Test Booklet Code &No.
प्रश्नपत्रिका कोड व क्र.

C

Paper-II

MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

MAY - 30216

Time Allowed : 1¼ Hours] **[Maximum Marks : 100**

Number of Pages in this Booklet : **28** Number of Questions in this Booklet : **84**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. (*a*) This paper consists of **Eighty Four (84)** multiple choice questions, each carrying **Two (2)** marks.
   (*b*) Answer to **first 16** questions is compulsory.
   (*c*) Answer any **34** out of remaining **68** questions. In case any candidate answers more than **34** questions, only first **34** questions will be evaluated.
   (*d*) Below each question, four alternatives or responses are given. Only one of these alternatives is the 'CORRECT' answer to the question.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.

   (A) (B) ⬤ (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. (*a*) या प्रश्नपत्रिकेत एकूण **चौऱ्यांशी (84)** बहुपर्यायी प्रश्न दिलेले आहेत, प्रत्येक प्रश्नाला **दोन (2)** गुण आहेत.
   (*b*) पहिले **सोळा (16)** प्रश्नांचे उत्तर आवश्यक आहेत.
   (*c*) उरलेले **अडुसष्ट (68)** प्रश्नांपैकी कोणतेही **चौतीस (34)** चे उत्तर द्या. जर कोणी परीक्षार्थी **चौतीस (34)** पेक्षा जास्त प्रश्नांचे उत्तर देतो, तर पहिले **चौतीस (34)** प्रश्नांचे मूल्यमापन होईल.
   (*d*) खाली दिलेल्या प्रश्नाचे चार पर्याय किंवा उत्तर दिलेले आहेत. प्रश्नाचे बहुपर्यायी उत्तरामधून केवळ एक 'बरोबर' आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.

   (A) (B) ⬤ (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहीलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

Mathematical Science
Paper II

Time Allowed : 75 Minutes] **[Maximum Marks : 100**

1. Let X be a rv with N(1, 1). Define the events :

$A_1 = \{-2, < X < 1\}$,

$B_1 = \{-1 < X < 1\}$,

$C_1 = \{0 < X < 2\}$.

Which of the following statements is *correct* ?

(A) $P(B_1) < P(A_1) < P(C_1)$

(B) $P(C_1) < P(B_1) < P(A_1)$

(C) $P(A_1) = P(B_1) < P(C_1)$

(D) $P(A_1) = P(B_1) = P(C_1)$

2. If $P(A|B) = P(A|\bar{B})$, then the events A and B are :

(A) Dependent events

(B) Mutually exclusive events

(C) Independent events

(D) Exhaustive events

3. In the canonical form of LPP if the objective function is of minimization, then all the constraints other than non-negative conditions are :

(A) greater than or equal to type

(B) lesser than type

(C) greater than type

(D) lesser than or equal to type

4. A feasible solution to an LPP :

(A) Must be a corner point of the feasible region

(B) Must satisfy all of the problems constraints and non-negative restrictions simultaneously

(C) need not satisfy all of the constraints

(D) must optimize the value of the objective function

5. Unbounded solution in an LPP is :

(A) Where the objective function can be decreased indefinitely

(B) Where the objective function can be increased or decreased indefinitely

(C) Which maximizes the objective function

(D) Where the objective function can be increased indefinitely

6. Using graphical method, the optimum solution of the LPP of :

Maximizing :

$$Z = 10x + 15y$$

Subject to :

$$2x + y \leq 26$$

$$x + 2y \leq 28$$

$$y - x \leq 5$$

$$x, y \geq 0$$

is obtained as :

(A) $x = 6$ and $y = 1$

(B) $x = 8$ and $y = 10$

(C) $x = 6$ and $y = 10$

(D) $x = 8$ and $y = 8$

7. The sequence of functions $f_n(x) = x^n$, $\forall\ x \in [0, 1]$, $n = 1, 2, \ldots\ldots\ldots\ldots$ is :

(A) pointwise convergent to a continuous function on [0, 1]

(B) not pointwise convergent on [0, 1]

(C) pointwise convergent but not uniformly convergent on [0, 1]

(D) uniformly convergent on [0, 1]

8. The radius of convergence of the series $\sum_{m=0}^{\infty} \frac{(-1)^m}{8^m} x^{3m}$ is :

(A) 8

(B) 2

(C) $\frac{1}{2}$

(D) $\frac{1}{8}$

9. What is the value of $\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3}+\ldots}}$?

(A) $\frac{1}{3+\sqrt{11}}$

(B) $\frac{3+\sqrt{13}}{2}$

(C) $\frac{2}{3+\sqrt{13}}$

(D) $3+\sqrt{11}$

10. Let $f(x) = x^4 + 2x^3 - 2, \ \forall \ x \in (0, \infty)$. Then :

(A) f has exactly four positive real roots

(B) f has exactly three positive real roots

(C) f has unique positive real root

(D) f has exactly two positive real roots

11. Let T : $\mathbf{R}^3 \to \mathbf{R}^2$ and S : $\mathbf{R}^2 \to \mathbf{R}^3$ be two linear transformations. Then which of the following is *true* ?

(A) SoT is not injective

(B) SoT is injective

(C) SoT is surjective

(D) SoT is bijective

12. Let W_1 and W_2 be two subspaces of a finite dimensional vector space V and let dim W_1 + dim W_2 > dim V. Then :

    (A) $W_1 \cap W_2 = \{0\}$

    (B) $W_1 \cap W_2 \neq \{0\}$

    (C) $W_1 \cap W_2 = \phi$

    (D) $W_1 \cup W_2 = V$

13. The eigenvalues of a real symmetric matrix are :

    (A) Purely imaginary

    (B) Need to be real numbers

    (C) Complex numbers with unit absolute value

    (D) Real numbers

14. Which of the following statements is *false* ?

    (A) The product of two orthogonal matrices is orthogonal

    (B) Inverse of a non-singular Hermitian matrix is Hermitian

    (C) The product of two Hermitian matrices is Hermitian

    (D) The product of two unitary matrices is unitary

15. If an event B is independent of itself, then P(B) is :

    (A) 0

    (B) 1

    (C) 1 or 0.5

    (D) 0 or 1

16. Let $P[X_1 = 2, X_2 = 3] = 1$, then EX_1, EX_2, $V(X_1)$ and $V(X_2)$ are as follows :

    (A) (1, 2, 3, 1)

    (B) (2, 3, 1, 1)

    (C) (2, 3, 0, 0)

    (D) (1, 1, 2, 3)

17. The ring of Gaussian integers, $\mathbf{Z}[i] = \{a + ib/a, b \in \mathbf{Z}]$ is :

(A) a Euclidean domain

(B) a unique factorization domain but not Euclidean domain

(C) a principle ideal domain but not Euclidean domain

(D) neither principle ideal domain nor Euclidean domain

18. Let T_n be such that $E(T_n) \to \theta$ and $Var(T_n) \to 0$. Then which of the following is not *true* ?

(A) Given condition is necessary for the consistency of T_n

(B) T_n is a consistent estimator of θ

(C) Given condition is sufficient for the consistency of T_n

(D) T_n converges weakly to θ

19. If G is a group of order 108, then :

(A) G is not simple

(B) G is cyclic

(C) G is non-abelian

(D) G has a unique normal subgroup

20. Suppose $X_1, X_2, ..., X_n$ is a random sample from $U(0, \theta), \theta > 0$. Let $X_{(1)}, X_{(2)}, ..., X_{(n)}$ be the corresponding order statistics. Define $T_1 = 2\bar{X}$ and $T_2 = \frac{n+1}{n} X_{(n)}$. Then which one of the following is *true* ?

(A) T_2 is uniformly minimum variance unbiased estimator

(B) Both T_1 and T_2 are biased for θ

(C) Only T_1 is biased

(D) T_1 is uniformly minimum variance unbiased estimator

21. Let F be a field. Then the number of ideals of F is :

(A) 2

(B) 1

(C) 3

(D) 4

22. Let $X_1, X_2, ..., X_n$ be a random sample from Poisson distribution with mean λ. Then $\sum_{i=1}^{n} a_i X_i$ gives UMVUE of λ if :

(A) $a_i = \frac{1}{n}$ for $i = 1, 2, ..., n$

(B) $\sum_{i=1}^{n} a_i = 1$

(C) $\sum_{i=1}^{n} a_i = 1, \sum_{i=1}^{n} a_i^2 = 1$

(D) $a_i = \frac{n-1}{n}$ for $i = 1, 2, ..., n$

23. Which of the following rings is *not* an integral domain ?

(A) $\mathbf{Z}[x]$

(B) $\mathbf{Z}[\sqrt{2}] = \{a + \sqrt{2}b \mid a, b \in \mathbf{Z}\}$

(C) $(\mathbf{Z}_p, +_p, \times_p)$

(D) $\mathbf{Z} \oplus \mathbf{Z}$

24. Consider Pitman family of distributions $\{f_X(x, \theta) : \theta > 0\}$ with :

$$f_X(x, \theta) = \begin{cases} u(x)/v(\theta) & \text{if } a(\theta) < x < b(\theta) \\ 0 & \text{otherwise} \end{cases}$$

where $u(x), v(\theta) > 0$.

Suppose $a(\theta)$ is increasing and $b(\theta)$ is decreasing function of θ.

Then minimal sufficient statistic based on a sample of size n is given by :

(A) $T = \max\{a^{-1}(X_{(1)}), b^{-1}(X_{(n)})\}$

(B) $T = \max\{a^{-1}(X_{(n)}), b^{-1}(X_{(1)})\}$

(C) $T = \min\{a^{-1}(X_{(n)}), b^{-1}(X_{(1)})\}$

(D) $T = \min\{a^{-1}(X_{(1)}), b^{-1}(X_{(n)})\}$

25. Suppose λ is an eigenvalue of a non-singular square matrix A. Then :

(A) $|A|$ is an eigenvalue of adj A

(B) $\frac{1}{\lambda}$ is an eigenvalue of adj A

(C) 0 is an eigenvalue of adj A

(D) $\frac{|A|}{\lambda}$ is an eigenvalue of adj A

26. Consider Cramer family of distributions $\{f(x, \theta) : \theta \in \Omega \subset R_1\}$. Let T be unbiased for $\psi(\theta)$. Which of the following is Cramer-Rao inequality ?

(A) $P(|T - \theta| > C) \leq \frac{\text{Var}(T)}{C^2}$

(B) $\text{Var}(T) \leq \frac{\left(\frac{d}{d\theta}\psi(\theta)\right)^2}{I_X(\theta)}$

(C) $P(\text{Sup}|T - \psi(\theta)| > C) \leq \frac{\text{Var}(T)}{C^2}$

(D) $\text{Var}(T) \geq \frac{\left(\frac{d\,\psi(\theta)}{d\theta}\right)^2}{I_X(\theta)}$

27. Consider the vector space $\mathbf{R}^3$ and its subsets $P = \{t(1, 0, 2) : t \in \mathbf{R}\}$ and $Q = \{(1, 1, 1)\}$. Then :

(A) P + Q is a subspace of $\mathbf{R}^3$

(B) P + Q is a plane passing through the point (1, 1, 1) and perpendicular to the line P

(C) P + Q is a plane passing through the point (1, 1, 1) and containing the line P

(D) P + Q is a line passing through the point (2, 1, 3) and parallel to the line P

28. Let X be Poisson with mean λ and $\psi(\lambda) = e^{-3\lambda}$. We define $T(X) = (-2)^X$. Then which of the following is *correct* ?

(A) T(X) is not a statistic

(B) T(X) is consistent for $\psi(\lambda)$

(C) T(X) is biased for $\psi(\lambda)$

(D) T(X) is unbiased for $\psi(\lambda)$

29. The dimension of the vector space $M = \{[a_{ij}]_{m \times n} | a_{ij} \in \mathbb{C}\}$ over the field **R** is :

(A) $m + n$

(B) $2mn$

(C) $2(m + n)$

(D) mn

30. Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, 5)$. Then the test statistic for testing $H_0 : \mu = 10$ against $H_1 : \mu = 14$ is :

(A) $T = \sqrt{\frac{n}{5}}(\bar{X} - 15)$

(B) $T = \sqrt{\frac{n}{5}}(\bar{X} - 10)$

(C) $T = \sqrt{\frac{n}{5}} \frac{(\bar{X} - 10)}{s}$,

$$s = \sqrt{\frac{\Sigma(X_i - \bar{X})^2}{n - 1}}$$

(D) $T = \frac{\bar{X} - 10}{\sqrt{(n - 1)}s}$

31. Let the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ be given by the reflection with respect to the origin. Then the matrix of T with respect to the standard basis is :

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

32. For testing $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$ we consider two independent random samples of sizes n_1 and n_2 drawn from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Let $\sigma_1^2 \neq \sigma_2^2$ are unknown. Then most suitable test is :

(A) Two-sample *t*-test

(B) Two-sample F-test

(C) Fisher-Behern's test

(D) Two-sample goodness-of-fit test

33. Which of the following matrices is *not* diagonalisable ?

(A) $\begin{bmatrix} -1 & 4 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

34. Let $H_0 : \theta = \theta_0$ and $H_1 : \theta = \theta_1$ where X follows a probability distribution with pdf $f(x, \theta)$. Then using N-P lemma we get :

(A) Most powerful consistent test

(B) Most powerful test

(C) Most powerful unbiased test

(D) Most powerful invariant test

35. The complete integral of the PDE $p - 3x^2 = q^2 - y$ is :

(A) $z = ax^2 + x^3 + \frac{2}{3}(a + y)^{3/2} + b$

(B) $z = ax + x^3 + \frac{2}{3}(a + y)^{3/2} + b$

(C) $z = ax + x^2 + \frac{2}{3}(a + y)^{3/2} + b$

(D) $z = ax^2 + x + \frac{2}{3}(a + y)^{3/2} + b$

36. Let X be a random variable with pdf $f(x, \theta), \theta \in \Omega$. Let $\Omega = \Omega_0 \cup \Omega_1$ where $\Omega_0 = \{\theta_0\}$ and $\Omega_1 = \{\theta_1\}$. Then likelihood ratio test statistics is :

(A) $\lambda(x) = \frac{f(x, \theta_0)}{f(x, \theta_1)}$

(B) $\lambda(x) = \frac{f(x, \theta_0)}{\max\{f(x, \theta_1), f(x, \theta_0)\}}$

(C) $\lambda(x) = \frac{f(x, \theta_0)}{\min\{f(x, \theta_0), f(x, \theta_1)\}}$

(D) $\lambda(x) = \frac{f(x, \theta_1)}{f(x, \theta_0)}$

37. The singular solution of the differential equation $y = px + p^3$ is :

(A) $4y^3 + 27x^2 = 0$

(B) $4y^2 - 27x^3 = 0$

(C) $4x^3 + 27y^2 = 0$

(D) $4y^2 + 27y = 0$

38. From a population of N = 100 units, a SRSWOR sample of $n = 12$ units is drawn. Denote the population by $U = \{u_1, u_2, \ldots\ldots\ldots, u_{100}\}$. Which of the following statements is *true* ?

(A) Probability that u_{24} is duplicated in the sample is non-zero

(B) Probability that both u_1 and u_2 are in the sample is 3/25

(C) u_1 is included in the sample with probability 3/25

(D) Probability that the sample will contain 12 distinct units is 1/2

39. The differential equation :

$$(\alpha x y^3 + y \cos x)\, dx + (x^2 y^2 + \beta \sin x) dy = 0$$

is exact if :

(A) $\alpha = \frac{3}{2}, \beta = 1$

(B) $\alpha = \frac{2}{3}, \beta = 1$

(C) $\alpha = 1, \beta = \frac{3}{2}$

(D) $\alpha = 1, \beta = \frac{2}{3}$

40. In stratified random sampling with a linear cost function $C = C_0 + \sum_{h=1}^{L} C_h n_h$, the variance of estimated mean is minimum for a specified cost C and the cost is minimum for a specified variance of $\bar{Y}_{st}$ when :

(A) $n_h \propto \frac{\sum_{h=1}^{L} w_h s_h}{C_h}$

(B) $n_h \propto \frac{w_h s_h}{\sqrt{C_h}}$

(C) $n_h \propto \frac{N_h s_h}{C_h}$

(D) $n_h \propto \frac{n s_h}{\sqrt{C_h}}$

41. The differential equation of all circles touching X-axis at the origin is :

(A) $(x^2 + y^2)\frac{dy}{dx} - 2xy = 0$

(B) $(x^2 - y^2)\frac{dy}{dx} + 2xy = 0$

(C) $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

(D) $(x^2 + y^2)\frac{dy}{dx} + 2xy = 0$

42. For SRSWR with population size 200, number of draws in the sample = 15, population variance s_y^2 = 180. Then variance of the sample mean is :

(A) 12.50

(B) 9.00

(C) 11.94

(D) 11.10

43. Let $y = \phi(x)$ and $y = \psi(x)$ be two solutions of the equation $y'' - 2xy' + (\sin^2 x)\, y = 0$ such that $\phi(0) = 1$, $\phi'(0) = 1$, $\psi(0) = 1$ and $\psi'(0) = 2$. Then the value of the Wronskian at $x = 0$ is :

(A) 1

(B) 0

(C) e

(D) e^2

44. When population size, N is very large then the approximate value of minimum sample size required for estimating population proportion (p) with desired accuracy (d) under SRSWOR design is :

(A) $\left\{\frac{p(1-p)\, z_{\alpha/2}}{d}\right\}^2$

(B) $\left\{\frac{z_{\alpha/2}}{d}\right\}^2$

(C) $\left\{\frac{p(1-p)\, z_{\alpha/2}}{d}\right\}$

(D) $\frac{(p(1-p)\, z_{\alpha/2})^2}{d}$

45. From a population of 40 units a sample of 5 units is selected by linear systematic sampling. Then which of the following statements is *true* ?

(A) Probability that 5th unit will be in the sample is 1/8

(B) If 12th unit is selected in the sample, then 17th unit will also be selected in the sample

(C) If 7th unit is selected in the sample, then 17th unit will also be selected in the sample

(D) Probability that 5th unit will be in the sample is 1/5

46. The following information is given regarding annual operating costs and resale values of two machines A and B. It has been found that :

(*i*) optimum replacement interval for machine A is 3 years with minimum total average cost as Rs. 5,200

(*ii*) Optimum replacement interval for machine B is 5 years with minimum total average cost as Rs. 4,000.

Considering above information, which of the following is *correct* ?

(A) Machine B is replaced

(B) Machine A is replaced

(C) Both machines A and B are replaced

(D) No need of replacing any machine

47. Consider a completely randomised design with treatment i replicated n_i times, $i = 1, 2, \ldots, v$. Let the linear model be $y_{ij} = \alpha + \tau_i + \epsilon_{ij}$, $j = 1, 2, \ldots n_i, i = 1, 2, \ldots, v$, where $\epsilon_{ij} \sim N(0, \sigma^2)$ for all i and j. Then a parametric function $\sum_{i=1}^{v} a_i \tau_i$ is estimable if and only if :

(A) $\Sigma a_i = 0$

(B) $a_i = \frac{n_i}{n}, i = 1, 2, \ldots, v,$

$$\sum_{i=1}^{v} n_i = n$$

(C) $a_i = 1$ for i odd and $a_i = -1$ if i even

(D) $a_i = 1 - \frac{n_i}{n}, i = 1, 2, \ldots, v$

48. With reference to inventory management which of the following is *correct* ?

(A) In ABC analysis, the inventory items are classified into three categories on the basis of their usage values

(B) HML classifies items according as they are high usage, medium usage or low usage

(C) VED analysis deals with classification of items on the basis of their availability

(D) XYZ analysis is based on the classification of items according to their unit costs

49. Consider a linear model :

$$Y_{ij} = \alpha + \tau_i + \beta_j + \in_{ij}, \; i = 1, 2, ..., \; v,$$

$$j = 1, 2, ..., b$$

and Y_{ij}'s are independent, τ_i is the effect of ith treatment and ϕ_j are effect jth block, $\in_{ij} \sim N(0, \sigma^2)$ for all i, j. The block sum of squares is :

(A) $\Sigma \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{v_b}, \; y_{i.} = \sum_{d=1}^{b} y_{ij},$

$$y_{..} = \sum_{i=1}^{v} \sum_{j=1}^{b} y_{ij}$$

(B) $\Sigma \frac{y_{i.}^2}{v} - \frac{y_{..}^2}{v_b}$

(C) $\Sigma \frac{y_{.j}^2}{b} - \frac{y_{..}^2}{v_b}, \; y_{.j} = \sum_{i=1}^{v} y_{ij}$

(D) $\Sigma \frac{y_{.j}^2}{v} - \frac{y_{..}^2}{v_b}$

50. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, find the mean queue size (line length) :

(A) 2 trains

(B) 4 trains

(C) 5 trains

(D) 3 trains

51. The assumption in a randomised block design with v treatments in b blocks with linear model $y_{ij} = \alpha + \tau_i + \beta_j + \in_{ij}$ is α : over all effect, τ_i : effect of treatment i, β_j : effect of jth block, $\in_{ij} \sim N(0, \sigma^2)$. Further yield of (i, j) plot is y_{ij} and y_{ij}'s are statistically independent. Which of the following assumptions are required ?

(A) $\sum_{i=1}^{v} \tau_i = \sum_{j=1}^{b} \beta_j$

(B) $\sum_{i=1}^{v} \tau_i = \sum_{j=1}^{b} \beta_j = \alpha$

(C) $\sum_{i=1}^{v} \tau_i = \sum_{j=1}^{b} \beta_j = \alpha = 0$

(D) $\sum_{i=1}^{v} \tau_i = \sum_{j=1}^{b} \beta_j = 0$

52. In the optimal simplex table, $c_j - z_j = 0$ value indicates :
    (A) Unbounded solution
    (B) Cycling
    (C) Infeasible solution
    (D) Alternative solution

53. With respect to Latin-square design which of following holds *true* ?
    (A) It is a block diagram
    (B) Total number of plots for v-treatments is v^2
    (C) Every treatment is replicated v^2 times
    (D) It is a two-factor design

54. In an assignment problem involving 5 workers and 4 jobs, total number of assignments possible are :
    (A) 5
    (B) 4
    (C) 9
    (D) 20

55. In a 2 × 2 factorial design which of the following gives interaction effect :
    (A) $ab - b + a - (1)$
    (B) $ab + b - a + (1)$
    (C) $ab + b - a - (1)$
    (D) $ab - a - b + (1)$

56. For the game with the following pay off matrix :

$$\begin{bmatrix} 8 & -3 \\ -3 & 1 \end{bmatrix}$$

value of the game is given by :
    (A) $\frac{17}{5}$
    (B) $\frac{-8}{15}$
    (C) $\frac{7}{15}$
    (D) $\frac{-1}{15}$

57. The number of limit points of the set $\left\{\frac{1}{2^m}+\frac{1}{2^n}=m, n \in \mathbf{N}\right\}$ is :

(A) infinitely many

(B) 2

(C) finitely many (more than 2)

(D) 1

58. When data are collected in a statistical study for only a portion or subset of all elements of interest we are using a :

(A) sample

(B) census

(C) sampling frame

(D) population

59. Which of the following functions $d : \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ is not a metric on $\mathbf{R}$?

(A) $d(x, y) = |x - y|$

(B) $d(x, y) = \frac{|x-y|}{1+|x-y|}$

(C) $d(x, y) = 1$ if $x \neq y$
$= 0$ if $x = y$

(D) $d(x, y) = x^2 - y^2$

60. Which of the following statements is *not* correct ?

(A) Color of ten automobiles recently purchased at a certain dealership is an example of a univariate data set

(B) Height and weight for each basketball player in Pune University is an example of bivariate data set

(C) The systolic blood pressure, diastolic blood pressure and serum cholesterol level for each patient participating in a research study is an example of multivariate data set

(D) Agricutlural fields in a country is an example of univariate data set

61. The total variation of the function $f(x) = \sin 2x$ on $[0, \pi]$ is :

(A) 2

(B) 4

(C) 0

(D) 1

62. Which of the following statements involves descriptive statistics as opposed to inferential statistics ?

(A) Based on a survey of 500 magazine readers, the magazine reports that 40% of its readers prefer double column articles

(B) The alcohol, tobacco and firearms department reported that New Delhi had 1,115 registered gun dealers in 2012

(C) The NSS samples 1000 traffic controllers in order to estimate the percent retiring due to job stress related illness

(D) Based on a sample of 250 professional tennis players, a tennis magazine reported that 25% of the parents of all professional tennis players did not play tennis

63. Let $\{x_n\}$ and $\{y_n\}$ be any two Cauchy sequences in a metric space (X, d). Then the sequence $\{d(x_n, y_n)\}$:

(A) always converges

(B) is Cauchy but need not converge

(C) need not be Cauchy

(D) converges to zero

64. The measure most unaffected by outliers is :

(A) median

(B) mean

(C) trimmed mean

(D) range

65. Let A be a class of subsets of a finite set X. For A ∈ A, # A denotes the number of elements in A. Then which of the following function $d : A \times A \to \mathbf{R}$ is a metric on A ?

(A) $d(A, B) = \#(A \cap B^C)$

(B) $d(A, B) = \#(A \cap B)$

(C) $d(A, B) = \min. \{\#(A - B), \#(B - A)\}$

(D) $d(A, B) = \#((A - B) \cup (B - A))$

66. The average score for a class of 25 students was 75. If the 15 female students in the class averaged 70, then the male students in the class averaged :

(A) 85.0

(B) 77.5

(C) 75.0

(D) 82.5

67. Which of the following statements is *true* ?

(A) Every Cauchy sequence in a metric space is convergent

(B) A sequence $\{a_n\}$ is Cauchy sequence in $\mathbf{R}$ if and only if $|a_{n+1} - a_n| \to 0$ as $n \to \infty$

(C) If $|a_{n+1} - a_n| \to 0$ as $n \to \infty$ then $\{a_n\}$ is a Cauchy sequence in $\mathbf{R}$

(D) If $\{a_n\}$ is Cauchy sequence in $\mathbf{R}$ then $|a_{n+1} - a_n| \to 0$ as $n \to \infty$

68. Which of the following is *not* a measure of central tendency ?

(A) mean

(B) median

(C) trimmed mean

(D) variance

69. The order of zero for the function $f(z) = z^3 \sin z$ at the point $z = 0$ is :

(A) 6

(B) 2

(C) 3

(D) 4

70. It is given that $P(A|B) = 0.4$ and $P(A|B^C) = 0.6$. Then :

(A) $0 \le P(A) \le 0.4$

(B) $P(A) = 0.5$

(C) $0.6 \le P(A) \le 1$

(D) $0.4 \le P(A) \le 0.6$

71. The value of the integral

$$\int_{|z|=1} \frac{\cos 2\pi z}{(2z-1)(z-3)} dz$$

is :

(A) $2\pi i$

(B) $\frac{2\pi i}{7}$

(C) $\frac{2\pi i}{5}$

(D) $\frac{\pi i}{5}$

72. Suppose the sample space $\Omega = \{1, 2, 3, 4\}$ and all the sample points are equally likely. Suppose $A = \{1, 2\}$, $B = \{1, 3\}$, and $C = \{1, 4\}$. Then the events A, B and C are :

(A) mutually independent

(B) neither pairwise independent nor mutually independent

(C) pairwise independent but not mutually independent

(D) not pairwise independent

73. An entire and bounded function $f : \mathbb{C} \to \mathbb{C}$ must be :

(A) non-constant

(B) unbounded

(C) constant

(D) discontinuous

74. Suppose

$$P(X = x + 1) = \frac{p(n - x)}{q(x + 1)} \ P(X = x).$$

Then the probability mass function of random variable X is given by :

(A) $P(X = x) = p^n(1 - q^n)^x$, $x = 0, 1, \ldots\ldots$

(B) $P(X = x) = e^{-np}(np)^x/x\,!$, $x = 0, 1, \ldots\ldots, n$

(C) $P(X = x) = \binom{n}{x} p^x q^{n-x}$, $x = 0, 1, \ldots\ldots, n$

(D) $P(X = x) = pq^{x-1}$

75. The function $f : \mathbb{C} \to \mathbb{C}$ defined by :

$$f(z) = |z| = x^2 + y^2$$

is :

(A) differentiable everywhere

(B) an entire function

(C) derivable only at the origin

(D) not continuous at the origin

76. Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample from standard Cauchy distribution. Then distribution of :

$$\bar{X} = \sum_{i=1}^{n} X_i / n$$

is :

(A) N(0, 1)

(B) Asymptotically N(0, 1)

(C) Standard Cauchy

(D) Asymptotically Cauchy distribution

77. Let C be the circle $|z| = 2$. Then the value of $\int_C \frac{3z^2 + z}{z^2 - 1} dz$ is :

(A) 0

(B) πi

(C) $4\pi i$

(D) $2\pi i$

78. Let X be a continuous random variable with cumulative distribution function $F_X(x)$. Define $Y = F_X(X)$. Then the distribution of $-\ln(1 - Y)$ is :

(A) standard normal

(B) uniform (0, 1)

(C) standard Laplace

(D) standard exponential

79. Consider the function $f(z) = \frac{\sin z}{z}$, $z \in \mathbb{C}$. Then

(A) f has a removable singularity at $z = 0$

(B) f has a pole at $z = 0$

(C) f has an essential singularity at $z = 0$

(D) f has a non-removable singularity at $z = 0$

80. Let $X_1, X_2, \ldots\ldots\ldots X_n$ be i.i.d. geometric random variables. Define $S_n = \sum_{i=1}^{n} X_i$. Then distribution of S_n is :

(A) Negative binomial with parameters n and p

(B) Binomial with parameters n and p

(C) Geometric with parameter np

(D) Geometric with parameter $n(1 - p)$

81. The number of elements of order 9 in the group $Z_3 \times Z_9$ is :

(A) 18

(B) 9

(C) 3

(D) 1

82. Let X be non-negative integer valued random variable with mean equal to 1 and variance equal to $\frac{5}{6}$. Then which of the following can be distribution of X ?

(A) Binomial with $n = 6$ and $p = \frac{1}{6}$

(B) Poisson with $\lambda = 1$

(C) Geometric with $p = \frac{1}{6}$

(D) Discrete uniform

83. The number of group homomorphisms from Z_{12} onto Z_8 is :

(A) 4

(B) 0

(C) 2

(D) 1

84. Let $X_1, X_2, \ldots\ldots, X_n$ be a sample from :

$$f(x, a, b) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{otherwise} \end{cases}$$

Then, by the method of moments, the estimators of a and b are :

(A) $\tilde{a} = \bar{X} - \frac{3\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}$, $\tilde{b} = \bar{X} + \frac{3\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}$

(B) $\tilde{a} = \bar{X} - \sqrt{\frac{3\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}}$, $\tilde{b} = \bar{X} + \sqrt{\frac{3\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}}$

(C) $\tilde{a} = \bar{X} - \sqrt{\frac{n}{3\Sigma(X_i - \bar{X})^2}}$, $\tilde{b} = \bar{X} + \sqrt{\frac{n}{3\Sigma(X_i - \bar{X})^2}}$

(D) $\tilde{a} = \bar{X} - \frac{n}{3\sum_{i=1}^{n}(X_i - \bar{X})^2}$, $\tilde{b} = \bar{X} + \frac{n}{3\Sigma(X_i - \bar{X})^2}$

ROUGH WORK

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