Test Booklet Code &No.
प्रश्नपत्रिका कोड व क्र.

C

Paper-III

MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

MAY - 30316

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **146**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. (*a*) This paper consists of **One hundred forty six (146)** multiple choice questions, each question carrying **Two (2)** marks.
   (*b*) Answer to first **Four** questions is compulsory.
   (*c*) Answer any **71** out of remaining **142** questions. In case any candidate answers more than **71** questions, only first **71** questions will be evaluated.
   (*d*) Below each question, four alternatives or responses are given. Only one of these alternatives is the 'CORRECT' answer to the question.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/ questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. (*a*) या प्रश्नपत्रिकेत एकूण **एकशेशेहेचाळीस (146)** बहुपर्यायी प्रश्न दिलेले आहेत, प्रत्येक प्रश्नाला **दोन (2)** गुण आहेत.
   (*b*) पहिले **चार (4)** प्रश्नांचे उत्तर आवश्यक आहेत.
   (*c*) उरलेले **एकशेबेचाळीस (142)** प्रश्नांपैकी कोणतेही **एकाहत्तर (71)** चे उत्तर द्या. जर कोणी परीक्षार्थी **एकाहत्तर (71)** पेक्षा जास्त प्रश्नांचे उत्तर देतो, तर पहिले **एकाहत्तर (71)** प्रश्नांचे मूल्यमापन होईल.
   (*d*) खाली दिलेल्या प्रश्नाचे चार पर्याय किंवा उत्तर दिलेले आहेत. प्रश्नाचे बहुपर्यायी उत्तरामधून केवळ एक 'बरोबर' आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहीलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

Mathematical Science

Paper III

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Note : This paper consists of **One hundred forty six (146)** multiple choice questions, each question carrying **Two (2)** marks. Answer to first **Four** questions is compulsory. Answer any **71** out of remaining **142** questions. In case any candidate answers more than **71** questions, only first **71** questions will be evaluated.

1. The optimal solution of the given linear programming problem is :

Max $z = 3x_1 + 5x_2$

Subject to constraints :

$$x_1 \leq 4$$
$$x_2 \leq 6$$
$$3x_1 + 2x_2 \leq 18 \text{ and}$$
$$x_1, x_2 \geq 0$$

(A) $x_1 = 2, x_2 = 6$

(B) $x_1 = 6, x_2 = 2$

(C) $x_1 = 0, x_2 = 6$

(D) $x_1 = 4, x_2 = 6$

2. Suppose that the two constraints do not intersect in the first quadrant. Consider the following statements :

(1) One of the constraint is redundant

(2) The problem is infeasible

(3) The solution is unbounded

Then :

(A) Only (1) is true

(B) Both (1) and (2) are true

(C) Both (1) and (3) are true

(D) Both (2) and (3) are true

3. For any primal problem and its dual, consider the following statements :

(1) Primal will have an optimal solution if and only if dual does too

(2) Optimal value of the objective functions is same

(3) Both primal and dual cannot be infeasible

Then :

(A) Both (1) and (2) are correct

(B) Only (2) is correct

(C) Only (3) is correct

(D) Both (2) and (3) are correct

4. In a mixed-integer programming problem, consider the following statements :

   (1) All the decision variables require integer solutions

   (2) Few of the decision variables require integer solutions

   (3) Different objective functions are mixed together

   Then :

   (A) Only (1) is correct

   (B) Only (2) is correct

   (C) Only (3) is correct

   (D) All are correct

5. Which of the following spaces is *not* complete ?

   (A) The open interval (0, 1)

   (B) $\mathbf{R}^n$

   (C) $\{0\} \cup \left\{\frac{1}{n} \,\middle|\, n \in \mathbf{N}\right\}$

   (D) $C(\mathbf{R})$

6. The number of adults (X) living in homes on a randomly selected city block is described by the following probability distribution :

X	**Prob.**
1	0.25
2	0.15
3	0.15
4	0.10

   What is the standard deviation of the probability distribution ?

   (A) 0.89

   (B) 0.79

   (C) 0.62

   (D) 0.50

7. Which of the following is *not* a complete space ?

(A) $C[a, b]$, $a, b \in \mathbf{R}$

(B) The space of continuous functions on [0, 1] taking positive values

(C) B(X), the space of bounded real valued functions on an infinite set X

(D) L_1, the space of Lebesgue integrable functions on $\mathbf{R}$

8. A crop insurance company establishes the following loss table based upon previous claims :

Percent loss	**Prob.**
0	0.90
25	0.05
50	0.02
100	?

If they write policy that pays a maximum of \$150/hectare, their expected loss in \$/hectare is approximately :

(A) 5.2

(B) 7.9

(C) 4.5

(D) 25.0

9. Suppose two permutations of a symmetric group have the same orders of their cyclic decompositions. Then they are always :

(A) identical

(B) Inverse of each other

(C) Conjugates of each other

(D) Commute with each other

10. The average length of stay in a hospital is useful for planning purposes. Suppose that the following is the distribution of the length of stay in a hospital after a minor operation :

Days	**Prob.**
2	0.05
3	0.20
4	0.40
5	0.20
6	?

The average length of stay is :

(A) 0.15

(B) 2.25

(C) 4.20

(D) 5.50

11. Let F be a proper subfield of the complex field **C** which is algebraically closed. Then :

(A) F is infinite dimensional over **Q**

(B) F is finite dimensional over **R**

(C) Characteristic of F is non-zero

(D) Such a field does not exist

12. Acme Toy company sells baseball cards in packages of 100. These types of players are represented in each package—rookies, veterans, and All-Stars. The company claims that 30% of the cards are rookies, 60% are veterans, and 10% are All-Stars. Cards from each group are rundomly assigned to packages.

Suppose you bought a package of cards and counted the players from each group. What method would you use to test Acme's claim that 30% of the production run are rookies; 60% veterans; and 10% All-Stars.

(A) Chi-square goodness of fit test

(B) Chi-square test for homogeneity

(C) Chi-square test for independence

(D) One sample t-test

13. Which of the following statements is *not* true ?

(A) Galois extension over a Galois extension is a Galois extension

(B) Separable extension over a separable extension is separable

(C) Algebraic extension over an algebraic extension is algebraic

(D) Inseparable extension over an inseparable extension is inseparable

14. Which of the following would be a reason to use a one-sample t-test instead of one-sample z-test ?

(*i*) The standard deviation of the population is unknown

(*ii*) The null hypothesis involves a continuous variable

(*iii*) The sample size is large (greater than 40)

Codes :

(A) (*i*) only

(B) (*ii*) only

(C) (*iii*) only

(D) (*i*) and (*iii*)

15. Let K be the splitting field of the polynomial $x^3 + \pi x + 6$ over $F = \mathbf{Q}(\pi)$ and K′ be the splitting field of $x^3 + ex + 6$ over $F' = Q(e)$. Then :

(A) $[K : F] \neq [K' : F']$

(B) $[K : F]$ is infinite

(C) $[K : F] = 6$

(D) $[K : F]$ is finite but $[K' : F']$ is infinite

16. Experience has shown that a certain lie detector will show a positive reading (indicates a lie) 10% of the time when a person is telling truth. Suppose that a random sample of 4 suspects is subjected to a lie detector test regarding a recent one-person crime. Then the probability of observing no positive reading if all suspects plead innocent and are telling the truth is :

(A) 0.4090

(B) 0.7350

(C) 0.6561

(D) 0.5905

17. Let

$$F : X \to Y$$

be a linear mapping from a normed linear space X to Y. Consider the following two statements.

(I) F is continuous at a point in X

(II) F is continuous at every point in X

Then :

(A) (I) is always true

(B) (II) is always true

(C) (II) is true only when X is Banach

(D) (II) is true only when (I) is true

18. In a triangle test a tester is presented with three fold samples, two of which are alike, and is asked to pick out the odd one by testing. If a tester has no well developed sense and can pick the odd one only, by chance, what is the probability that in five trials he will make four or more correct decisions ?

(A) $\frac{1}{243}$

(B) $\frac{10}{243}$

(C) $\frac{233}{243}$

(D) $\frac{11}{243}$

19. Let x be an element in a normed linear space X. Then there always exists a bounded linear functional f on X such that :

(A) $f(x) = -1$

(B) $f(x) = 1$

(C) $f(x) = ||x||$

(D) $f(x) = ||x|| + 1$

20. A study was conducted to investigate the effectiveness of a new drug. A group of patients was randomly divided into two groups. One group received the new drug; the other group received a placebo. The difference in mean subsequent surival (patients receiving drugs-patients not receiving drugs) was found to be 1.04 years and a 95% confidence interval was found to be 1.04 ± 2.37 years. Based upon this information :

(A) We can conclude that the drug was effective because those receiving the drug lived, on average, 1.04 years longer

(B) We can conclude that the drug was ineffective because those receiving the drug lived, on average, 1.04 years less

(C) We can conclude that there is no evidence to conclude that the drug was effective because the 95% confidence interval covers zero

(D) We can arrive at no conclusion because we do not know the sample size nor the actual mean survival of each group

21. Let $<T_n>$ be a sequence of continuous linear operators on a Banach space X to a normed linear space Y. Suppose that for each x in X the sequence $<T_n x>$ converges to a value Tx. Then :

(A) T is bounded but need not be linear

(B) T is bounded and linear

(C) T is linear but need not be bounded

(D) T may not be bounded and may not be linear

22. A normal population distribution is needed for the following statistical procedure :

(A) Chi-squared test

(B) Variance estimation

(C) Student's t-test

(D) Kendall's rank coefficient

23. In a separable Hilbert space every orthonormal system is :

(A) infinite

(B) complete

(C) finite

(D) countable

24. In statistics :

(A) null hypothesis describes the probability that a relationship exists between two samples

(B) the mode is the measurement which lies exactly between each end of a range of values ranked in order

(C) skewed data invalidates further statistical analysis

(D) analytical statistics are the same as inferential statistics

25. Let X be locally path connected topological space. Then :

(A) every connected open set in X is path connected

(B) every connected set in X is path connected

(C) every connected closed set in X is path connected

(D) every open set in X is path connected

26. Which of the following holds for Student's *t*-test ?

(A) It can be used to study the effect of an eye drop on intraocular pressure

(B) Its critical value is independent of the degrees of freedom

(C) It can be approximated by the *z*-test

(D) It is especially useful for multivariate analysis

27. The one-point compactification of $\mathbf{R}^2$ is homeomorphic with :

(A) $\mathbf{R}^2$

(B) $\mathbf{R}$

(C) S^2

(D) S^1

28. Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample from $U(0, \theta)$. Let

$$X_{(n)} = \max\{X_1, \ldots\ldots, X_n\}$$

then $X_{(n)}$ is :

(A) unbiased for θ

(B) unbiased and consistent for θ

(C) consistent for θ

(D) UMVUE of θ

29. Which of the following spaces need not be normal ?

(A) Metrizable space

(B) Compact Hausdorff space

(C) Well ordered set with order topology

(D) Product of two normal spaces

30. The pdf

$$f(x) = \frac{1}{2} e^{-|x - \theta|} \quad -\infty < x < \infty$$

has :

(A) MLR in x

(B) MLR in $-x$

(C) MLR in x^2

(D) MLR in $|x|$

31. Which of the following statements is not equivalent to any two of the remaining statements for a topological space X, where one point sets are closed ?

(A) X is completely regular

(B) X is metrizable

(C) X is homeomorphic to a subspace of a compact Hausdorff space

(D) X is homeomorphic to a subspace of a normal space

32. Suppose X and Y are independent r.v.s. each with a normal distribution, and

$$E(X) = \theta, \; E(Y) = 3\theta,$$

$$V(X) = V(Y) = 1.$$

Let $\hat{\theta}$ be MLE of θ based on X and Y. Then :

(A) $\hat{\theta}$ is biased and $V(\hat{\theta}) = \frac{1}{100}$

(B) $\hat{\theta}$ is unbiased and $V(\hat{\theta}) = \frac{10}{100}$

(C) $\hat{\theta}$ is biased and $V(\hat{\theta}) = \frac{10}{100}$

(D) $\hat{\theta}$ is unbiased and $V(\hat{\theta}) = \frac{1}{100}$

33. Let L be a lattice. Consider the following statements :

(1) Any two elements in L are comparable

(2) Any two elements in L are incomparable

(3) Every ideal of L is a maximal ideal

(4) Every ideal of L is a prime ideal

Then L is a chain iff :

(A) Any of (1) and (4) holds

(B) Both (1) and (3) hold

(C) Any of (2) and (3) holds

(D) Both (3) and (4) hold

34. A test ϕ is said to be similar if :

(A) $E_{HO}[\phi(X)] = \alpha$

(B) $E_{HO}[\phi(X)] \leq \alpha$

(C) $E_{HO}[\phi(X)] \geq \alpha$

(D) $E_{HO}[\phi(X)] > \alpha$

35. Consider the set N of natural numbers. Let ρ be a binary relation on N defined by $a \rho b$ iff a divides b.

Consider the following statements :

(1) ρ is a partial ordering relation on N

(2) N is a lattice under the relation ρ

(3) N is a chain under the relation ρ

(4) N is a complete lattice under the relation ρ

Then :

(A) Only (1) is true

(B) Both (2) and (3) are true

(C) None of (1) to (4) is true

(D) Both (1) and (2) are true

36. Significance of partial correlation coefficient is tested using :

(A) F-test

(B) t-test

(C) chi-square test

(D) z-test

37. Let R be a commutative ring. Let I(R) be the set of all ideals of R. Which of the following statements is necessarily *true* ?

(A) I(R) is a distributive lattice

(B) I(R) is the empty set

(C) I(R) is a modular lattice

(D) I(R) is not a lattice

38. Let $\{T_n\}$ be a sequence of unbiased consistent estimators of θ, then :

(A) $\left\{T_n^2\right\}$ is a sequence of unbiased consistent estimators of θ^2

(B) $\left\{T_n^2\right\}$ is a sequence of unbiased inconsistent estimators of θ^2

(C) $\left\{T_n^2\right\}$ is a sequence of biased consistent estimators of θ^2

(D) $\left\{T_n^2\right\}$ is a sequence of biased and inconsistent estimators of θ^2

39. Let G be a graph. Consider the following statements :

(1) G is acyclic

(2) There is a unique path between every pair of vertices in G

(3) G is connected and every edge is a cut edge

(4) G contains a cycle

Then G is a tree iff :

(A) Either (2) or (3) holds

(B) (3) does not hold

(C) (2) does not hold

(D) (1) holds

40. In a sample of size 100 there were 50 smokers and 50 non-smokers. Out of 50 smokers 30 suffered from cancer and out of 50 non-smokers 20 sufferred from cancer. The value of chi-square statistic to test the association between smokers and cancer is :

(A) 4

(B) 10

(C) 4.16

(D) 1

41. Let R be the region defined by

$$R : |x| \leq a, \ |y| \leq b.$$

Then the function

$$f(x, y) = x \sin y + y \cos x$$

satisfies the Lipschitz condition with Lipschitz constant K equal to :

(A) a

(B) $a + 1$

(C) b

(D) $a + b$

42. If $x_1, x_2, ..., x_n$ is a sample from the Cauchy distribution with location parameter θ. Q_i denotes ith quartile $i = 1, 2, 3$. Which of the following is a consistent estimator of θ.

(A) Sample mean

(B) Q_2

(C) Q_1

(D) Q_3

43. The integral surface satisfying the partial differential equation

$$xp + yq = z$$

and passing through the circle

$$x^2 + y^2 + z^2 = 4$$

$$x + y + z = 2$$

is :

(A) $x + y + z = 0$

(B) $xy + yz + zx = 0$

(C) $x^2 + y^2 + z^2 = 0$

(D) $\frac{x}{y} + \frac{y}{x} + z = 0$

44. Multiple correlation between X_1 and (X_1, X_3) is :

(A) $\max(\rho_{12}, \rho_{13})$

(B) Correlation coefficient between X_1 and best linear predictor of X_1 based on X_2 and X_3

(C) Correlation coefficient between X_1 and $X_2 + X_3$

(D) Correlation coefficient between X_1 and best predictor of X_1 based on X_2 and X_3

45. Let a string of uniform density ρ be stretched to a length l and fixed at points $x = 0$ and $x = l$ and executes a small transverse vibrations. Then the total kinetic energy of the string is given by :

(A) $T = \frac{\rho}{2} \int_0^l \left[\left(\frac{\partial y}{\partial x} \right)^2 + \left(\frac{\partial y}{\partial t} \right)^2 \right] dx$

(B) $T = \frac{\rho}{2} \int_0^l \left(\frac{\partial y}{\partial x} \right)^2 dx$

(C) $T = \frac{\rho}{2} \int_0^l \left(\frac{\partial^2 y}{\partial x^2} \right) dx$

(D) $T = \frac{\rho}{2} \int_0^l \left(\frac{\partial y}{\partial t} \right)^2 dx$

46. Let z_1 and z_2 be independent standard normal variables. Let

$$Y_1 = Z_1 Z_2^2.$$

The correlation coefficient between Z_1 and Y_1 is :

(A) 0

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{5}}$

(D) $\frac{1}{\sqrt{3}}$

47. If $J_n(x)$ and $J_{-n}(x)$ be Bessel functions of first and second kinds. Consider the following relations :

(1) $J_{-n}(x) = (-1)^n \ J_n(x)$

(2) $J_n(-x) = (-1)^n \ J_n(x)$

(3) $J_{-n}(x) = J_n(-x)$

Then :

(A) Only (3) is true

(B) Only (1) and (2) are true

(C) Only (2) and (3) are true

(D) All (1), (2) and (3) are true

48. Suppose (X, Y) has Bivariate normal distribution with parameters

$$\mu_1 = \frac{1}{2},\ \mu_2 = \frac{3}{2},$$

$$\sigma_1^2 = 4,\ \sigma_2^2 = 9,\ \rho = \frac{1}{2}.$$

Then correlation coefficient between

$$\frac{1}{2}X - \frac{1}{2} \text{ and } \frac{1}{3}Y - \frac{1}{2}$$

is :

(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{1}{2}$

49. Which of the following is a primitive root of 23 ?

(A) 1

(B) 2

(C) 3

(D) 4

50. Let $(X_1, X_2, X_3)'$ follow three-variate Normal distribution with covariance matrix Σ, where :

$$\Sigma = \begin{bmatrix} 3.0 & 0.5 & 0.6 \\ 0.5 & 2.0 & 0.6 \\ 0.6 & 0.6 & 4.0 \end{bmatrix}$$

Let Y_i denote ith principal component of Σ, $i = 1, 2, 3$. Then correlation coefficient between (Y_1, Y_2) is :

(A) $\frac{2}{9}$

(B) $\frac{4}{9}$

(C) 0

(D) $\frac{3}{9}$

51. For which of the following prime numbers p, both 2 and -1 are quadratic residues modulo p ?

(A) 17

(B) 7

(C) 11

(D) 13

52. Given that we have collected pairs of observations on two variables X and Y, we would consider fitting a straight line with X as an explanatory variable if :
    (A) the change in Y is a constant for each unit change in X
    (B) The change in Y is an additive constant
    (C) The change in Y is a fixed percent of Y
    (D) The change in Y is exponential

53. Which of the following integers can be written as a sum of two squares ?
    (A) 720
    (B) 39
    (C) 56
    (D) 143

54. For children, there is approximately a linear relationship between "height" and "age". One child was measured monthly. Her height was 75 cm at 3 years of age and 85 cm when she was measured 18 months later. A least square line was fit to her data. The slope of this line is approximately :
    (A) 0.55 cm/m
    (B) 10 cm/m
    (C) 25 cm/m
    (D) 1.57 cm/m

55. $\sum_{d/108} \phi(d)$ is equal to :
    (A) 216
    (B) 100
    (C) 108
    (D) 156

56. The yield of a grain, $Y\left(\frac{\text{t}}{\text{ha}}\right)$, appears to be linearly related to the amount of fertilizer applied, $X\left(\frac{\text{kg}}{\text{ha}}\right)$. An experiment was conducted by applying different amounts of fertilizer (0 to 10 kg/ha) to plots of land and the resulting yields were measured. The following estimated regression line was obtained.

$$Y = a + bX$$

Which of the following is *not* true ?

(A) If no fertilizer was used, the yield is estimated to be 4.85 $\frac{\text{t}}{\text{ha}}$

(B) If fertilizer is applied at 10 $\frac{\text{kg}}{\text{ha}}$, the estimated yield is 5.35 $\frac{\text{t}}{\text{ha}}$

(C) If the current level of fertilizer is changed from 7.0 to 9.0 $\frac{\text{kg}}{\text{ha}}$ the yield is estimated to increase by 0.20 $\frac{\text{t}}{\text{ha}}$

(D) To obtain an estimated yield of 5.2 $\frac{\text{t}}{\text{ha}}$, you need to apply 7.0 $\frac{\text{kg}}{\text{ha}}$ of fertilizer

57. If L is a Lagrangian of a particle, the Lagrange's equations of motion are given by :

(A) $\dot{q}_j = \frac{\partial L}{\partial q_j}$

(B) $\frac{\partial L}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = 0$

(C) $\frac{\partial L}{\partial t} - \frac{d}{dt}\left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = 0$

(D) $\frac{\partial L}{\partial q_j} - \frac{d}{dt}\left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = 0$

58. In single factor ANOVA, MSTr is the mean square for treatments, and MSE is the mean square error. Which of the following statements are *not* true ?

(A) MSE is a measure of within sample variation

(B) MSTr is a measure of between sample variation

(C) MSE is a measure of between sample variation

(D) The value of MSTr is affected by the status of (true or false)

59. The Hamiltonian of a system represents total energy when it is :

(A) Non-conservative and scleronomic

(B) Conservative and rheonomic

(C) Conservative and scleronomic

(D) Conservative and kinetic energy is a homogeneous quadratic function of generalised velocities

60. In stratified sampling, the strata :

(A) are equal to each other in size

(B) are proportionate to the units in the target population

(C) are disproportionate to the units in the target population

(D) Can be proportionate or disproportionate to the units in the target population

61. Let

$$A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

be the matrix of orthogonal transformation, then its inverse is given by :

(A) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \end{pmatrix}$

(C) $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta \\ 0 & 1 & 0 \end{pmatrix}$

62. Cluster sampling is more efficient than simple random sampling if :

(A) Clusters are small and homogeneous

(B) Clusters are formed by grouping similar sampling units

(C) Clusters are internally heterogeneous

(D) Clusters are formed randomly

63. A particle is thrown horizontally from the top of a building of height h with initial velocity u, then :

(A) angular momentum of the particle is conserved

(B) linear momentum along horizontal direction is conserved

(C) linear momentum along vertical direction is conserved

(D) neither linear momentum nor angular momentum is conseved

64. Ratio estimator is more efficient than the sample mean if :

(A) The sample ratio is small

(B) The concomitant variable is strongly and positively correlated with the variable of interest

(C) The concomitant variable is independent of the variable of interest

(D) The variable of interest is positive

65. Which of the following is *not* a surface ?

(A) $x^2 + y^2 = z^2$

(B) $x^2 + y^2 + z^2 = 1$

(C) $\frac{x^2 - y^2}{z} = 4$

(D) $(x^2 + y^2)^2 + 3z^2 = 1$

66. Horvitz-Thompson estimator has a variance smaller than the sample mean because :

(A) Horvitz-Thompson estimator assumes probabilities proportional to the size of sampling units

(B) Horvitz-Thompson estimator assumes simple random sampling for selection of the sample

(C) Horvitz-Thompson estimator assigns unequal weights to sampling units

(D) Horvitz-Thompson estimator is similar to ratio estimator

67. Let I be an open interval in the real line **R**. A curve $\alpha : \mathrm{I} \to \mathrm{E}^3$ in 3-dim. Euclidean space, defined by :

$$\alpha(s) = \left(\frac{4}{5}\cos(s),\ 1 - \sin(s),\ -\frac{3}{5}\cos(s)\right)$$

represents :

(A) a parabola

(B) a hyperbola

(C) an ellipse

(D) a circle

68. Consider a symmetric BIBD with parameters $v = b = 7$, $r = k = 3$ and $\lambda = 1$. If C denotes the incidence matrix of the design, then det(C) is :

(A) 12

(B) 36

(C) 48

(D) 24

69. For a unit speed curve c : $(x_1(s), x_2(s))$ in E^2, the unit normal vector N is given by :

(A) $(x'_2(s), x'_1(s))$

(B) $(x'_2(s), -x'_1(s))$

(C) $(-x'_2(s), x'_1(s))$

(D) $(-x'_2(s), -x'_1(s))$

70. If

[(1), *ae*, *abc*, *bce*, *acd*, *cde*, *bd*, *abde*]

is the key block of a 2^5 factorial experiment, then the confounded interactions are :

(A) *ace*, *bed*, *abcd*

(B) *abc*, *cde*, *abcd*

(C) *ace*, *bcd*, *abde*

(D) *ade*, *bcd*, *abce*

71. A curve α in $M \subset E^3$ is a geodesic of M if :

(A) its acceleration vector is always tangent to the surface M

(B) its acceleration vector is always normal to the surface M

(C) its acceleration vector is zero

(D) its acceleration vector and velocity vector are orthogonal

72. The total number of Latin squares that can be obtained from a 4×4 square is :

(A) 16

(B) 144

(C) 24

(D) 64

73. The first integral of the Euler-Lagrange's differential equation of the functional

$$f = f(y, y')$$

is :

(A) $\dfrac{\partial f}{\partial y} - \dfrac{d}{dx}\left(\dfrac{\partial f}{dy'}\right) = k$

(B) $\dfrac{\partial^2 f}{\partial y'^2} = k$

(C) $f - y' \dfrac{\partial f}{\partial y'} = k$

(D) $y' \dfrac{\partial f}{\partial y'} = k$,

where k arbitrary constant

74. The total number of factorial effects in a 2^5 factorial experiment is :

(A) 10

(B) 32

(C) 31

(D) 5

75. Let $y = y(x)$ be the extremal of the functional

$$I(y(x)) = \frac{1}{2}\int_0^2 (y'')^2\, dx$$

and satisfy

$$y(0) = 1,\ y'(0) = 1,$$

$$y(2) = 1,\ y'(2) = 0,$$

then :

(A) $y = -\dfrac{x^2}{4}$

(B) $y = -\dfrac{x^2}{4} + x + 1$

(C) $y = \dfrac{x^2}{4} + 1$

(D) $y = \dfrac{x^2}{4} - x$

76. Consider a time series

$X_t = \sin(2\Pi\ U_t),\ t = 1, 2, \ldots..$

where $U_t \sim$ uniform (0, 1). Then, which of the following statements are *correct* ?

(*i*) $\{X_t\}$ is weakly stationary

(*ii*) $\{X_t\}$ is strictly stationary

(*iii*) $\{X_t\}$ is a periodic time series

(*iv*) $\{X_t\}$ has infinite variance

Codes :

(A) (*i*) and (*ii*)

(B) (*i*) and (*iii*)

(C) (*ii*) and (*iv*)

(D) (*iii*) and (*iv*)

77. The condition(s) for the extremal of the integral

$$I(y(x)) = \int_{x_1}^{x_2} f(x, y, y')\,dx$$

where y is not prescribed at the end points is (are) :

(A) $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{dy'}\right) = 0$

(B) $f - y' \frac{\partial f}{\partial y'} = 0$

(C) $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ and

$\left(\frac{\partial f}{\partial y'}\right)_{x = x_1} = 0$

(D) $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0,$

$\left(\frac{\partial f}{\partial y'}\right)_{x=x_1} = 0,\ \left(\frac{\partial f}{\partial y'}\right)_{x=x_2} = 0$

78. Consider a stationary time series Y_t given by

$$Y_t - \phi\, Y_{t-1} = \mu + Z_t - \theta\, Z_{t-1},$$

where Z_t is a white noise process with mean zero and variance σ^2 ? Which of the following statements is *true* ?

(A) $E(Y_t) = \mu$, $\mathrm{Var}(Y_t) = \dfrac{\sigma^2}{(1-\phi^2)}$

(B) $E(Y_t) = \dfrac{\mu}{(1-\phi)}$,

$$\mathrm{Var}(Y_t) = \frac{\sigma^2}{(1-\phi^2)}$$

(C) $E(Y_t) = \dfrac{\mu}{(1-\theta)}$,

$$\mathrm{Var}(Y_t) = \frac{1 - 2\phi\,\theta + \phi^2}{1-\theta^2}\sigma^2$$

(D) $E(Y_t) = \dfrac{\mu}{(1-\phi)}$,

$$\mathrm{Var}(Y_t) = \frac{1 - 2\phi\,\theta + \theta^2}{1-\phi^2}\sigma^2$$

79. The extremal of the functional

$$I = \int_{\theta_0}^{\theta} \sqrt{r^2 + r'^2}\, d\theta,\ r^1 = \frac{dr}{d\theta}$$

is :

(A) a catenary

(B) a straight line

(C) an arc of the great circle

(D) a helix

80. Suppose Z_t's are iid random variables with mean zero and variance one. Let

$$X_t = \begin{cases} Z_t & \text{if } t \text{ is even} \\ \left(Z_{t-1}^2 - 1\right)/\sqrt{2} & \text{if } t \text{ is odd} \end{cases}$$

Which of the following statements are *correct* ?

(*i*) X_t is an iid (0, 1) sequence

(*ii*) X_t is a white noise (0, 1) sequence

(*iii*) The autocorrelation function ρ_h of X_t vanishes for all h, except at $h = 0$

(*iv*) X_t is a weak stationary process

Codes :

(A) (*i*), (*ii*) and (*iii*)

(B) (*ii*), (*iii*) and (*iv*)

(C) (*i*), (*iii*) and (*iv*)

(D) (*i*), (*ii*) and (*iv*)

81. The integral equation :

$$x(t) = \sin t + \int_0^t e^{(t-s)} x(s)\, ds,$$

is a :

(A) linear integral equation of Volterra type with convolution kernel

(B) linear integral equation of Fredholm type with convolution kernel

(C) linear integral equation of Volterra type with symmetric kernel

(D) linear integral equation of Fredholm type with symmetric kernel

82. Consider a random walk process

$$X_t = \mu + X_{t-1} + Z_t,$$

where Z_t ~ white noise $(0, \sigma^2)$, then which of the following statements is *true* ?

(A) $\{X_t\}$ is a non-stationary process

(B) $\{X_t\}$ is a stationary process

(C) $\{X_t - t_\mu\}$ is a stationary process

(D) $\dfrac{\{X_t - t_\mu\}}{\sqrt{t}}$ is a stationary process

83. The solution of the integral equation

$$x(t) = \sec^2 t - 1 + \int_0^{\pi/4} x(s)\, ds,$$

satisfies :

(A) $x(0) + x\left(\frac{\pi}{4}\right) = 3$

(B) $x\left(\frac{\pi}{3}\right) + x\left(\frac{\pi}{6}\right) = 2$

(C) $x(0) + x\left(\frac{\pi}{6}\right) = 1$

(D) $x\left(\frac{\pi}{3}\right) + x\left(\frac{\pi}{4}\right) = 0$

84. The transition probability matrix of a three-state Markov chain with states 0, 1 and 2 is :

$$P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

Also, $P[X_0 = i] = \frac{1}{3}$, $i = 0, 1, 2.$

Then $P[X_2 = 2, X_1 = 1 | X_0 = 2]$ equals :

(A) $\frac{3}{16}$

(B) $\frac{3}{64}$

(C) $\frac{3}{4}$

(D) $\frac{1}{4}$

85. The solution of integral equation

$$\pi = \int_0^t \frac{1}{\sqrt{t-s}} x(s)\, ds$$

is :

(A) t

(B) $\frac{1}{t}$

(C) $\frac{1}{t^2}$

(D) $\frac{1}{\sqrt{t}}$

86. If

$$\{N(t),\ t \geq 0\}$$

is a Poisson process and

$$X = T_n - T_{n-1},$$

where T_n and T_{n-1} are two consecutive epochs at which occurrences of Poisson process take place, then the distribution of X is :

(A) Poisson

(B) Binomial

(C) Geometric

(D) Exponential

87. The solution of the Volterra integral equation of first kind

$$f(t) = \int_0^t e^{t-s} x(s)\, ds$$

where $f(t)$ is known function with $f(0) = 0$ is :

(A) $f(t)$

(B) $f'(t)$

(C) $f'(t) - f(t)e^t$

(D) $f'(t) - f(t)$

88. If

$$\{X_n, n \geq 0\}$$

is a branching process with

$$E(X_1) = m, m < 1,$$

then

$$E\left[\sum_{n=1}^{\infty} X_n\right]$$

is :

(A) m^n

(B) $\dfrac{m^n}{1 - m}$

(C) $m^n(m^n - 1)$

(D) $\dfrac{m}{1 - m}$

89. If $f(x)$ is a real continuous function in $[a, b]$ and $f(a)\, f(b) < 0$, then for $f(x) = 0$, there is (are) .... in the domain $[a, b]$.

(A) One root

(B) an undeterminable number of roots

(C) at least one root

(D) no root

90. The transition probability matrix of a four-state Markov chain is :

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.6 & 0.2 & 0.2 & 0 \end{bmatrix}$$

and the four states are 1, 2, 3, 4. Then the closed set is :

(A) {1, 3, 4}

(B) {2, 3, 4}

(C) {1, 2, 3}

(D) {1, 2, 4}

91. Which of the following is *incorrect* ?

(A) $\nabla = \delta \ E^{-1/2}$

(B) $\delta^2 E = \nabla^2$

(C) $\Delta - \nabla = \delta$

(D) $\mu E = E\mu$

92. Vital statistics are available generally in the following form :

(A) Mean of the vital events

(B) Median of the vital events

(C) Frequencies of the vital events

(D) Mode of the vital events

93. A rod is rotating in a plane. The following table gives the angle θ(radians) through which the rod has turned for various values of the time *t* second :

t	**θ**
0	0
0.2	0.12
0.4	0.49
0.6	1.12
0.8	2.02
1.0	3.20
1.2	4.67

The angular velocity of the rod, when $t = 0.6$ second is ....

(A) 3.82 rad/sec

(B) 2.62 rad/sec

(C) 2.72 rad/sec

(D) 3.92 rad/sec

94. Crude rate of natural increase of the population growth in the context of vital statistics is :

(A) The difference between the crude birth rate per thousand and the crude death rate per thousand

(B) Ratio of the crude birth rate per thousand and the crude death rate per thousand

(C) Crude birth rate per thousand multiplied by the crude death rate per thousand

(D) Crude birth rate per thousand, plus crude death rate per thousand

95. The largest eigen value of the matrix :

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

by power method is ....

(A) 2.08

(B) 5.38

(C) 7.15

(D) 3.47

96. Given the following table for L_x the number of rabbits living at age x, complete the life table for rabbits :

x	L_x
0	100
1	90
2	80
3	75
4	60
5	30
6	0

Let x, y and z be three rabbits of ages 1, 2 and 3 years respectively, then what is the probability that, at least one of them will be alive for one year more :

(A) $\frac{321}{400}$

(B) $\frac{719}{720}$

(C) $\frac{719}{919}$

(D) $\frac{321}{523}$

97. If

$$L^{-1}\{f(p)\} = F(t) \text{ and}$$

$$L^{-1}\{g(p)\} = G(t),$$

then

$$L^{-1}\{f(p)g(p)\}$$

is :

(A) $\int_0^t F(t-s)G(s-t)\,ds$

(B) $\int_0^t F(s)G(t+s)\,ds$

(C) $\int_0^t F(s)G(s)\,ds$

(D) $\int_0^t F(s)G(t-s)\,ds$

98. The values in the following life table which are marked with question marks in the context of vital statistics are :

Age x :	L_x	d_x	q_x	p_x
20 :	693435	?	?	?
21 :	690673	—	—	—

(A) $d_x = 6920$, $q_x = 0.99602$, $p_x = 0.00398$

(B) $d_x = 2762$, $q_x = 0.99602$, $p_x = 0.00398$

(C) $d_x = 6920$, $q_x = 0.00398$, $p_x = 0.99602$

(D) $d_x = 2762$, $q_x = 0.00398$, $p_x = 0.99602$

99. The Fourier sine transform of

$$\frac{1}{e^{\pi t} - e^{-\pi t}}$$

is :

(A) $\left[\frac{e^{-p} + 1}{e^{p} + 1}\right] \cdot \frac{1}{2\sqrt{2\pi}}$

(B) $\frac{1}{2\sqrt{2\pi}}\left[\frac{e^{-p} - 1}{e^{p} - 1}\right]$

(C) $\frac{1}{2\sqrt{2\pi}}\left[\frac{e^{p} - 1}{e^{p} + 1}\right]$

(D) $\frac{1}{2\sqrt{2\pi}}\left[\frac{e^{p} + 1}{e^{p} - 1}\right]$

100. Two components each with reliability ½ are arranged to form a series system with reliability $R_S(t)$, and then to form a parallel system with reliability $R_P(t)$. Then the difference $R_P(t) - R_S(t)$ is :

(A) $\frac{1}{4}$

(B) $\frac{3}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

101. The Fourier transform of the second derivative of $u(x, t)$ is :

(A) $p\ \tilde{u}\ (p, t)$

(B) $p^2\ \tilde{u}\ (p, t)$

(C) $-p\ \tilde{u}\ (p, t)$

(D) $-p^2\ \tilde{u}\ (p, t)$

102. Which of the following relations is *true* ?

(A) $\text{MTSF} = \int_0^t R(x)\, dx$

(B) $\text{MTSF} = \exp\left\{-\int_0^\infty R(t)\, dt\right\}$

(C) $\text{MTSF} = \exp\left\{\int_0^\infty r(t)\, dt\right\}$

(D) $\text{MTSF} = \int_0^\infty R(x)\, dx$

103. The solution of the integral equation :

$$x(t) = e^{-t} - 2\int_0^t \cos\ (t - s)\ x(s)ds$$

by using Laplace transform is :

(A) $x(t) = e^{-t^2}(1 + t)^2$

(B) $x(t) = e^{-t}(1 + t)^2$

(C) $x(t) = e^{-t^2}(1 - t)^2$

(D) $x(t) = e^{-t}(1 - t)^2$

104. For what value of r, the component reliability, a 2-out-of-3 system will have reliability ½ ?

(A) $\frac{1}{4}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{1}{2}$

105. Let $\overline{f}$ be the vector field given by

$$\overline{f}(x, y) = y\overline{i} + (x^2 + y)\overline{j}$$

Then the line integral of $\overline{f}$ from A(0, 0) to B(1, 1) along the line AB is :

(A) $\frac{4}{3}$

(B) $\frac{5}{3}$

(C) 2

(D) 1

106. A double sampling plan has the following parameters : N = 2000, n_1 = 40, C_1 = 2, n_2 = 80 and C_2 = 4. It is observed that the first sample contains 3 defectives, then in order that plan accepts the lot, the number of defectives in the second sample should not exceed :

(A) 1

(B) 2

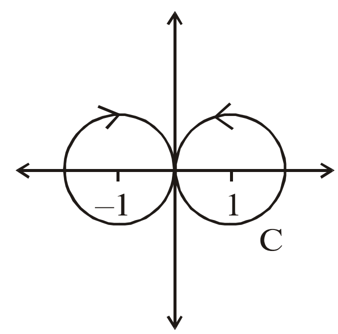
(C) 3

(D) Cannot be determined

107. The value of the integral

$$\int_C \frac{dz}{z^2 - 1},$$

where C is shown in the figure is :



(A) $-4\pi i$

(B) $4\pi i$

(C) $2\pi i$

(D) zero

108. If small orders and placed frequently rather than placing large orders infrequently, then total inventory cost gets :

(A) Reduced

(B) Increased

(C) Either reduced or increased

(D) Minimized

109. The value of the integral

$$\int_C \frac{e^z}{(z+1)^3} dz$$

where C : $|z| = 3$ is :

(A) πie

(B) $\frac{\pi i}{e}$

(C) $\frac{1}{3}\frac{\pi i}{e^2}$

(D) $2\pi ie$

110. In the context of inventory models service level is defined as the probability of :

(A) Stocking during each order cycle

(B) Not stocking during each order cycle

(C) Stocking during lead time

(D) Not stocking during lead time

111. The value of the integral

$$\int_C \frac{e^z}{z(1-z)^3} dz$$

where C is $|z - 1| = \frac{1}{2}$ is equal to :

(A) πie

(B) πi

(C) $-\pi ie$

(D) ie

112. At certain health care centre, patients arrive at a mean rate of 4 per hour and they are checked by doctor at a mean rate of 5 per hour. The centre feels that service times have some unspecified positive skewed unimodal two-tails distribution with a standard deviation of 0.05 hours. What is the average queue length and average waiting time of the patients spend in the health care centre ?

(A) 2.5 and 0.625 hours

(B) 1.7 and 0.425 hours

(C) 1.7 and 0.625 hours

(D) 2.5 and 0.425 hours

113. Which of the following is a set of positive measure ?

(A) The set of rational number in [0, 1]

(B) The set of numbers in [0, 1] not containing the digit 2 in their decimal expansion

(C) The Cantor set

(D) The set of transcendental numbers in [0, 1]

114. For a queueing system, which of the following statements is *correct* ?

(1) For a workable queueing system it is necessary that the arrival rate of customers per unit of time should be grated than the service rate

(2) Queues in series are those where input of a queue is derived from the output of another queue

(3) Circular queue are those where a customer after being served joins back the queue again

(4) When the queueing systems are allowed to depend upon time and are controlled, then these system are known as dynamic control systems

Codes :

(A) (1) and (2)

(B) (1) and (3)

(C) (1), (2) and (4)

(D) (2), (3) and (4)

115. Which of the following is *false* ?

(A) If f is absolutely continuous on [0, 1] then f is bounded variation on [0, 1]

(B) If f is non-negative and absolutely continuous on [0, 1], then so is f^2

(C) If f is non-negative and absolutely continuous on [0, 1], then $\sqrt{f}$ is absolutely continuous on [0, 1]

(D) If f is convex on (0, 1) then f is absolutely continuous on every closed subinterval $[a, b] \subset (0, 1)$

116. Which of the following statements are *correct* in decision analysis ?

(1) Organisational decisions are the decisions taken by the executives in their official capacity

(2) Tactical decisions are the decisions taken by the executives in their individual capacity

(3) Strategic decisions which have far reaching effect on the future course of action

(4) Major decisions are the decisions which involve large sum of money and require prior sanction

Codes :

(A) (1), (2) and (3)

(B) (2), (3) and (4)

(C) (1), (3) and (4)

(D) (1), (2) and (4)

117. Let $\{f_n\}_{n \geq 1}$ be a sequence of non-negative measurable functions. Then :

(A) $\lim \inf \int f_n \, dx \geq \int \lim \inf f_n \, dx$

(B) $\lim \inf \int f_n \, dx \leq \int \lim \inf f_n \, dx$

(C) $\lim \inf \int f_n \, dx = \int \lim \inf f_n \, dx$

(D) $\lim \sup \int f_n \, dx \geq \int \lim \sup f_n \, dx$

118. When a positive q is divided into five parts, the maximum value of their product is :

(A) $\left(\frac{q}{5}\right)^5$

(B) $5q$

(C) $(5q)^5$

(D) $5\left(\frac{q}{5}\right)$

119. Which of the following is *false* ?

(A) If μ and ν are signed measures and ν is absolutely continuous w.r.t. μ (i.e. $\nu \ll \mu$), then $|\nu|$ is absolutely continuous w.r.t. $|\mu|$

(B) If μ is a measure, $\int f \, d\mu$ exists and $\nu(E) = \int_E f \, d\mu$, then ν is absolutely continuous w.r.t. μ

(C) If $[[X, S, \mu]]$ is a σ-finite measure space and ν is any σ-finite measure on S, then there is a finite-valued non-negative measurable function f on X such that for each $E \in S$, $\nu(E) = \int_E f \, d\mu$

(D) If $[[X, S, \mu]]$ is a σ-finite measure space and ν is a σ-finite signed measure such that $\nu \ll \mu$, then there exists a finite-valued measurable function f on X such that for each $E \in S$, $\nu(E) = \int_E f \, d\mu$

120. Using dynamic programming, the maximum value of

$$3x_1 + 5x_2$$

subject to

$$x_1 \le 4$$
$$x_2 \le 6$$
$$3x_1 + 2x_2 \le 18 \text{ and}$$
$$x_1, x_2 \ge 0$$

is obtained as :

(A) 46

(B) 63

(C) 36

(D) 32

121. Which of the following is a compact subset of $\mathbf{R}^2$?

(A) The x-axis

(B) The set of all points in $\mathbf{R}^2$ having integer coordinates

(C) The set $\{(x, y) \mid |x| + |y| \le 1\}$

(D) $\{(x, y) \mid y^2 = x\}$

122. A set of values (x_0, y_0), (x_1, y_1), (x_2, y_2), ..., (x_n, y_n) satisfying the relation $y = f(x)$ is given, where the explicit nature of $f(x)$ is not known. It is required to find a function $\phi(x)$ such that $f(x)$ and $\phi(x)$ agree on the set of tabulated points. The method used for this is called :

(A) extrapolation

(B) polynomial extrapolation

(C) interpolation

(D) polynomial interpolation

123. Consider the following integrals :

(1) $\int_0^{\infty} \frac{(-1)^{[x]}\, dx}{1+x}$

(2) $\int_0^{\infty} \frac{\sin x\, dx}{(1+x)^2}$

Here $[x]$ is the integral part of x.

Then :

(A) Both (1) and (2) converge absolutely

(B) None of (1) and (2) converges absolutely

(C) only (1) converges absolutely

(D) only (2) converges absolutely

124. The certain corresponding values of x and $\log_{10} x$ are :

x	$\log_{10} x$
300	2.4771
304	2.4829
305	2.4843
307	2.4871

The value of $\log_{10} 301$ by Aitken's method (interpolation by iteration) is :

(A) 3.4785

(B) 3.4786

(C) 2.4785

(D) 2.4786

125. The Taylor series for $(1 - \cos x)e^{x^2}$ about 0 :

(A) contains only even powers of x

(B) contains only odd powers of x

(C) contains even as well as odd powers of x

(D) contains only powers of x of the type x^{k^2}, $k = 1, 2, ...$.

126. The value of

$$\int_0^1 \frac{1}{1 + x} dx$$

correct to three decimal places by the trapezoidal rule, with $h = 0.5$ and :

x	$\frac{1}{1+x} = y$
0.0	1.000
0.5	0.6667
1.0	0.5000

(A) 0.708

(B) 0.608

(C) 0.806

(D) 0.807

127. For which of the following functions the equation $f(x, y) = 0$ can be solved for y in terms of x in a neighbourhood of (0, 0) ?

(A) $f(x, y) = (x - 1)^2 + y^2 - 1$

(B) $f(x, y) = x^3 - y^2$

(C) $f(x, y) = x - y^2 \cos y$

(D) $f(x, y) = x \sin y + y \cos x$

128. The solution to the following simulataneous equations is :

$$0.0002x + 0.3003y = 0.1002$$

$$2.0000x + 3.0000y = 2.0000$$

(A) $x = 0.5000; y = 0.4444$

(B) $x = 0.4444; y = 0.5000$

(C) $x = 0.3333; y = 0.5000$

(D) $x = 0.5000; y = 0.3333$

129. Let

$$f : \mathbf{C} \to \mathbf{R}$$

be a function defined by

$$f(z) = |z|$$

for every $z \in \mathbf{C}$.

Then :

(A) $f'(0) = 0$

(B) $f'(z)$ exists only at $z = 0$

(C) $f'(z) = 0$ for $z \neq 0$

(D) $f'(z)$ does not exit at any $z \in \mathbf{C}$

130. Which of the following terms refers to a statistical method that can be used to statistically equate groups on a pretest or some other variable ?

(A) Experimental control

(B) Interaction effect

(C) Treatment contrasts

(D) Analysis of covariance

131. The value of the integral

$$\int_C \frac{z^2}{z^3 - 2} dz$$

where

$$C : |z| = 3,$$

is equal to :

(A) 0

(B) $-\pi i$

(C) πi

(D) $2\pi i$

132. When interpreting a correlation coefficient expressing the relationship between two variables, it is very important to avoid ..... .

(A) Checking the strength of relationship

(B) Expressing a relationship with a correlation coefficient

(C) Checking the direction of the relationship

(D) Jumping to the conclusion of causality

133. The number of non-isomorphic abelian groups of order pqr, where p, q, r are distinct primes is :

(A) 1

(B) 2

(C) 3

(D) 4

134. Fill in the missing words in the following sentence : "Statistical methods draw conclusions about .... based on .... computed from the ...."

(A) Populations, statistics, samples

(B) Populations, parameters, samples

(C) Parameters, statistics, populations

(D) Parameters, populations, samples

135. Let F be a field with p^n elements, p is prime. Consider the following statements :

(1) $(F, +) \cong z_p \oplus .... \oplus z_p$ (n copies of z_p)

(2) $F^* \cong z_{p^n - 1}$

(3) $F - \{0\}$ is a cyclic group under multiplication.

Then which of the following is *true* ?

(A) Only (1) is true

(B) Only (2) is true

(C) Only (3) is true

(D) All the statements are true

136. The following table represents the relative frequency of accidents per day in a city :

Accidents	Relative Frequency
0	0.55
1	0.20
2	0.10
3	0.15
4 or more	0

Which of the following are *true* ?

(I) The mean and modal number of accidents are equal

(II) The mean and median number of accidents are equal

(III) The median and modal number of accidents are equal

(A) (I), (II) and (III)

(B) (I) only

(C) (II) only

(D) (III) only

137. Let T_1 and T_2 be linear operators on a finite dimensional vector space V over a field F. Suppose T_1 and T_2 are similar. Consider the following statements :

(1) T_1^2 and T_2^2 are similar

(2) T_1^{-1} and T_2^{-1} are similar (if inverse exist)

(3) T_1T_2 and T_2T_1 are similar, given that at least one of T_1 and T_2 is invertible

(4) $T_1 + T_2$ and $T_1 - T_2$ are similar

Then :

(A) Only (1) is true

(B) Only (4) is true

(C) None of (1) to (4) is true

(D) Each of (1), (2) and (3) are true

138. Let X be a uniform (0, 1) and Y be a Bernoulli $\left(\frac{1}{2}\right)$ random variable, independent of each other. Define :

$$Z = \begin{cases} 1 \text{ if } Y = 0 \\ X \text{ if } Y = 1 \end{cases}$$

Which of the following statements are *correct* ?

(*i*) The cumulative distribution function of z is

$$F_z(z) = 0.5 \ I_{(0 \leq z \leq 1)} + 0.5 \ I_{\{z = 1\}}$$

(*ii*) F_z is absolutely continuous

(*iii*) The z-induced measure is the Lebesgue-Stieltje's measure generated by F_z

(*iv*) A dominating measure is $\lambda + \nu$, where λ is the Lebesgue measure and ν is the counting measure on $\{1\}$

Codes :

(A) (*i*) and (*ii*)

(B) (*ii*), (*iii*) and (*iv*)

(C) (*ii*) and (*iii*)

(D) (*i*), (*iii*) and (*iv*)

139. For a matrix A, the characteristic polynomial is $\Delta(\lambda) = (\lambda - 2)^4 (\lambda - 3)^2$ and the minimal polynomial is $m(\lambda) = (\lambda - 2)^2 (\lambda - 3)^2$. Then a Jordan canonical form of A is :

(A) $\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 3 & 2 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

140. Let $X_2, X_3, ...$ be random-variables such that for $\alpha > 0$

$$P[X_n = 1] = 1 - \frac{1}{n^\alpha},$$

$$P[X_n = n] = \frac{1}{n^\alpha}, \; n \geq 2.$$

Which of the following statements are *correct* ?

(*i*) $X_n \xrightarrow{d} 1$

(*ii*) $X_n \xrightarrow{p} 1$

(*iii*) $X_n \xrightarrow{r} 1$

(*iv*) $X_n \xrightarrow{a.s.} 1$

Codes :

(A) (*i*), (*ii*) and (*iii*)

(B) (*i*), (*ii*) and (*iv*)

(C) (*i*) and (*ii*)

(D) (*i*), (*ii*), (*iii*) and (*iv*)

141. Which of the following groups has a subgroup which is not normal ?

(A) Z_{20}

(B) $Z_3 \times Z_6$

(C) Q_8, the group of quaternions with 8 elements

(D) S_3

142. Let $\{X_k\}$ be a sequence of independent random variables with :

$$P[X_k = k] = \frac{k^{-\lambda}}{2} = P[X_k = -k]$$

$$P[X_k = 0] = 1 - k^{-\lambda},\ \lambda > 0$$

Let $S_n = \sum_{i=1}^{n} X_i$. Which of the following statements are *correct* ?

(*i*) $E(X_k) = 0,\ S_n^2 = \text{Var}(S_n) \to \infty$ for all λ

(*ii*) $E(X_k) = 0,\ \text{Var}(S_n) \to \infty$ for $\lambda \leq 3$

(*iii*) Central limit theorem $\mathbf{F}\left(\frac{S_n}{S_n}\right) \to N(0, 1)$ hold for $0 < \lambda \leq 1$

(*iv*) Central limit theorem $\mathbf{F}\left(\frac{S_n}{S_n}\right) \to N(0, 1)$ hold for $0 < \lambda < 1$

Codes :

(A) (*i*) and (*iii*)

(B) (*ii*) and (*iii*)

(C) (*i*) and (*iv*)

(D) (*ii*) and (*iv*)

143. Let X be a metric space. Consider the following statements :

(1) X is compact

(2) X has Bolzano-Weierstrass property

(3) X is sequencially compact

(4) X is complete

Then which of the following is *true* ?

(A) (4) $\Leftrightarrow$ (1)

(B) (3) $\Leftrightarrow$ (4)

(C) (2) $\Leftrightarrow$ (4)

(D) (1) $\Leftrightarrow$ (2)

144. Let X_n be a sequence of random variables such that

$$P[X_n = 0] = 1 - \frac{1}{n^2},$$

$$P[X_n = e^n] = \frac{1}{n^2}.$$

Which of the following statements are *correct* ?

(*i*) $X_n \xrightarrow{r} 0$

(*ii*) $X_n \xrightarrow{a.s.} 0$

(*iii*) $X_n \xrightarrow{p} 0$

(*iv*) $X_n \xrightarrow{d} 0$

Codes :

(A) (*i*), (*iii*) and (*iv*)

(B) (*i*), (*ii*) and (*iii*)

(C) (*i*), (*ii*) and (*iv*)

(D) (*ii*), (*iii*) and (*iv*)

145. Which of the following metric spaces is *not* separable ?

(A) R with usual metric

(B) **C**[0, 1]

(C) **R** with discrete metric

(D) The space l_2 of square summable real sequences

146. A coin is tossed three times. What is the probability that it lands on heads exactly one time ?

(A) 0.125

(B) 0.275

(C) 0.375

(D) 0.500

ROUGH WORK

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