ?
(A) 0.3
(B) 0.5
(C) 0.6
(D) 0.1

140. Let X be a random variable with
$$E[|X|^3] = 3$$
Then :
(A) $P[|X| \leq 2] \leq \frac{3}{8}$
(B) $P[|X| \leq 2] \leq \frac{3}{5}$
(C) One cannot say anything about $P[|X| \leq 2]$ from the given information
(D) $P[|X| \leq 2] \geq \frac{5}{8}$

141. Let the joint distribution of (X, Y) be given by :

| Y \ X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | $1/10$ | $1/20$ | $1/20$ | $1/20$ |
| 1 | $3/20$ | $1/5$ | $3/20$ | $5/20$ |

Then $P[X \leq 1]$ is :
(A) $3/20$
(B) $1/20$
(C) $1/2$
(D) $1/5$

142. Which of the following is a characteristic function of a random variable ?

(A) $\Phi(t) = \cos(t^2)$

(B) $\Phi(t) = \cos\left(t/2\right)$

(C) $\Phi(t) = \dfrac{2}{1+2t}$

(D) $\Phi(t) = \dfrac{t}{1+t^2}$

143. Suppose E, F and G are 3 events such that $P(E \cap F \cap G) = P(E).P(F).P(G)$ and $P(E \cap F \cap G) = P(E).P(F \cap G)$.

Consider the statements :

(I) F and G are independent events

(II) E and $F^C \cup G^C$ are independent events

(III) $E^C$ and $F \cap G$ are independent events

(IV) E, F and G are mutually independent events

(A) Only statements I, II and III are correct

(B) Only statements II, III and IV are correct

(C) Only statements II and III are correct

(D) Only statement I is correct

144. Suppose $X_1, X_2, X_3$ are independent and identically distributed random variables each having exponential distribution with mean 1. Then the probability density function of sample range is given by,

(A) $h(r) = e^{-r},\ r > 0$

(B) $h(r) = re^{-r},\ r \in \Re$

(C) $h(r) = (1 - e^{-r})e^{-r},\ r \in \Re$

(D) $h(r) = 2(1 - e^{-r})e^{-r},\ r > 0$

145. The distribution with moment generating function $\dfrac{1}{3-2e^t}$ is :

(A) Exponential distribution

(B) Cauchy distribution

(C) Geometric distribution

(D) Logarithmic series distribution

146. Which of the following statements about an exponential distribution with mean $\lambda$ is true ?

(A) Its variance is $\lambda$

(B) Its moment generating function is $1/(1 - \lambda t)$ exists for all $t \in R$

(C) $E(X) = \int_0^\infty e^{-\lambda x} dx.$

(D) $\bar{F}(t+s) = \bar{F}(t)\bar{F}(s), \quad \forall t, s \in R^+$, where $\bar{F}(x)$ is the survival function of X.

147. Suppose $X_1$, $X_2$, $X_3$ and $X_4$ are independent and identically distributed tri-variate random vectors having normal distribution. Then the distribution of $a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4$, where $a_1$, $a_2$, $a_3$, $a_4$ are real numbers, is :

(A) Univariate normal

(B) Four variate normal

(C) Trivariate normal

(D) Not normal

148. Let $X_1, X_2, ......., X_n$ be independently and identically distributed random variables with $N(\mu, 1)$ distribution. Assume that $\mu \geq 0$. Let $\hat{\mu}$ denote MLE of $\mu$. Then which of the following statements is *true* ?

(A) $\hat{\mu} = \max\{\bar{X}_n, 0\}$

(B) $\hat{\mu}$ is unbiased for $\mu$

(C) The sample mean $\bar{X}_n$ is sufficient for $\mu$

(D) $\hat{\mu}$ may not exist

149. Let X be a single observation from $\theta e^{-\theta x} I_{[0,\infty)}(x)$, $\theta > 0$. Then (X, 2X) is a confidence interval for $1/\theta$ with confidence coefficient :

(A) $\frac{1}{2}$

(B) $e^{-1}$

(C) $e^{-\frac{1}{2}}$

(D) $e^{-\frac{1}{2}} - e^{-1}$

150. Let $X_1$, $X_2$, ........, $X_n$ be a random sample from uniform $\left(\theta - \frac{1}{2}, \theta + \frac{1}{2}\right)$ distribution. Consider the problem of testing $H_0 : \theta = -\frac{1}{2}$ *Vs* $H_1 : \theta = \frac{1}{2}$. Let $X_{(1)} = \min\{X_1, X_2, ......., X_n\}$. Consider the test, reject $H_0$ if $X_{(1)} > 0$ and accept otherwise. Then the power ($p$) and size ($s$) of the test ($p$, $s$) are given by :

(A) (0, 0)

(B) (0, 1)

(C) (1, 0)

(D) (1, 1)

151. Which of the following theorems is useful for obtaining a sufficient statistics ?

(A) Neyman-Pearson theorem

(B) Neyman-factorization theorem

(C) Basu's theorem

(D) Rao-Blackwell theorem

152. Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample from the density $f(x, \theta) = c(\theta)\ h(x)\ e^{\pi(\theta)d(x)},\ \theta \in Ⓗ$.

Let $T(\underset{\sim}{x}) = \sum_{i=1}^{n} d(x_i)$.

Then which of the following statements is *not* true ?

(A) $T(\underline{X})$ is a complete sufficient statistics for $\theta$

(B) $T(\underline{X})$ is uniformly minimum variance unbiased estimator of its expected value

(C) $T(\underline{X})$ is unbiased for $\theta$

(D) The probability distribution depends on $\theta$

153. Suppose $t$ test is applied to test the equality of means of two random variables X and Y, where X and Y are independent. Then :

(A) Distribution of both X and Y must be normal with equal variances

(B) Distribution of both X and Y must be normal

(C) Distribution of X and Y may not be normal

(D) Both X and Y must have the same variance

154. X is a normal $(\mu, \sigma^2)$ random variable and a 95% confidence interval for $\mu$ based on a random sample of size $n$ on X is given by (15, 25]. Hence it can be interpreted that :

(A) The true value of $\mu$ is 20

(B) The true value of $\mu$ lies in (15, 25] with probability 0.95

(C) The true value of $\mu$ is between 15 and 25

(D) The interval (15, 25] covers the true value of $\mu$ with probability 0.95

155. Suppose $r_{12.3}$ is a partial correlation coefficient between $(X_1, X_2)$ and $X_3$. Then which of the following statements is *not* correct ?

(A) $r_{12.3}$ is a correlation coefficient between $X_1 - \hat{X}_1$ and $X_2 - \hat{X}_2$, where $\hat{X}_1$ is a line of best fit based on $X_3$ and $\hat{X}_2$ is a line of best fit based on $X_3$.

(B) $-1 \leq r_{12.3} \leq 1$

(C) $r_{12.3} \geq 0$ always

(D) $r_{12.3} = \dfrac{r_{12} - r_{13}.r_{23}}{\sqrt{\left(1 - r_{13}^2\right)\left(1 - r_{23}^2\right)}}$

156. A frequency data is classified in 10 classes and normal $N(\mu,\sigma^2)$ distribution is fitted to data after estimating the parameters. To use $\chi^2$ goodness of fit test, two classes are combined into one class. Hence the degrees of freedom associated with $\chi^2$ test are :

(A) 9

(B) 8

(C) 7

(D) 6

157. Let $(X_i, Y_i)$ $i = 1, ......, 5$ be the bivariate pair of observations and $\mathbf{Z}_i = X_i - Y_i$ $i = 1, ......, 5$. For a given sample the number of $Z_i > 0$ is 2. Hence the $p$-value to test the null hypothesis that $H_0$ : median (Z) = 0 against $H_1$ : median (Z) < 0 is :

(A) 0.5

(B) 0.25

(C) 0.10

(D) 0.05

158. Suppose that there are 1000 components in operation. The number of failed replacements at the end of each week are as follows :

| W1 | W2 | W3 | W4 |
|---|---|---|---|
| 50 | 82 | 128 | 199 |

If the cost of replacing an individual failed component is ₹ 125 and the cost of group replacement of all 1000 components is ₹ 300, what is the best interval between group replacements ?

(A) One week

(B) Two weeks

(C) Three weeks

(D) Four weeks

159. A newspaper vendor buys papers for ₹ 3 each and sells them for ₹ 5 each. The unsold papers have no value. The daily demand for the paper has the following probability distribution :

| **# of customers** | **Probabilities** |
|---|---|
| 200 | 0.2 |
| 250 | 0.3 |
| 300 | 0.3 |
| 350 | 0.1 |
| 400 | 0.1 |

If each day's demand is independent of the other days, the average profit per day is :

(A) ₹ 75

(B) ₹ 120

(C) ₹ 93

(D) ₹ 50

160. A canteen has a single cash counter. A customer has to buy coupons from the cash counter before getting the food items. Customers arrive at the counter according to a Poisson process with rate of 2 per 5 minutes. It takes 1.5 minutes on average to buy coupons. What is the long-run probability that the cash counter is free ?

(A) $1 - \frac{1.5}{2}$

(B) $1-\frac{1.5}{5}$

(C) $1-\frac{1.5}{2.5}$

(D) 0

161. In a survey of finite population, the following method is applied to obtain a sample of size $n$.

(*i*) In the 1st draw, the $i$th unit is selected with probability $P_i, > 0, i = 1 \ldots\ldots\ldots\ldots N, \sum_{i=1}^{N} Pi = 1$ and $P_i \neq \frac{1}{N}$ for some $i$.

(*ii*) The remaining $(n - 1)$ units are obtained by SRSWOR method from remaining $N - 1$ units of the population.

Let $\bar{Y}$ denote the sample mean, then :

(A) $\bar{Y}$ is an unbiased estimator of population mean

(B) There exists an unbiased estimator of the population mean and it is different from the sample mean

(C) This is a stratified sampling procedure

(D) There does not exist an unbiased estimator of the population mean

162. Let the variances of the regression estimator and the sample mean, both computed from SRSWOR of the same sample size, be denoted by $V^2_{Reg}$ and $V^2_{SRS}$ respectively. Then :

(A) $V^2_{Reg} \leq V^2_{SRS}$ for a large $n$.

(B) $V^2_{Reg} = V^2_{SRS}$

(C) Both the estimators of the population mean are unbiased

(D) $V^2_{Reg} \leq V^2_{SRS}$ for any $n$

163. To examine whether five different skin creams A, B, C, D and E have different effect on the human body, $n$ randomly chosen persons were enrolled in an experiment. The five creams were applied on each of one randomly chosen fingres of the same person; and the procedure is repeated for each person. The resulting design is :

(A) a completely randomized design with 5 treatments

(B) a randomized block design with 5 blocks and $n$ treatments

(C) a randomized block design with $n$ blocks and five treatments

(D) a repeated measures design

164. For one-way classification model the F-test is valid under the assumption that :

(A) The observations on response variable are uncorrelated with common mean and variance

(B) The errors are uncorrelated with common mean and variance

(C) The errors are normally distributed with common mean zero and common positive variance

(D) The errors are independently normally distributed with common mean zero and common +ve variance

165. Under a $2^3$ factorial design with factors A, B, C arranged in a single replicate of two blocks, the containts of block-I (in Yate's notation) are $\{1, a, ab, b\}$. Which of the following effects is confounded with blocks ?

(A) AB

(B) AC

(C) ABC

(D) C

166. Let $\mathbf{F_1}$ and $\mathbf{F_2}$ be two σ-fields of subsets of Ω.

Then which of the following is *not* true :

(A) Both $\mathbf{F}_1 \cap \mathbf{F}_2$ and $\mathbf{F}_1 \cup \mathbf{F}_2$ are σ-fields

(B) $\mathbf{F}_1 \cap \mathbf{F}_2$ is not a σ-field but $\mathbf{F}_1 \cup \mathbf{F}_2$ is a σ-field

(C) Neither $\mathbf{F}_1 \cap \mathbf{F}_2$ nor $\mathbf{F}_1 \cup \mathbf{F}_2$ are σ-fields

(D) $\mathbf{F}_1 \cap \mathbf{F}_2$ is a σ-field but $\mathbf{F}_1 \cup \mathbf{F}_2$ is not a σ-field

167. Let Ω = {1, 2, 3, 4}.

Consider the two σ-fields :

$\mathbf{F}_1 = \{\phi, \Omega, \{1\}, \{2\}, \{1,2\}, \{3,4\}, \{2,3,4\}, \{1,3,4\}\}$

$\mathbf{F}_2 = \{\phi, \Omega, \{1,2\}, \{3,4\}\}$

Let $f$ denote a function from $(\Omega, \mathbf{F}_1)$ to $(\Omega, \mathbf{F}_2)$.

Then :

(A) Any $f$ is measurable

(B) $f$ defined by $f(k) = k$, $k = 1, 2, 3, 4$ is measurable

(C) $f$ defined by $f(1) = f(3) = 1$ and $f(2) = f(4) = 4$ is measurable

(D) $f$ defined by $f(1) = f(2) = f(3) = 1$ and $f(4) = 4$ is measurable

168. Let R denote the real line, **B** the Borel σ-field and $m$ the Lebesgue measure on R :

For $A \in B$, define $\mu(A) = \int_A \exp\left(-\frac{x^2}{2}\right) dm(x)$.

Consider the statements :

(I) μ is a measure on **B**

(II) The Radon-Nikodym derivative of μ with respect to $m$,

$$\frac{d\mu}{dm}(x) = \exp\left(-\frac{x^2}{2}\right) \text{ a.e. } m.$$

(III) $m << \mu$, ($m$ is absolutely continuous with respect to μ).

Which of the above statements are *true* ?

(A) All the three

(B) (I) and (II) only

(C) (I) and (III) only

(D) (I) only

169. Let $\{X_n\}$ be a sequence of random variables and X a random variable such that :

$$\sum_{n=1}^{\infty} P[|X_n - X| > \varepsilon] < \infty \text{ for every}$$

$\varepsilon > 0$.

Consider the statements :

(I) $P\left[\lim_{n\to\infty} X_n = X\right] = 1$

(II) $P\left[\lim_{n\to\infty} X_n^2 = X^2\right] = 1$

(III) $P\left[\lim_{n\to\infty} X_n = X\right]$ only if $\{X_n\}$ is a sequence of independent random variables.

Which of the above are always *correct* ?

(A) (I) only

(B) (I) and (II) only

(C) (III) only

(D) None of the three

170. Let $\{X_n\}$ be i.i.d. random variables with mean zero and finite variance $\sigma^2 > 0$.

Which of the following is *false* ?

(A) $\frac{1}{n}(\exp(X_1) + \exp(X_2) + X_3 + \ldots\ldots + X_n) \to 0$ in probability as $n \to \infty$

(B) $\frac{1}{n^{5/8}}(\exp(X_1) + X_2 + \ldots\ldots. + X_n) \to 0$ in probability as $n \to \infty$

(C) $\frac{1}{n^{3/8}}(X_1 + \ldots\ldots. + X_n)$ does not converge in probability as $n \to \infty$

(D) $\frac{1}{n^{3/8}}(X_1 + \ldots\ldots. + X_n)$ converges in distribution to a r.v. as $n \to \infty$

171. Suppose the sequence of random variables $\{X_n\}$ converges in distribution to X.

Then which of the following is *not* always true ?

(A) $\lim_n E[X_n] = E[X]$

(B) $E[|X|] \le \lim_n \inf E[|X_n|]$

(C) $\lim_n E\left[\frac{|X_n|}{1+|X_n|}\right] = E\left[\frac{|X|}{1+|X|}\right]$

(D) $\lim_n \frac{X_n}{\sqrt{n}} = 0$ in probability

172. Suppose $\{X_n\}$ and $\{Y_n\}$ are sequences of random variables and X and Y are random variables such that as $n \to \infty$, $X_n$ converges in quadratic mean to X and $Y_n$ converges in quadratic mean to Y.

Consider the following statements :

(I) The sequence $(X_n + Y_n)$ converges in quadratic mean to $(X + Y)$ as $n \to \infty$

(II) $\lim_{n\to\infty} P\left[|X_n Y_n - XY| > \varepsilon\right] = 0,$ $\forall \varepsilon > 0$

(III) $\lim_{n\to\infty} E\left[e^{iX_n}\right] = E\left[e^{iX}\right]$

Which of the above are *true* ?

(A) All the three

(B) (I) and (II) only

(C) (I) and (III) only

(D) (II) and (III) only

173. Let $\{X_n\}$ be a sequence of independent random variables.

Let $E = \left\{w \mid \sum_{k=1}^{\infty} X_k(w) \text{ converges}\right\}$

Then which of the following is always *correct* ?

(A) $P(E) = 0$

(B) $P(E) = 1$

(C) $0 < P(E) < 1$

(D) P(E) is either '0' or 1

174. Let $\{Y_n\}$ be a sequence of independent random variables with mean zero and variance 1.

Let $\mathbf{F}_n = \sigma(Y_1, \ldots\ldots, Y_n)$ the σ-field generated by $\{Y_1, Y_2, \ldots\ldots, Y_n\}$

Consider the statements :

(I) The sequence $X_n = Y_1 + Y_2 + \ldots\ldots + Y_n$ is a $\mathbf{F}_n$ - martingale.

(II) The sequence $X_n = \prod_{k=1}^{n} Y_k$ is a $\mathbf{F}_n$ - martingale.

(III) The sequence $X_n = \left(\sum_{k=1}^{n} Y_k\right)^2 - n$ is a $\mathbf{F}_n$ - martingale.

Which of the following is *correct* ?

(A) (I) and (III) only

(B) (I) and (II) only

(C) (II) and (III) only

(D) All the three

175. Suppose a distribution function $F : \mathbf{R} \to [0, 1]$ is as follows :

$$F(x) = \begin{cases} 0, & \text{if} \quad x < 0, \\ 1/4, & \text{if} \quad 0 \le x < 1, \\ 1/2, & \text{if} \quad 1 \le x < 2, \\ 1/2 + (x-2)/2, & \text{if} \quad 2 \le x < 3, \\ 1, & \text{if} \quad x \ge 3. \end{cases}$$

Then $F(x) = l_1F_1(x) + l_2F_2(x)$ where :

(A) $l_1 = 1/4$, $l_2 = 3/4$, $F_1$ is a distribution function of a discrete random variable with support {0, 1} with respective probabilities 1/4, 3/4 and $F_2$ is a distribution function of uniform U(2, 3) distribution.

(B) $l_1 = 1/2$, $l_2 = 1/2$, $F_1$ is a distribution function of a discrete random variable with support {0, 1} with respective probabilities 1/2, 1/2 and $F_2$ is a distribution function of uniform U(2, 3) distribution.

(C) $l_1 = 1/4$, $l_2 = 3/4$, $F_1$ is a distribution function of a discrete random variable with support {0, 1} with respective probabilities 1/2, 1/2 and $F_2$ is a distribution function of uniform U(2, 3) distribution.

(D) $l_1 = 1/4$, $l_2 = 3/4$, $F_1$ is a distribution function of a discrete random variable with support {1, 2} with respective probabilities 1/4, 3/4 and $F_2$ is a distribution function of uniform U(2, 3) distribution.

176. Which of the following statements is *not* correct ? The set of points of discontinuities of a distribution function may be :

(A) empty

(B) finite

(C) uncountable

(D) countable

177. Suppose X is a non-negative random variable whose mean exists. Then :

(A) $E(\log(X)) < \log(E(X))$

(B) $E(\log(X)) \ge \log(E(X))$

(C) $E(\log(X)) = \log(E(X))$

(D) None of (A), (B), (C) is true

178. Suppose $\phi(\cdot)$ is a characteristic function of a random variable with distribution function F. Then :

(A) $F(b) - F(a) =$

$$\lim_{u\to\infty} \frac{1}{2\pi} \int_{-u}^{u} \frac{[\exp(-iua) - \exp(-iub)]\, \phi(u)\, du}{iu}$$

(B) $F(b) - F(a) =$

$$\lim_{u\to\infty} \int_{-u}^{u} \frac{[\exp(-iua) - \exp(-iub)]\, \phi(u)\, du}{iu}$$

(C) $F(b) - F(a) =$

$$\lim_{u\to\infty} \frac{1}{2\pi} \int_{-u}^{u} \frac{[\exp(iua) - \exp(iub)]\, \phi(u)\, du}{iu}$$

(D) $F(b) - F(a) =$

$$\lim_{u\to\infty} \frac{1}{2\pi} \int_{-u}^{u} \frac{[\exp(-iub) - \exp(-iua)]\, \phi(u)\, du}{iu}$$

179. Suppose X has bivariate normal distribution with mean vector (3, 4)' and
$V(X_1) = 2$. Further, it is known that $X_1$ and $X_2$ are positively correlated. Which of the following can be the conditional distribution of $(X_1 | X_2 = 2)$ ?
(A) N(1.5, 2.64)
(B) N(2.6, 1.84)
(C) N(3.5, 1.52)
(D) N(3.5, 2.84)

180. Let $X_1, X_2, ........, X_n$ be *i.i.d.* Bernoulli ($\theta$). Let the loss function for the problem of estimation of $\theta$ be given by
$L(\theta, a) = \frac{(\theta - a)^2}{\theta(1-\theta)}$. Let the pdf of the prior for $\theta$ ($\pi(\theta)$) be given by $\pi(\theta) = 1 \quad 0 \leq \theta \leq 1$.
Let $\bar{X}$ be the sample mean. Then :
(A) $\bar{X}$ is Bayes but not minimax
(B) $\bar{X}$ is not Bayes but it is minimax
(C) $\bar{X}$ is both Bayes and minimax
(D) $\bar{X}$ is not admissible

181. Let $X_1, X_2, ........, X_n$ be iid Bernoulli ($\theta$) random variables. We wish to follow the Bayes procedure for estimation of $\theta$. Then, a conjugate prior for $\theta$ :
(A) is $\pi(\theta) = 1 \quad 0 < \theta < 1$
(B) is $\pi(\theta) \propto \theta^{\alpha - 1} (1 - \theta)^{\beta - 1}$
$\alpha > \theta, \beta > 0$
(C) is $\pi(\theta) \propto \frac{e^\theta}{1 + e^\theta}$
(D) Cannot be decided since the loss function is not given

182. Suppose $X_1, ......, X_n$ are independent and identically distributed random variables each having $U(-\theta, \theta)$ distribution. Then the minimal sufficient statistic for $\theta$ is given by :
(A) $-X_{(1)}$
(B) $X_{(n)}$
(C) max $\{-X_{(1)}, (X_{(n)}\}$
(D) min $\{-X_{(1)}, (X_{(n)}\}$

183. Suppose $\{X_1, X_2, ......, X_n\}$ is a random sample from uniform $U(\theta - 1/2, \theta + 1/2)$ distribution, $\theta \in \mathbf{R}$. Then :

(A) Sample mean is unbiased for $\theta$

(B) Sample median is unbiased for $\theta$

(C) $X_{(1)} + 1/2$ is unbiased for $\theta$

(D) $X_{(n)} - 1/2$ is unbiased for $\theta$

184. Suppose $\{X_1, X_2, ......, X_n\}$ are independent and identically distributed random variables each having $U(-\theta, \theta)$ r.vs. Then the maximum likelihood estimator of $\theta$ is :

(A) $X_{(n)}$

(B) $(X_{(1)} + X_{(n)})/4$

(C) $-X_{(1)}$

(D) $\max \{|X_1|, ...... |X_n|\}$

185. A random variable X follows $N(\mu, 1)$ distribution. A conventional test based on sample mean is used to test the hypothesis $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1 > \mu_0$. Consider the following statements :

(*i*) If $H_0$ is rejected at $\alpha = 0.05$, it is also rejected at $\alpha = 0.10$.

(*ii*) If $H_0$ is accepted at $\alpha = 0.05$, it is also accepted at $\alpha = 0.01$. Then :

(A) (*i*) is correct but (*ii*) is wrong

(B) (*i*) and (*ii*) both are wrong

(C) (*i*) and (*ii*) both are correct

(D) (*i*) is wrong but (*ii*) is correct

186. Suppose $\{X_1, X_2, ......, X_n\}$ is a random sample of size $n$ from uniform $U(\theta - 1, \theta + 1)$ distribution. Then which of the following is *not* true ?

(A) $X_{(n)}$ is consistent for $\theta$

(B) Sample median is consistent for $\theta$

(C) Sample mean is consistent for $\theta$

(D) $X_{(n)} - 1$ is consistent for $\theta$

187. Suppose $X_1, X_2, \ldots\ldots, X_n$ are independent and identically distributed random variables each having exponential distribution with mean $\theta$. Then the asymptotic distribution of $\sqrt{n}\left(\frac{\bar{X}}{S}-1\right)$ where $S^2 = \frac{1}{n}\sum_1^n X_i^2 - (\bar{X})^2$ is :

(A) N(0, 1)

(B) N(0, $\theta$)

(C) N(1, 1)

(D) N(0, $\theta^2$)

188. Let $X_n$ be asymptotically normally distributed with mean $\mu$ and variance $\sigma_n^2$, such that $\sigma_n^2 \to 0$ as $n \to \infty$. Which of the following statements is *not* true ?

(A) For $\mu \neq 0$, $1/X_n$ ~ AN $(1/\mu, \sigma_n^2/\mu^2)$

(B) For any $\mu$, $e^{X_n}$ ~ AN $(e^{\mu}, e^{2\mu}\ \sigma_n^2)$

(C) For $\mu = 0$, $\log\ |X_n|\sigma_n| \xrightarrow{d} \log\ |N(0, 1)|$

(D) For $\mu \neq 0$, $\log\ |X_n|$ ~ AN $(\log\ |\mu|, \sigma_n^2/\mu^2)$

189. Let $X_1, X_2, \ldots\ldots, X_n$ be iid $U(0, \theta)$. Consider $T_1$ = sample median (= $M_n$) and $T_2 = 2\bar{X}$, where $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$.

Then, ARE $(T_1, T_2)$ is :

(A) 3/4

(B) 2/3

(C) 1

(D) 4/3

190. Let $X_1, X_2, \ldots\ldots, X_n$ be iid observations from a continuous distribution F. Define :

$$F_n(x) = \begin{cases} 0 & x < X_{(1)} \\ k/n & X_{(k)} \le x < X_{(k+1)} \\ 1 & x \ge X_{(n)} \end{cases}$$

Then for any fixed $x$,

$\sqrt{n}\ (F_n(x) - F(x)) \xrightarrow{d} Z$, where Z is a normal r.v. with mean 0 and variance :

(A) $F^2(x)/2$

(B) $(1 - F(x))^2$

(C) $F^2(x)$

(D) $F(x)\,(1 - F(x))$

ROUGH WORK

**ROUGH WORK**