Test Booklet Code & Serial No. **B**

प्रश्नपत्रिका कोड व क्रमांक

Paper-III

MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

JAN - 30318

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **145**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. (*a*) This paper consists of **One hundred forty five (145)** multiple choice questions, each question carrying **Two (2)** marks.
   (*b*) There are *three* sections, **Section-I, II, III** in this paper.
   (*c*) Students should attempt all questions from **Sections I and II** *or* **Sections I and III**.
   (*d*) Below each question, four alternatives or responses are given. Only one of these alternatives is the 'CORRECT' answer to the question.
   (*e*) The OMR sheets with questions attempted from both the Sections viz. II & III, will not be assessed.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. (*a*) या प्रश्नपत्रिकेत एकूण **एकशेपंचेचाळीस (145)** बहुपर्यायी प्रश्न दिलेले आहेत, प्रत्येक प्रश्नाला **दोन (2)** गुण आहेत.
   (*b*) या प्रश्नपत्रिकेत **खण्ड**-I, II, III असे तीन खण्ड आहेत.
   (*c*) विद्यार्थ्यांनी **खण्ड-I आणि II** किंवा **खण्ड I आणि III** यांचे सगळे प्रश्न सोडावे.
   (*d*) खाली दिलेल्या प्रश्नाचे चार पर्याय किंवा उत्तर दिलेले आहेत. प्रश्नाचे बहुपर्यायी उत्तरामधून केवळ एक 'बरोबर' आहे.
   (*e*) ओ.एम.आर. उत्तरपत्रिकेच्या क्रमशः दोन्ही खण्ड-II व III मधील सोडवलेले प्रश्नाची आकारणी नाही केली जाईल.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहीलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गांचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

Mathematical Science

Paper III

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Note : Attempt all questions either from Sections I & II or from Sections I & III only. The OMR sheets with questions attempted from both the Sections viz. II & III, will not be assessed.
Section I : Q. Nos. 1 to 5, Section II : Q. Nos. 6 to 75,
Section III : Q. Nos. 76 to 145.

Section I

1. Let $f(x, y) = x^2 + 5xy^2$, then the directional derivative of f at the point $(-2, 1)$ in the direction of vector $v = (12, 5)$ is :

(A) 88/13

(B) –88/13

(C) 78/7

(D) –88

2. Let

$$f(x) = x, \quad \text{if } x \text{ is rational}$$
$$= 0, \quad \text{if } x \text{ is irrational}$$

then :

(A) f is continuous on **R**

(B) f is continuous on **R** except at origin

(C) f is discontinuous at all points except at origin

(D) f is discontinuous at all points of **R**

3. Let $\mathbf{R}_4[t]$ be the vector space of polynomials with degree ≤ 4. Find the matrix of the linear operator $L(f(t)) = (f(t) - f(0))/t$ with respect to the standard basis $\{1, t, t^2, t^3, t^4\}$.

(A) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

4. Suppose that the columns of an $n \times n$-matrix M over $\mathbf{R}$ are orthonormal. Then which of the following is *not* true ?

(A) For every $x \in \mathbf{R}^n$, $\|Mx\| = \|x\|$

(B) For every $x, y \in \mathbf{R}^n$,

$\langle Mx, My \rangle = \langle x, y \rangle$

(C) The rows of M are orthonormal

(D) M is symmetric

5. Let

$$f(x, y) = \frac{x^2 + y}{\sqrt{x^2 + y^2}}, \quad (x, y) \neq (0, 0)$$

$$= 0, \qquad (x, y) = (0, 0).$$

Then :

(A) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at (0, 0)

(B) $f(x, y)$ is continuous at (0, 0)

(C) $f(x, y)$ is differentiable at (0, 0)

(D) $f(x, y)$ is continuously differentiable at (0, 0)

Section II

6. Which of the following subsets of $\mathbf{R}^2$ are not compact ?

(*i*) a circle

(*ii*) a parabola

(*iii*) $S = \left\{(x, y) \in \mathbf{R}^2 \middle/ y = \sin\frac{1}{x}, 0 < x \leq 1\right\}$

(*iv*) $T = \left\{(x, y) \in \mathbf{R}^2 / |x| + |y| = 4\right\}$

(A) (*i*) and (*ii*)

(B) (*ii*) and (*iii*)

(C) (*iii*) and (*iv*)

(D) (*ii*), (*iii*) and (*iv*)

7. A metric space is compact if and only if it is complete and :

(A) bounded

(B) totally bounded

(C) closed

(D) countable

8. Consider the following statements :

I. A function f on $[a, b]$ is of bounded variation if, and only if, f is the difference of finite-valued monotone increasing functions on $[a, b]$.

II. If the function f is of bounded variation on $[a, b]$, then f is measurable.

(A) Only I is true

(B) Only II is true

(C) Both I and II are not true

(D) Both I and II are true

9. Consider the functions

$f, g : [0, 1] \to \mathbf{R}$ defined by :

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

and

$$g(x) = \begin{cases} 1/x & \text{if } 0 < x \leq 1 \\ 0 & \text{if } x = 0 \end{cases}$$

then :

(A) both f and g are not Lebesgue integrable

(B) f is Riemann integrable and g is Lebesgue integrable

(C) f is not Riemann integrable and g is Lebesgue integrable

(D) f is Lebesgue integrable and g is not Lebesgue integrable

10. The Lebesgue integral of the function $f : [0, 1] \to \mathbf{R}$ defined by :

$$f(x) = \begin{cases} 1/\sqrt[3]{x} & \text{if } 0 < x \le 1 \\ 0 & \text{if } x = 0 \end{cases}$$

is :

(A) $\frac{3}{4}$

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $\frac{4}{3}$

11. For which of the following values of n is every group of order n abelian ?

(A) $n = 100$

(B) $n = 121$

(C) $n = 24$

(D) $n = 120$

12. Which of the following groups is *not* solvable ?

(A) $\mathbf{Z}_{150}$

(B) S_4

(C) S_5

(D) a non-abelian group of order 27

13. Which of the following rings is *not* Noetherian ?

(A) $\mathbf{R}[x_1, x_2, ...]$

(B) $\mathbf{Z}[i]$

(C) $\mathbf{Z}$

(D) $\mathbf{Z}_{205}[x]$

14. The degree of the splitting field of the polynomial $x^3 - 2 \in \mathbf{Q}[x]$ is :

(A) 3

(B) 6

(C) 5

(D) 4

15. Which of the following constructions is *not* possible by ruler and compass ?

(A) Trisection of an angle of 90°

(B) Construction of a regular polygon of 7 sides

(C) Construction of length equal to $1 + \sqrt{2} + \sqrt[4]{2}$

(D) Construction of the complex number $e^{\pi i/40}$

16. Let S be a subset of a normed space X and [S] be the span of S in X. Then which of the following need *not* be true ?

(A) [S] is a subspace of X

(B) [S] is a closed subspace of X

(C) $\overline{[S]}$ is a subspace of X

(D) $\overline{[S]}$ is a closed subset of X

17. Let X be a normed space such that every closed and bounded set in X is compact. Then which of the following is *true* ?

(A) X has a countably infinite basis

(B) Every bounded subset of X is complete

(C) X is finite dimensional

(D) Every proper subspace of X is bounded

18. Let X be a separable inner product space and Y be any orthonormal set in X. Then :

(A) Y is linearly dependent

(B) Y is countably infinite

(C) Y is a basis (Hamel) of X

(D) Y is countable

19. An infinite dimensional, separable complex Hilbert space is congruent to :

(A) l_∞

(B) l_2

(C) l_1

(D) l_3

20. Denote the dual space of the normed linear space X by $\tilde{X}$. Then which of the following is *true* ?

(A) $\tilde{X}$ is separable $\Rightarrow$ X is separable

(B) X is separable $\Rightarrow$ $\tilde{X}$ is separable

(C) $\tilde{\tilde{\tilde{X}}} \cong X$

(D) $\tilde{\tilde{\tilde{X}}}$ is always separable

21. Let f: [0, 1] $\to$ [0, 1] be a continuous function. Then which of the following is *true* ?

(A) For all $x, y \in [0, 1]$,

$x \le y \Rightarrow f(x) \le f(y)$

(B) $\exists\ x \in [0, 1]$ such that $f(x) = x$

(C) If for some $x, y \in [0, 1]$ with $x \le y$, $f(x) \le f(y)$ then for all $x \le y$, $f(x) \le f(y)$

(D) If there is $x_0 \in [0, 1]$, $f(x_0) = x_0$ then $f(x) = x$ for all $x \in X$

22. Which of the following spaces need not be Hausdorff ?
    - (A) X is a topological space such that $\Delta = \{x \times x / x \in X\}$ is closed in $X \times X$
    - (B) X is a metric space
    - (C) X is an Indiscrete Topological space
    - (D) X is the product of two Hausdorff spaces

23. Let X be a non-empty subset of **R** such that for every positive integer M there is $x \in X$ such that $|x| \geq M$. Then which of the following statements is *true* ?
    - (A) X is bounded with respect to some metric which induces the same topology as induced by the standard metric on **R**
    - (B) X is unbounded with respect to every metric on **R**
    - (C) The set X contains rationals as well as irrationals
    - (D) X is countably infinite

24. Let X be a locally connected topological space. Then which of the following is *true* ?
    - (A) X is connected
    - (B) X is locally path connected
    - (C) Every component of an open set of X is open in X
    - (D) Every component of X is also a path component of X

25. What is the coefficient of $x^2y^2z^3$ in $(x + y + z)^7$?
    - (A) 630
    - (B) 210
    - (C) 420
    - (D) 179

26. The dual statement of $p \Rightarrow q \wedge r$ is :

(A) $(p \wedge q) \vee (p \wedge r)$

(B) $(\sim p) \vee (q \vee r)$

(C) $p \vee (\sim q) \vee (\sim r)$

(D) $((\sim p) \wedge q) \vee ((\sim p) \wedge r)$

27. Find the number of integers between 1 and 10,000 both inclusive which are divisible by none of 5, 6 or 8 :

(A) 3666

(B) 5000

(C) 6250

(D) 6000

28. Find a_{12} if $a_{n+1}^2 = 5a_n^2$, where $a_n > 0$ for $n \geq 0$ and $a_0 = 2$:

(A) 31250

(B) 31000

(C) 30350

(D) 30250

29. Eigenvalues corresponding to the following Sturm-Liouville problem :

$$\frac{d^2X}{dt^2} + \lambda X = 0, \ X(0) = 0, \ X'(1) = 0$$

are λ_n, $n \in \mathbf{N}$, where $\lambda_n =$

(A) $\left[\frac{(2n-1)\pi}{2}\right]^2$

(B) $(2n\pi)^2$

(C) $\left(\frac{n\pi}{2}\right)^2$

(D) $(n + 1)\pi$

30. Consider the boundary value problem :

$u_t(x, t) = u_{xx}(x, t), 0 < x < c, t > 0,$

with conditions :

$u_x(0, t) = 0 = u_x(c, t), t > 0,$

$u(x, 0) = f(x), 0 < x < c.$

Substituting $u(x, t) = \mathrm{X}(x)\,\mathrm{T}(t)$ in the above equation we generate the following Strüm-Liouville problem for X(x) :

(A) $\mathrm{X}''(x) + \lambda \mathrm{X}(x) = 0, \mathrm{X}(0) = \mathrm{X}(c) = 0$

(B) $\mathrm{X}''(x) + \lambda \mathrm{X}(x) = 0, \mathrm{X}'(0) = \mathrm{X}'(c) = 0$

(C) $\mathrm{X}''(x) + \lambda \mathrm{X}(x) = 0, \mathrm{X}(0) = 1, \mathrm{X}(c) = 0$

(D) $\mathrm{X}''(x) + \lambda \mathrm{X}(x) = 0, \mathrm{X}'(0) = 0, \mathrm{X}(c) = 0$

31. The differential equation by eliminating arbitrary constant a from :

$$y = a(x - a)^2$$

is :

(A) $\left(\frac{dy}{dx}\right)^3 - 4xy\frac{dy}{dx} + 8y = 0$

(B) $\left(\frac{dy}{dx}\right)^3 - 4xy\frac{dy}{dx} + 8y^2 = 0$

(C) $\left(\frac{dy}{dx}\right)^2 - 4xy\frac{dy}{dx} + 8y^2 = 0$

(D) $\left(\frac{dy}{dx}\right)^2 - 4xy\frac{dy}{dx} + 8y = 0$

32. The general solution of the partial differential equation :

$$xzp - yzq = y^2 - x^2, \left(\frac{\partial z}{\partial x} = p, \frac{\partial z}{\partial y} = q\right)$$

is :

(A) $\phi(xy, x^2 + y^2 + z^2) = 0$

(B) $\phi(x^2 - y^2, x^2 + y^2 + z^2) = 0$

(C) $\phi(xyz, x^2 - y^2) = 0$

(D) $\phi(xyz, x^2 + y^2) = 0$

33. The following partial differential equation :

$$\left(x^2 + z^2\right)\frac{\partial z}{\partial x} - xy\frac{\partial z}{\partial y} = z^3 x + y^2$$

is :

(A) linear

(B) non-linear

(C) semi-linear

(D) Quasi-linear

34. Which of the following natural numbers cannot be written as a sum of 3 squares of non-negative integers ?

(A) 2013

(B) 2015

(C) 2017

(D) 2019

35. For which of the following primes p, is the quadratic congruence $x^2 \equiv 2(\text{mod } p)$ solvable ?

(A) $p = 5$

(B) $p = 13$

(C) $p = 19$

(D) $p = 23$

36. Let $\varphi(n)$ denote Euler's function. Then :

$$\sum_{d|24} \varphi(d)$$

is equal to :

(A) 24

(B) 25

(C) 23

(D) 20

37. The degrees of freedom of a rigid body are :

(A) 1

(B) 2

(C) 3

(D) 6

38. If a particle of mass m moves in a plane under the influence of gravitational force of magnitude $\frac{m}{r^2}$ directed towards origin. Then Lagrangian of the system is :

(A) $\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{m}{r}$

(B) $\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{m}{r}$

(C) $\frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{m}{r^2}$

(D) $\frac{m}{2}\left(\dot{r}^2 + r\dot{\theta}^2\right) + \frac{m}{r}$

39. If the Lagrangian $L(x, \dot{x})$ corresponding to Atwood's machine is given as :

$$L(x, \dot{x}) = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + m_1 gx + m_2 g(l - x),$$

where m_1, m_2, l and g are constants.

Then the equation of motion is :

(A) $\dot{x} = \frac{m_1 - m_2}{m_1 + m_2} g$

(B) $\dot{x} = \frac{m_1 + m_2}{m_1 - m_2} g$

(C) $\ddot{x} = \frac{m_1 - m_2}{m_1 + m_2} g$

(D) $\ddot{x} = \frac{m_1 + m_2}{m_1 - m_2} g$

40. If Lagrangian of a system is :

$$L(\theta, \dot{\theta}) = \frac{1}{2} m\left(l^2\dot{\theta}^2 - gl\theta^2\right),$$

where g, l are constants. Then the Hamiltonian of the system is :

(A) $\dfrac{P_\theta^2}{2ml^2} - \dfrac{1}{2} mgl\theta^2$

(B) $\dfrac{P_\theta^2}{2ml^2} + \dfrac{1}{2} mgl\theta^2$

(C) $\dfrac{P_\theta^2}{ml^2} + mgl\theta^2$

(D) $\dfrac{P_\theta^2}{2ml^2} + mgl\theta^2$

41. Let $\alpha : (0, 1) \to \mathbf{R}^3$ given by $\alpha(s) = (s, s + 1, s^2)$ be a curve parametrised by arc length s. Then the curvature of α at s is :

(A) 2

(B) $2s$

(C) $\sqrt{2 + 4s^2}$

(D) s

42. Let $\alpha : \left(\frac{1}{2}, 2\right) \to \mathbf{R}^3$ be defined by $\alpha(t) = (t, -t, t^3)$. Then the torsion of α at $t = 1$ is :

(A) 1

(B) –1

(C) 0

(D) 2

43. The right cylinder over the circle $x^2 + y^2 = 1$ has the parametrization $\bar{x} : U \to \mathbf{R}^3$, where :

$$U = \left\{(u, v) \in \mathbf{R}^2 \mid 0 < u < 2\pi, -\infty < v < \infty\right\},$$

$$\bar{x}(u, v) = (\cos u, \sin u, v)$$

Then the coefficients E, F, G in the first fundamental form of the cylinder are given by :

(A) E = 1, F = 1, G = 1

(B) E = 1, F = 0, G = 1

(C) $E = \sin^2 u$, F = 0, G = 1

(D) $E = \cos u$, F = 0, G = 1

44. Consider the functional :

$$I\left(u(x, y)\right) = \iint_{G} F\left(x, y, u, u_x, u_y\right) dx\,dy$$

over the region of integration G, where u is continuous and has continuous derivatives upto second order. Further u takes prescribed values on the boundary of G. Then the differential equation for the extremization of the functional is :

(A) $\frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_y}\right) = 0$

(B) $\frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right) = 0$

(C) $\frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_x}\right) = 0$

(D) $\frac{\partial F}{\partial u} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial u_y}\right) = 0$

45. The extremal of the functional :

$$I\left(y(x)\right) = \int_1^2 \frac{x^3}{(y')^2}\, dx$$

subject to the conditions $y(1) = 0$, $y(2) = 3$ is :

(A) $y = (x^2 - 1)x$

(B) $y = (x^2 - 1)$

(C) $y = (x - 1)x$

(D) $y = (x - 1)$

46. The extremal of the isoperimetric problem :

$$I\left(y(x)\right) = \int_1^4 (y')^2\, dx$$

subject to the conditions :

$$\int_1^4 y\, dx = 36$$

and $y(1) = 3$, $y(4) = 24$ is :

(A) a parabola

(B) an ellipse

(C) a hyperbola

(D) a circle

47. The integral equation :

$$e^t + 2\int_0^1 e^{(t-s)} x(s) = 0$$

is a :

(A) linear Fredholm integral equation of second kind

(B) linear Volterra integral equation of second kind

(C) linear Fredholm integral equation of first kind

(D) linear Volterra integral equation of first kind

48. The initial value problem corresponding to the integral equation :

$$x(t) = x - \cos x + \int_0^x (x-t)\,x(t)\,dt$$

is :

(A) $x''(t) + x(t) = \sin t,\ x(0) = 1,\ x'(0) = 0$

(B) $x''(t) + x'(t) + x(t) = \sin t,\ x(0) = 1,\ x'(0) = 0$

(C) $x''(t) - x(t) = \cos t,\ x(0) = -1,\ x'(0) = 1$

(D) $x''(t) - x'(t) - x(t) = \cos t,\ x(0) = -1,\ x'(0) = 1$

49. The solution of the singular integral equation :

$$\pi t = \int_0^t \frac{1}{\sqrt{t-s}}\, x(s)\, ds$$

is :

(A) 2π

(B) $2\pi\sqrt{x}$

(C) $2\sqrt{x}$

(D) $\pi\sqrt{x}$

50. The value of $\left(\frac{\Delta^2}{E}\right) x^2$ (taking $h = 1$) is :

(A) -1

(B) 0

(C) 1

(D) 2

where :

Δ — forward difference operator

E — shift operator.

51. The cubic polynomial which takes the following values :

x	$f(x)$
0	1
1	2
3	1
3	10

by Newton's forward interpolation formula is :

(A) $4x^3 - 2x^2 + 5x + 1$

(B) $2x^3 - 7x + 6x + 1$

(C) $2x^3 - x^2 + 8x + 1$

(D) $4x^3 - 3x^2 + 6x + 1$

52. A curve passing through the points as given in the table :

x	$y(x)$
1	0.2
2	0.7
3	1
4	1.3
5	1.5
6	1.7
7	1.9
8	2.1
9	2.3

the area bounded by the curve, the x-axis, $x = 1$ and $x = 9$ is :

(A) 11.5 sq. unit

(B) 12.0 sq. unit

(C) 10.5 sq. unit

(D) 10.2 sq. unit

53. If $u_0 = 1$, $u_1 = 5$, $u_2 = 8$, $u_3 = 3$, $u_4 = 7$, $u_5 = 0$, then the value of $\Delta^5 u_0$ is :

(A) 24

(B) –60

(C) 60

(D) –61

where Δ forward difference operator.

54. If $L^{-1}\{F(s)\} = f(t)$, then

$L^{-1}\{F^{(n)}(s)\} =$ is :

(A) $t^n f(t)$

(B) $(-1)^n t^n f(t)$

(C) $(-1)^n f(t)$

(D) $(-1)^n f^{(n)}(t)$

where L^{-1} be the inverse Laplace transform

55. The solution of the integral equation :

$$x(t) = e^{-t} + \int_0^t \sin(t-\tau)\, x(\tau)\, d\tau$$

by using Laplace transform method is :

(A) $x(t) = e^{-2t} + 2t - 1$

(B) $x(t) = 2e^{-2t} + t - 1$

(C) $x(t) = e^{-t} + 2t - 1$

(D) $x(t) = 2e^{-t} + t - 1$

56. If $\tilde{F}(s)$ is the Fourier transform of $f(t)$, then the Fourier transform of :

$$f(t) \cos at$$

is :

(A) $\tilde{F}(s+a)$

(B) $\tilde{F}(s-a)$

(C) $\frac{1}{2}\left[\tilde{F}(s+a) + \tilde{F}(s-a)\right]$

(D) $\frac{1}{2}\left[\tilde{F}(s+a) - \tilde{F}(s-a)\right]$

57. Let μ^* be an outer measure on $H(\mathbf{R})$.

Then $E \in H(\mathbf{R})$ is μ^*-measurable if :

(A) for each $A \in H(\mathbf{R})$

$$\mu^*(A) = \mu^*(A \cap E) + \mu^*(A \cap E^C)$$

(B) for some $A \in H(\mathbf{R})$

$$\mu^*(A) = \mu^*(A \cap E) + \mu^*(A \cap E^C)$$

(C) for each $A \in H(\mathbf{R})$

$$\mu^*(A) \le \mu^*(A \cap E) + \mu^*(A \cap E^C)$$

(D) for some $A \in H(\mathbf{R})$

$$\mu^*(A) \le \mu^*(A \cap E) + \mu^*(A \cap E^C)$$

where $H(\mathbf{R})$-hereditary σ-ring.

58. Consider the following statements :

I. A measure μ on a ring $\mathbf{R}$ is complete if $E \in \mathbf{R}$, $F \subseteq E$ and $\mu(E) = 0$ then $F \in \mathbf{R}$.

II. A measure μ on a ring $\mathbf{R}$ is σ-finite if, for every set $E \in \mathbf{R}$, $E = \bigcup_{n=1}^{\infty} E_n$ for some sequence $\{E_n\}$ such that $E_n \in \mathbf{R}$.

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) Both I and II are not true

59. Let E and F be measurable sets, $f \in L(E, \mu)$ and $\mu(E \,\Delta\, F) = 0$, then :

(A) $f \in L(F, \mu)$ and

$$\int_E f\,d\mu = \int_F f\,d\mu$$

(B) $f \in L(F, \mu)$ and

$$\int_E f\,d\mu < \int_F f\,d\mu$$

(C) $f \in L(F, \mu)$ and

$$\int_E f\,d\mu > \int_F f\,d\mu$$

(D) $f \notin L(F, \mu)$

60. Consider the following statements :

I. If v_1, v_2 and μ are measures and $v_1 \perp \mu$, $v_2 \perp \mu$, then $v_1 + v_2 \perp \mu$.

II. If v and μ are measures such that $v << \mu$ and $v \perp \mu$, then v is identically zero.

(A) Only I is true

(B) Only II is true

(C) Both I and II are not true

(D) Both I and II are true

61. Suppose $f(x, y)$ is of class c^2, then under the coordinate transformation $x = r\cos\theta$, $y = r\sin\theta$, the expression $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ transforms to :

(A) $\frac{\partial^2 f}{\partial r^2} + \frac{\partial^2 f}{\partial \theta^2}$

(B) $\frac{\partial^2 f}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$

(C) $\frac{\partial^2 f}{\partial r^2} + \frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$

(D) $\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$

62. For the function $f(x, y) = (x - 1)(x^2 - y^2)$, the point $\left(\frac{2}{3}, 0\right)$:

(A) is not a critical point

(B) is a local maximum

(C) is a local minimum

(D) is a saddle point

63. The improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges :

(A) $\forall\ n$

(B) if $n > 1$

(C) if n is an integer

(D) only if $0 < n < 1$

64. The functions :

$f_1(x, y, z) = x + y + z$

$f_2(x, y, z) = xy + yz + zx$

$f_3(x, y, z) = x^2 + y^2 + z^2$

(A) are functionally independent on $\mathbf{R}^3/\{x\text{-axis}\}$

(B) are functionally independent on the set $\{(x, y, z) \in \mathbf{R}^3/x > 0\}$

(C) are functionally independent on the set $\{(x, y, z) \in \mathbf{R}^3/x > 0, y > 0, z > 0\}$

(D) are functionally dependent on $\mathbf{R}^3$

65. Let R be a Boolean ring with identity having n elements, then which of the following values of n is possible ?

(A) $n = 32$

(B) $n = 23$

(C) $n = 27$

(D) $n = 15$

66. Let R be a ring with identity 1 ($\neq 0$) and m be the characteristic of R, then which of the following is *true* ?

(A) wherever R is infinite, $m = 0$

(B) wherever $m \neq 0$, R is finite

(C) wherever R is finite, $m \neq 0$

(D) wherever $m \neq 0$, R is infinite

67. Let F be a field with at least two elements. Then which of the following statements need *not* be *true* ?

(A) Either $\mathbf{Z}$ or $\mathbf{Z}_p$ for some integer $p \geq 2$ is embedded in F

(B) If n is not prime, then $\mathbf{Z}_n$ is not embedded in F

(C) If $\mathbf{Z}$ is embedded in F, then the field Q is also embedded in F

(D) If F is infinite, then $\mathbf{Z}_p$ cannot be embedded in F for any prime p

68. Which of the following rings is a Unique Factorization domain but not a PID ?

(A) $\mathbf{Z}$

(B) F[x] (F a field)

(C) $\mathbf{Z}[x]$

(D) $\mathbf{Z}[i]$

69. Which of the following statements is *false* for $\mathbf{Q}\left[\sqrt{2}\right]$ and $\mathbf{Z}\left[\sqrt{2}\right]$?

(A) There are infinitely many units in $\mathbf{Z}\left[\sqrt{2}\right]$

(B) $\mathbf{Q}\left[\sqrt{2}\right]$ is not a subfield of complex numbers

(C) $\mathbf{Q}\left[\sqrt{2}\right]$ is isomorphic to $\mathbf{Q}[x]/\left(x^2 - 2\right)$

(D) There are infinitely many primes in $\mathbf{Z}\left[\sqrt{2}\right]$

70. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{\left(n^2+1\right)2^n}$ is :

(A) $\frac{1}{2}$

(B) 1

(C) ∞

(D) 2

71. For which of the following functions does the function take complex values arbitrarily close to any complex number, inside any arbitrary neighbourhood of 0 ?

(A) $f(z) = e^z$

(B) $f(z) = \sin\left(\frac{1}{z}\right)$

(C) $f(z) = z^3 + z^2 + \frac{1}{z}$

(D) $f(z) = \cosh z + \sinh^2 z$

72. Which of the following continuous functions is *not* an open mapping from $\mathbf{C}$ to $\mathbf{C}$?

(A) $f(z) = e^{z^2}$

(B) $f(z) = |z|$

(C) $f(z) = \sin z + \cos z^2$

(D) $f(z) = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + ...$

73. The bilinear transformation :

$f(z) = \dfrac{3z+4}{5z+6}$ from $\mathbf{C} - \left\{-\dfrac{6}{5}\right\}$ to $\mathbf{C}$

assumes all complex values, *except* :

(A) $\dfrac{5}{3}$

(B) $-\dfrac{4}{3}$

(C) $-\dfrac{6}{5}$

(D) $\dfrac{3}{5}$

74. Which of the following complex functions has an infinite number of poles ?

(A) e^z

(B) $\sec z$

(C) $\cos\left(\dfrac{1}{z}\right)$

(D) $\dfrac{1}{z^5 + z + 1}$

75. Let

$A = \left\{(x, y) \in \mathbf{R}^2 / x = 0, -1 \le y \le 1\right\}$

$B = \left\{(x, y) \in \mathbf{R}^2 \Big/ 0 < x \le \dfrac{1}{\pi}, \ y = \sin\dfrac{1}{x}\right\}$

and X = A ∪ B. Then :

(A) X is connected and path connected

(B) X is connected but not path connected

(C) X is not connected but path connected

(D) X is neither connected nor path connected

Section III

76. Let $X_1, X_2, \ldots\ldots\ldots\ X_n$ be a random sample from $N(\mu, \sigma^2)$, where μ and σ^2 are unknown. Then which of the following statements is *not* correct ?

(A) $\left(\sum_1^n X_i, \sum_1^n X_1^2\right)$ is jointly sufficient for (μ, σ^2)

(B) $\left(\bar{X}, s^2\right)$ is jointly sufficient for (μ, σ^2)

(C) $\bar{X}$ is sufficient for μ and s^2 is sufficient for σ^2

(D) If σ^2 is known then $\bar{X}$ is sufficient for μ

77. Let X be a r.v. having the pmf,

$$P[X = x] = \left(\frac{\theta}{2}\right)^{|x|} (1 - \theta)^{1 - |x|};$$

$$x = -1, 0, 1, 0 < \theta < 1$$

The complete statistics for θ :

(A) is X

(B) is $|X|$

(C) is X^2

(D) Does not exist

78. Let $X_1, X_2, \ldots\ldots\ldots\ X_n$ be iid r.v.s with $U(\theta, \theta + 1)$, then $E[X_{(n)} - X_{(1)}]$, where $X_{(n)} = \text{Max } X_i$, and $X_{(1)} = \text{Min } X_i$, is given by :

(A) $\frac{n+1}{n-1}$

(B) $\frac{1}{n+1}$

(C) $\frac{n-1}{n+1}$

(D) $\frac{n}{n-1}$

79. Let $X_1, X_2, \ldots\ldots\ldots\ldots X_n$ be iid r.v.s satisfying the following regression equation :

$X_i = \alpha z_i + e_i;\ i = 1, 2, \ldots\ldots\ n$

where $z_1, z_2, \ldots\ldots\ z_n$ are fixed and e_i's ($i = 1, 2, \ldots\ldots n$) are iid r.v.s with $N(0, \sigma^2)$, σ^2 is unknown MLE of α is given as :

(A) $\dfrac{\sum_1^n x_i z_i}{\sum_1^n x_i^2}$

(B) $\dfrac{\sum_1^n x_i z_i}{n}$

(C) $\dfrac{\sum z_i^2}{n}$

(D) $\dfrac{\sum x_i z_i}{\sum z_i^2}$

80. If, for a given α, $0 < \alpha < 1$, non-randomized Neyman-Pearson and likelihood ratio tests of a simple hypothesis against a simple alternative exists, then :

(A) They are equivalent

(B) They are one and the same

(C) They are exactly opposite

(D) One can't say anything about it

81. A sample of size n is obtained from a Poisson distribution with parameter m. The most powerful (MP) test of less than size α to test $H_0 : m \leq m_0$ against $H_1 : m > m_0$ is given as :

(A) $\phi(x) = \begin{cases} 1 & ; \quad T > t_0 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(B) $\phi(x) = \begin{cases} 1 & ; \quad T > t_0 \\ \gamma & ; \quad T = t_0 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(C) $\phi(x) = \begin{cases} 1 & ; \quad T < t_0 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(D) $\phi(x) = \begin{cases} 1 & ; \quad T < t_0 \\ \gamma & ; \quad T = t_0 \\ 0 & ; \quad \text{otherwise} \end{cases}$

where $T = \sum_1^n X_i$

82. Let $X_1, X_2, \ldots\ldots\ldots\ldots X_n$ be iid random sample of size n from exponential distribution with mean θ. The MP test of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 < \theta_0$, is :

(A) $\phi(x) = \begin{cases} 1 \ ; & T < \dfrac{2\chi^2_{2n, 1-\alpha}}{\theta_0} \\ 0 \ ; & \text{otherwise} \end{cases}$

(B) $\phi(x) = \begin{cases} 1 \ ; & T < \dfrac{\theta_0\chi^2_{2n, 1-\alpha}}{2} \\ 0 \ ; & \text{otherwise} \end{cases}$

(C) $\phi(x) = \begin{cases} 1 \ ; & T < \dfrac{\theta_0\chi^2_{2n, \alpha}}{2} \\ 0 \ ; & \text{otherwise} \end{cases}$

(D) $\phi(x) = \begin{cases} 1 \ ; & T < \dfrac{2\chi^2_{2n, \alpha}}{\theta_0} \\ 0 \ ; & \text{otherwise} \end{cases}$

where $T = \sum_{1}^{n} X_i$

83. A test $\phi(x)$ is called an unbiased test if :

(A) $E_{H_0}\phi(X) \le \alpha$

(B) $E_{H_1}\phi(X) \ge \alpha$

(C) $E_{H_0}\phi(X) \le \alpha$ and $E_{H_1}\phi(X) \ge \alpha$

(D) $E_{H_0}\phi(X) = \alpha = E_{H_1}\phi(X)$

84. Let $X_1, X_2, \ldots\ldots\ldots\ldots X_n$ be a random sample of size n observed from Cauchy distribution with location parameter θ. Define T_1 = sample mean and T_2 = sample median. Then :

(A) T_1 and T_2 are consistent estimator of θ

(B) T_1 is asymptotically normal

(C) T_2 is asymptotically normal

(D) $(T_1 + T_2)/2$ is asymptotically normal

85. Let $F_n(\cdot)$ be empirical cumulative distribution function, based on a random of size n from a continuous distribution with cumulative distribution function $F(\cdot)$. Then :

(A) F_n is not a consistent estimator for F

(B) Variance of F_n converges to a positive constant

(C) Asymptotic distribution of F_n is normal

(D) Asymptotic mean of F_n is 1/2

86. Based on a random sample of size n from Poisson distribution with mean λ, asymptotic variance of $\bar{X} e^{-\bar{X}}$ is :

(A) $\dfrac{\lambda e^{-\lambda}}{n}$, $\lambda > 0$

(B) $\dfrac{\lambda}{n}(1-\lambda)^2 e^{-2\lambda}$, $\lambda \neq 1$

(C) $\dfrac{\lambda}{n}(1-\lambda) e^{-\lambda}$, $\lambda \neq 1$

(D) $\dfrac{\lambda}{n}$, $\lambda > 0$

87. Consider the problem of testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$, where θ_0 is specified value of θ and is the parameter of one parameter Cramer family. If $\lambda(\underset{\sim}{x})$ is the likelihood ratio statistic based on sample of size n,

(A) $-2 \log \lambda(\underset{\sim}{x}) \to \chi_1^2 \;\forall\; \theta$ as $n \to \infty$

(B) $-2 \log \lambda(\underset{\sim}{x}) \to \chi_n^2 \;\forall\; \theta$ as $n \to \infty$

(C) $-2 \log \lambda(\underset{\sim}{x}) \to \chi_1^2$, for $\theta = \theta_0$, as $n \to \infty$

(D) $-2 \log \lambda(\underset{\sim}{x}) \to N(0, 1)$, for $\theta = \theta_0$ as $n \to \infty$

88. Let $X_1, X_2, \ldots\ldots\ldots X_n$ be the random sample observed from Poisson distribution with mean λ. Then consistent estimator of $1 - e^{-\lambda}$ is :

(A) unique

(B) not unique

(C) function of $\bar{X}$ only

(D) not asymptotically normally distributed

89. If $\underset{\sim}{X}$ be a p-component random vector with $E(\underset{\sim}{X}) = \underset{\sim}{0}$ and variance covariance matrix Σ, positive definite. If $\underset{\sim}{X}$ is partitioned into $\underset{\sim}{X}^{(1)}$ of p_1 components and $\underset{\sim}{X}^{(2)}$ of p_2 components such that $p_1 + p_2 = p$ and $p_1 \leq p_2$. Then the square of canonical correlation are the characteristic roots of the matrix :

(A) Σ

(B) $\Sigma_{11}^{-1} \Sigma_{12}$

(C) $\Sigma_{22}^{-1} \Sigma_{21}$

(D) $\Sigma_{11}^{-1} \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$

90. If T^2 is Hotelling T^2-statistic, then the distribution of $\frac{T^2}{n-1}\frac{n-p}{p}$ would be :

(A) Non-central F-distribution

(B) Central F-distribution

(C) Chi-square distribution

(D) Student t-distribution

91. If $\underset{\sim}{x}_1, \underset{\sim}{x}_2, ..., \underset{\sim}{x}_n$ $(n \geq p + 1)$ are distributed independently each according to $N_p(\underset{\sim}{\mu}, \Sigma)$, then the distribution of

$$S = \frac{1}{n-1}\sum_{\alpha=1}^{n}\left(\underset{\sim}{x}_\alpha - \bar{\underset{\sim}{x}}\right)\left(\underset{\sim}{x}_\alpha - \bar{\underset{\sim}{x}}\right)'$$

is :

(A) $W_p(n - 1, \Sigma/(n - 1))$

(B) $W_p(n, \Sigma/n)$

(C) $N_p(\underset{\sim}{\mu}, \Sigma)$

(D) $N_p(\underset{\sim}{\mu}, S)$

92. If $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, then the distribution of $N(\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})' \Sigma^{-1}(\bar{\underset{\sim}{X}} - \underset{\sim}{\mu})$ would be :

(A) Multivariate normal distribution

(B) χ^2 with $N - p$ degree of freedom

(C) χ^2 with p degree of freedom

(D) χ^2 with N degree of freedom

93. Let $\underset{\sim}{X}$ a p-component random vector is partitioned into $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$ where $\underset{\sim}{X}^{(1)}$ has q-components and $\underset{\sim}{X}^{(2)}$ has $(p - q)$ components and $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$. If $\underset{\sim}{\mu}$ and Σ are also partitioned as of $\underset{\sim}{X}$, then the variance covariance matrix of the conditional distribution of $\underset{\sim}{X}^{(1)}$ given $\underset{\sim}{X}^{(2)}$ is :

(A) $\Sigma_{11} + \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$

(B) $\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$

(C) $\Sigma_{22} + \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$

(D) $\Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$

94. Given $(X, Y) \sim N_2(5, 10, 1, 25, \rho)$ and $P[4 < Y < 16 \mid X = 5] = 0.954$, then ρ is equal to :

(A) +0.80

(B) –0.80

(C) ±0.8

(D) 0.96

(It is given that

$$\frac{1}{\sqrt{2\pi}} \int_0^2 e^{-\frac{1}{2}z^2} dz = 0.477)$$

95. In a multiple regression with 3 regressors and 10 observations, the total variation is found to be 25.549, whereas the explained variation by the regression is 24.875. What is the value of adjusted-R^2 ?

(A) 0.964

(B) 0.962

(C) 0.974

(D) 0.982

96. For a standard multiple regression model $\{Y, X\beta, \sigma^2 I\}$, which of the following statements is *not* true ?

(A) $E(\hat{Y}) = E(HY) = X\beta$, where $H = X(X'X)^{-1}X'$

(B) $Cov(\hat{Y}) = \sigma^2 H$

(C) $Cov(\hat{Y}, Y - \hat{Y}) = 0$

(D) $\hat{Y} \sim N_n(\beta, \sigma^2 H)$, under normality

97. Under the standard multiple regression model $\{Y, X\beta, \sigma^2\Omega\}$, which of the following statements is *not* true ?

(A) $E(\hat{Y}) = X\beta$, $E(\hat{\beta}) = \beta$

(B) $Cov(\hat{Y}) = \sigma^2\Omega$

(C) $Cov(\hat{\beta}) = \sigma^2(X'\Omega^{-1}X)^{-1}$

(D) $Cov(HY, MY) = \sigma^2 H\Omega M$

98. Consider the multiple regression set-up $\{Y, X\beta, \sigma^2 I\}$ with *p*-regressors. The ordinary least squares estimator (OLSE) $\hat{\beta}$ is the vector which minimizes $E(Y - X\beta)(Y - X\beta)'$. Which of the following statements is *not* true under this set-up ?

(A) $X\hat{\beta}$ is always unique

(B) $\hat{\beta}$ is unique if and only if rank $(X) = p$

(C) $\hat{\beta}$ is unique if and only if rank $(X) < p$

(D) OLSE $K'\hat{\beta}$ of $K'\beta$ is unique if and only if $K'\beta$ is estimable

99. Which of the following statements is *not* true in the context of the transformation of a response variable under a regression set-up ?

(A) When $E(Y) = V(Y) = C$ (a constant), square root transformation is more appropriate

(B) When $E(Y) = C$ and $V(Y) = C^2$, log transformation is more appropriate

(C) Scale transformation preserve the directions of the association between Y and X

(D) The Box-Cox transformation $Y \rightarrow g(Y, \lambda)$ is a discontinuous function of λ

100. Which of the following statements is *not* true in the context of a simple linear regression with one predictor ?

(A) The ratio SSReg/SSTot will be same whether Y is regressed on X or X is regressed on Y

(B) A value $R^2 = 0.02$ indicates that X and Y are not related

(C) When the fitted regression line is horizontal, then SSE = SSTot and $R^2 = 0$

(D) When the fitted regression line is horizontal, then $Y_i = \bar{X}$.

101. If the regression estimator of $\bar{Y}$ is $\bar{y} + b(\bar{X} - \bar{x})$ where $b = \frac{S_{xy}}{S_x^2}$, then an exact expression for bias of regression estimator is :

(A) $-\text{Cov}(\bar{y}, b)$

(B) $\text{Cov}(\bar{y}, b)$

(C) $\text{Cov}(b, \bar{x})$

(D) $-\text{Cov}(b, \bar{x})$

102. The bais in ratio estimator decreases with :

(A) increasing the sample size n

(B) decreasing the sample size n

(C) both (A) and (B)

(D) increase in the population size

103. A simple random sample of n clusters is selected from a population of N clusters each of size M, then cluster sampling will be less efficient than SRSWOR if (ρ_d = intra class correlation coefficient between elements belonging to same cluster) :

(A) $M > 1$ and $\rho_d > 0$

(B) $M > 1$ and $\rho_d < 0$

(C) $M = 1$ and $\rho_d = 0$

(D) $M > 1$ and $\rho_d = 0$

104. If π_i and π_{ij} are respectively the first order and second order inclusion probabilities of a sampling design in PPSWOR, then which of the following relation is *true* ?

(A) $\sum_{j} \pi_{ij} = \pi_i$

(B) $\sum_{j} \pi_{ij} = (n-1)\pi_i$

(C) $\sum_{j} \pi_{ij} \neq \pi_i$

(D) $\sum_{j} \pi_{ij} = n(n-1)$

105. PPSWR sampling reduced to SRS if the probability of proportion to size *i.e.* p_i is :

(A) $1/n$

(B) 1

(C) 1/N

(D) n/N

106. Consider a two-factor factorial, fixed effect model with main effects A and B used at a and b levels respectively. Then the estimate of operating characteristic curve parameter for A is :

(A) $\dfrac{na \sum_{i=1}^{a} \tau_i^2}{b\sigma^2}$

(B) $\dfrac{nb \sum_{i=1}^{a} \tau_i^2}{a\sigma^2}$

(C) $\dfrac{na \sum_{i=1}^{b} \beta_j^2}{b\sigma^2}$

(D) $\dfrac{nb \sum_{i=1}^{b} \beta_j^2}{a\sigma^2}$

107. Consider the following statements about BIBD(a, b, k, r, λ) :

(1) If $a = b$, the design is said to be symmetric

(2) $\lambda(k - 1) = r(a - 1)$

(3) The adjusted treatment sum of squares is free from block effects.

Which of the above are *correct* ?

(A) Only 1 is correct

(B) Only 1 and 2 are correct

(C) Only 3 is correct

(D) Only 2 and 3 are correct

108. Consider the following statements :

(1) If the presence of interaction inflates the error mean square, one should use factorial designs.

(2) Two contrasts with coefficient $\{c_i\}$ and $\{d_i\}$ are orthogonal if $\sum_{i=1}^{a} c_i d_i = 0$.

(3) Confounding is a design technique for arranging a complete factorial experiment in blocks.

Which of the above statements are *correct* ?

(A) Only 1 and 2 are correct

(B) Only 1 and 3 are correct

(C) Only 2 and 3 are correct

(D) All are correct

109. Consider the following statements :

(1) In Resolution-III designs main effects are aliased with two-factor interactions.

(2) In Resolution-IV designs, two-factor interactions are aliased with each other.

(3) In Resolution-V designs two-factor interactions cannot be aliased with three-factor interactions.

Which of the above statements are *correct* ?

(A) 1 and 2 are correct

(B) 1 and 3 are correct

(C) 2 and 3 are correct

(D) All are correct

110. A 2^3 design with 4 replicates are under consideration. Two of the replicates are shown below :

Replicate 1	
(1)	a
c	b
ab	ac
abc	bc

Replicate 2	
(1)	b
a	c
bc	ab
abc	ac

Identify which of the treatment combinations are partially confounded in each replicate ?

(A) AB and AC

(B) AB and BC

(C) AC and BC

(D) A and BC

111. Consider the time series model :

$X_t = 0.2X_{t-2} - 0.6X_{t-1} + Z_t + 1.2Z_{t-1}$,

where $Z_t \sim$ iid Normal $(0, \sigma^2)$. Which of the following statements is *true* ?

(A) $\{X_t\}$ is stationary and invertible

(B) $\{X_t\}$ is invertible, but not causal

(C) $\{X_t\}$ is causal but not invertible

(D) $\{X_t\}$ is neither causal nor invertible

112. The sample autocorrelation of certain time series data was found to be significant at lags one and five, whereas the sample partial autocorrelations were not significants for first 30 lags. What model would you suggest for such a time series ?

(A) ARMA(1, 5)

(B) ARMA(5, 1)

(C) MA(5)

(D) AR(5)

113. In an AR(1) model $X_t = 0.5X_{t-1} + Z_t$, $Z_t \sim$ iid normal (0, 1), the best linear predictor $aX_1 + bX_3$ of X_2 using (X_1, X_3) will have :

(A) $a = b$

(B) $a < b$

(C) $a > b$

(D) Cannot be determined

114. Let the time series $\{Y_t\}$ be an ARIMA(2, 1, 2). Then $\{Y_t\}$ is :

(A) a stationary model

(B) having one unit root for the AR polynomial

(C) having one unit root for the MA polynomial

(D) having unit roots for both AR and MA polynomials

115. Consider the MC consisting of the three states 0, 1, 2 and having TPM :

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Which of the following is *correct* ?

(A) All states do not communicate

(B) The stationary distribution does not exist

(C) $P_{00}^{(2)} = \frac{1}{4}$

(D) MC is irreducible

116. Let $\{X_n\}$ be a MC on [0, 1] with TPM :

$$P = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix},$$

$0 < \alpha < 1,\ 0 < \beta < 1.$

Then $\lim_{n \to \infty} P_{11}^{(n)}$ is :

(A) $\frac{\alpha}{\alpha+\beta}$

(B) $\frac{\beta}{\alpha+\beta}$

(C) $\frac{\alpha\beta}{\alpha+\beta}$

(D) $\frac{1}{\alpha+\beta}$

117. There are n units in the system at time t and number of arrivals take places during the time interval Δt, the probability of the event is :

(A) $P_n(t)\ (1 - \mu\Delta t)$

(B) $P_n(t)\ (1 - \lambda\Delta t)$

(C) $P_{n-1}(t)\ (1 - \lambda\Delta t)$

(D) $P_n(t)\ (1 - \lambda\Delta t) + P_{n-1}(t)\ \Delta t$

118. The probability generating function of a particular random variable is given as :

$$P(s) = \frac{1}{2}(s + s^2)$$

What is the variance of the random variable ?

(A) $\frac{1}{3}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

119. Given the following table :

Age group of child bearing female	Number of Women ('000)	Total Births
15 — 19	16.0	260
20 — 24	16.4	2244
25 — 29	15.8	1894
30 — 34	15.2	1320
35 — 39	14.8	916
40 — 44	15.0	280
45 — 49	14.5	145

What is the value of TFR ?

(A) 2251.75 per thousand

(B) 2135.48 per thousand

(C) 2106.96 per thousand

(D) 2018.88 per thousand

120. Consider the following statements :

(1) Changing basic objective function coefficient c_j will affect the entire 0-raw to change.

(2) Changing right-hand side of a constraint will retain the current basis to be optimal even if the constraint is negative.

(3) Changing the column of a non-basic variable x_j will affect, the coefficient of x_j in row-0 is still non-negative and the current basis is optimal.

Which of the above are *correct* ?

(A) 1 and 2 are correct

(B) 1 and 3 are correct

(C) 2 and 3 are correct

(D) All are correct

121. Let X_1, X_2, ..., X_n be a random sample of size n from Bernoulli distribution with parameter θ. Then variance of conditional expectation of $\left(\frac{X_1 + X_2}{2}\right)$ given $\sum_{i=1}^{n} X_i$ is :

(A) 0

(B) θ

(C) $\theta(1 - \theta)$

(D) $\theta(1 - \theta)/n$

122. Let X be Poisson variate with mean λ. Then distribution function of X, evaluated at 1.5 is :

(A) 0

(B) less than $e^{-\lambda}$

(C) equal to $(\lambda + 1)e^{-\lambda}$

(D) greater than $\lambda(\lambda + 1)e^{-\lambda}$

123. Let X be a r.v. with cdf, F(x), where :

$$F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ x/2 & \text{if} \quad 0 \le x < 1 \\ 3/4 & \text{if} \quad 1 \le x < 2 \\ 1 & \text{if} \quad 2 \le x \end{cases}$$

Then E(X) is given by :

(A) 3/8

(B) 1/2

(C) 3/2

(D) 1

124. Suppose (X, Y) have joint pdf $f(x, y)$, where :

$$f(x, y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then $E(X/y)$ is given by :

(A) $2y$

(B) $y/2$

(C) 1

(D) y

125. Let X be a degenerate random variable, at $X = c$. Then characteristic function of X at 't' is :

(A) c

(B) 0

(C) $\exp(itc)$

(D) $\exp(-itc)$

126. Consider the following statements :

1. Sensitivity analysis provides a single value within which a parameter may change without affecting optimality.
2. While performing sensitivity analysis, the upper bound infinity on the right hand side of a constraint means that the constraint is redundant.
3. When an additional constraint is added in the LP models, the existing optimal solution can further be improved if $z_i - c_j \geq 0$.

Which of the above are *correct* ?

(A) Only 1 is correct

(B) Only 2 is correct

(C) Only 3 is correct

(D) All are wrong

127. Consider the following LPP :

Max. : $Z = 2x_1 + x_2$

Subject to :

$$3x_1 + 4x_2 \le 6$$

$$6x_1 + x_2 \le 3$$

$$x_1 \ge 0,\ x_2 \ge 0$$

What is the solution of this LPP ?

(A) (1/3, 2/3)

(B) (3/11, 14/11)

(C) (5/7, 5/7)

(D) (2/7, 9/7)

128. Consider the following statements :

1. The use of cutting-plane method reduces the number of constraints in the given problem.
2. In a Branch and Bound minimization tree, the lower bounds on objective function value do not decrease in value.
3. The 0-1 integer programming problem requires the decision variables to have values between zero and one.

Which of the above are *correct* ?

(A) Only 1 and 2 are correct

(B) Only 1 and 3 are correct

(C) Only 2 and 3 are correct

(D) All are correct

129. A quadratic programming problem is given as follows :

Max. : $Z_x = 2x_1 + 3x_2 - 2x_1^2$

Subject to :

$$x_1 + 4x_2 \leq 4$$

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

If we apply Wolfe's method to solve, then the correct Kuhn-Tucker condition will be :

(A) $4x_1 = \lambda_1 + \lambda_2 + \mu_1$ and

$x_2 = 3 - 4\lambda_1 - \lambda_2 + \mu_2$

(B) $4x_1 = 2 - \lambda_1 - \lambda_2 - \mu_1$ and

$4x_2 = 3 + \lambda_1 + \lambda_2 - \mu_2$

(C) $4x_1 = -2 - \lambda_1 + \lambda_2 - \mu_1$ and

$x_2 = 3 - 4\lambda_1 - \lambda_2 + \mu_2$

(D) $4x_1 = 2 - \lambda_1 - \lambda_2 + \mu_1$ and

$3 + \mu_2 = 4\lambda_1 + \lambda_2$

130. Let X and Y be two random variables defined on the same probability space such that for each $y > 0$, the conditional density function of X given Y = y is :

$$f(x/y) = \sqrt{\frac{y}{2\pi}}\, e^{-yx^2/2},\ y > 0,\ x \in \mathrm{R}.$$

Let

$$g(y) = \frac{1}{\sqrt{2\pi y}}\, e^{-y/2},\ y > 0.$$

Which of the following statements is more appropriate ?

(A) $E(X \mid Y) = 0$

(B) $E(E(X \mid Y)) = 0$

(C) $E(X) = 0$

(D) $E(X) \neq E(E(X \mid Y))$

131. Let Y be a Bernoulli random variable with :

$$P(Y = 0) = P(Y = 1) = 1/2.$$

Let $X_n = (1 - Y/n^{\alpha})^n$, $\alpha \geq 0$,

then, which of the following statements are *correct* ?

(*i*) $X_n \xrightarrow{d} Y$ when $0 \leq \alpha < 1$

(*ii*) $X_n \xrightarrow{d} e^{-Y}$ when $\alpha = 1$

(*iii*) $X_n \xrightarrow{d} Y$ when $\alpha > 1$

(A) (*i*) and (*ii*)

(B) (*ii*) and (*iii*)

(C) (*i*) and (*iii*)

(D) Only (*i*)

132. Let $\{X_n\}$ be a sequence of random variables such that :

$P(X_n = -n^{1/2}) = P(X_n = n^{1/2}) = 1/2n$

and $P(X_n = 0) = (n - 1)/n$.

Let $S_n = \sum_{i=1}^{n} X_i$.

Then, which of the following statements is *not* true ?

(A) $E(X_n) = 0$

(B) $V(X_n) = 1$

(C) $S_n/\sqrt{n} \xrightarrow{d} Z$, $Z \sim N(0, 1)$

(D) $S_n/\sqrt{n} \xrightarrow{d} Z$, Z is not Gaussian

133. Let $\{(X_n, F_n)\}_{n=1}^{\infty}$ be a martingale. Let $\zeta_n = \sigma(X_1, X_2, ..., X_n)$. Then, $\{(X_n, \zeta_n)\}_{n=1}^{\infty}$ is a ....

(A) Martingale

(B) Sub Martingale

(C) Super Martingale

(D) White noise

134. Let Ω be a countably infinite set and let F consists of all subsets of Ω. Define :

$$\mu(A) = \begin{cases} 0 & \text{if A is finite} \\ \infty & \text{if A is infinite} \end{cases}$$

Which of the following statements are *true* ?

(*i*) μ is finitely additive

(*ii*) μ is countably additive

(*iii*) $\{A_n\}$ is an $\uparrow$ sequence of sets with $\mu(A_n) = 0 \ \forall \ n$, but $\mu(\Omega) = \infty$.

(A) (*i*) and (*ii*)

(B) (*i*) and (*iii*)

(C) (*ii*) and (*iii*)

(D) (*i*), (*ii*) and (*iii*)

135. Let X and Y be two independent zero-mean unit variance Gaussian random variables defined on a common probability space. Define $U = X + Y$ and $V = X - Y$. Let $F = \sigma(X)$. Then, which of the following statements is *not* true ?

(A) $E(U|F) = X$ a.s.

(B) $E(V|F) = X$ a.s.

(C) $E(U + V|F) = 2X$ a.s.

(D) $E(U|F)$ and $E(V|F)$ are independent

136. Let X_1 and X_2 be two iid random variables with :

$$P(X_1 = 1) = P(X_1 = -1) = 1/2.$$

Let $Z = X_1 + X_2$, $A_i = X_i^{-1}(\{1\})$, $i = 1, 2$.

Which of the following is *not* true on $Z^{-1}(\{0\})$?

(A) $P(A_1|Z) = 1/2$

(B) $P(A_2|Z) = 1/2$

(C) $P(A_1 \cap A_2|Z) = 0$

(D) $P(A_1|Z)\,P(A_2|Z) = P(A_1 \cap A_2|Z)$

137. Let X be a single observation with unknown mean $E(X) = \mu \in (-\infty, \infty)$ and variance $Var(X) = \mu^2 + 1$. Consider the problem of estimating μ based on X under the squared error loss $L(\mu, a) = (a - \mu)^2$. Let T_1 and T_2 be defined as follows :

$$T_1 = X,\ T_2 = X/2 + 1/2.$$

Which of the following statements is *true* ?

(A) Risk of $T_1 = \mu^2$

(B) Risk of $T_2 = (1 - \mu + \mu^2)/2$

(C) Risk of T_2 is larger than that of T_1

(D) T_1 is an admissible estimator

138. Suppose X_1 ...... X_n are independent observations from a Poisson distribution with probability mass function :

$$f(x/\lambda) = \frac{e^{-\lambda}\lambda^x}{x!},\ x = 0, 1, 2, \ldots\ldots.,$$

$\lambda > 0$ unkonwn.

Suppose the prior distribution of λ is desired as :

$$g(\lambda) \propto \lambda,\ e^{-2\lambda},\ \lambda > 0.$$

Let $n = 3$ and the data $(X_1, X_2, X_3) = (1, 1, 2)$. Then, suppose we estimate λ under squared error loss $L(\lambda, a) = (\lambda - a)^2$. Then, the Bayes estimator based on the given data is :

(A) 1.5

(B) 1.65

(C) 2.25

(D) 1.2

139. Suppose two control charts have the same in-control average run length (ARL). Then which of the following is *true* ?

(A) Both the charts will perform in similar way

(B) The charts can be compared using out-of-control ARL

(C) Two charts cannot have the same in-control ARL

(D) The charts must be only to monitor population mean

140. Let L_q be the average number of customers in the queue, λ be the customer arrival rate and μ be the average service rate. Then average waiting time for a customer in the queue for all infinite source queueing models is :

(A) L_q/μ

(B) μ/λ

(C) λ/μ

(D) L_q/λ

141. Let X be a random variable with distribution function :

$$F_\alpha(t) = 1 - \exp(-(\lambda t)^\alpha),\ \lambda,\quad \alpha > 0$$

$$t \geq 0$$

Then :

(A) F_α is IFR

(B) F_α is DFR

(C) F_α is IFR if $0 < \alpha \leq 1$

(D) F_α is IFR if $1 \leq \alpha$

142. Suppose that in a K-unit parallel system each unit has life time distribution with distribution function F. Then reliability function of the system at time t is given by :

(A) $(F(t))^K$

(B) $(1 - F(t))^K$

(C) $1 - (F(t))^K$

(D) $1 - (1 - F(t))^K$

143. Let X and Y be independent Poisson variables with $E(X) = \lambda$ and $E(Y) = \lambda + 1$, when $\lambda > 0$ is an unknown parameter. Based on single observations X and Y; $(XY) = (2, 3)$, the MLE of λ will be :

(A) 2

(B) 2.5

(C) 3

(D) 3.5

144. Let X_1 and X_2 be independent observations with $X_1 \sim N(\mu, 2\sigma^2)$ and $X_2 \sim N(2\mu, \sigma^2)$. Suppose we define $T_1 = X_1 - 4X_2$ and $T_2 = 2X_1 - X_2$. Which of the following statements is *not* true ?

(A) (T_1, T_2) is sufficient

(B) When σ^2 is known, T_2 is ancillary

(C) When σ^2 is known, T_1 and T_2 are independent

(D) When σ^2 is known, T_1 is not sufficient

145. Let X_1, X_2, X_3 be iid r.v.s with $U(\theta, \theta^2)$; $\theta > 1$. The maximum likelihood estimator (mle) of θ is :

(A) $X_{(1)}^2$

(B) $\left(X_{(3)}\right)^{1/2}$

(C) $X_{(1)}^2 + X_{(3)}$

(D) $0.2X_{(1)} + 0.8\left(X_{(3)}\right)^{1/2}$

where :

$X_{(1)} = \text{Min}\ (X_1, X_2, X_3)$

$X_{(3)} = \text{Max}\ (X_1, X_2, X_3)$.

ROUGH WORK

ROUGH WORK