Test Booklet Code & Serial No.
प्रश्नपत्रिका कोड व क्रमांक

C

Paper-III
MATHEMATICAL SCIENCE

Signature and Name of Invigilator

1. (Signature) ....

(Name) ....

2. (Signature) ....

(Name) ....

Seat No. | | | | | | |

(In figures as in Admit Card)

Seat No. ....

(In words)

OMR Sheet No. | | | | | | |

(To be filled by the Candidate)

APR - 30317

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Number of Pages in this Booklet : **48** Number of Questions in this Booklet : **145**

Instructions for the Candidates

1. Write your Seat No. and OMR Sheet No. in the space provided on the top of this page.
2. (*a*) This paper consists of **One hundred forty five (145)** multiple choice questions, each question carrying **Two (2)** marks.
   (*b*) There are *three* sections, Section-I, II, III in this paper.
   (*c*) Students should attempt all questions from Sections I and II *or* Sections I and III.
   (*d*) Below each question, four alternatives or responses are given. Only one of these alternatives is the 'CORRECT' answer to the question.
3. At the commencement of examination, the question booklet will be given to the student. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as follows :
   (*i*) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal or open booklet.
   (*ii*) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to missing pages/questions or questions repeated or not in serial order or any other discrepancy should not be accepted and correct booklet should be obtained from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given. The same may please be noted.**
   (*iii*) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each question has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : where (C) is the correct response.
   (A) (B) ● (D)
5. Your responses to the items are to be indicated in the **OMR Sheet given inside the Booklet only.** If you mark at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done at the end of this booklet.
8. If you write your Name, Seat Number, Phone Number or put any mark on any part of the OMR Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
9. You have to return original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry the Test Booklet and duplicate copy of OMR Sheet on conclusion of examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table, etc., is prohibited.**
12. **There is no negative marking for incorrect answers.**

विद्यार्थ्यांसाठी महत्त्वाच्या सूचना

1. परिक्षार्थींनी आपला आसन क्रमांक या पृष्ठावरील वरच्या कोपऱ्यात लिहावा. तसेच आपणांस दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. (*a*) या प्रश्नपत्रिकेत एकूण **एकशेपंचेचाळीस (145)** बहुपर्यायी प्रश्न दिलेले आहेत, प्रत्येक प्रश्नाला **दोन (2)** गुण आहेत.
   (*b*) या प्रश्नपत्रिकेत खण्ड-I, II, III असे तीन खण्ड आहेत.
   (*c*) विद्यार्थ्यांनी खण्ड-I आणि II किंवा खण्ड I आणि III यांचे सगळे प्रश्न सोडावे.
   (*d*) खाली दिलेल्या प्रश्नाचे चार पर्याय किंवा उत्तर दिलेले आहेत. प्रश्नाचे बहुपर्यायी उत्तरामधून केवळ एक 'बरोबर' आहे.
3. परीक्षा सुरू झाल्यावर विद्यार्थ्याला प्रश्नपत्रिका दिली जाईल. सुरुवातीच्या 5 मिनीटांमध्ये आपण सदर प्रश्नपत्रिका उघडून खालील बाबी अवश्य तपासून पहाव्यात.
   (*i*) प्रश्नपत्रिका उघडण्यासाठी प्रश्नपत्रिकेवर लावलेले सील उघडावे. सील नसलेली किंवा सील उघडलेली प्रश्नपत्रिका स्विकारू नये.
   (*ii*) **पहिल्या पृष्ठावर नमूद केल्याप्रमाणे प्रश्नपत्रिकेची एकूण पृष्ठे तसेच प्रश्नपत्रिकेतील एकूण प्रश्नांची संख्या पडताळून पहावी. पृष्ठे कमी असलेली/कमी प्रश्न असलेली/प्रश्नांचा चूकीचा क्रम असलेली किंवा इतर त्रुटी असलेली सदोष प्रश्नपत्रिका सुरुवातीच्या 5 मिनिटातच पर्यवेक्षकाला परत देऊन दुसरी प्रश्नपत्रिका मागवून घ्यावी. त्यानंतर प्रश्नपत्रिका बदलून मिळणार नाही तसेच वेळही वाढवून मिळणार नाही याची कृपया विद्यार्थ्यांनी नोंद घ्यावी.**
   (*iii*) वरीलप्रमाणे सर्व पडताळून पहिल्यानंतरच प्रश्नपत्रिकेवर ओ.एम.आर. उत्तरपत्रिकेचा नंबर लिहावा.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा/निळा करावा.
   उदा. : जर (C) हे योग्य उत्तर असेल तर.
   (A) (B) ● (D)
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे **ओ.एम.आर. उत्तरपत्रिकेतच दर्शवावीत.** इतर ठिकाणी लिहिलेली उत्तरे तपासली जाणार नाहीत.
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. प्रश्नपत्रिकेच्या शेवटी जोडलेल्या कोऱ्या पानावरच कच्चे काम करावे.
8. जर आपण ओ.एम.आर. वर नमूद केलेल्या ठिकाणा व्यतिरीक्त इतर कोठेही नाव, आसन क्रमांक, फोन नंबर किंवा ओळख पटेल अशी कोणतीही खूण केलेली आढळून आल्यास अथवा असभ्य भाषेचा वापर किंवा इतर गैरमार्गाचा अवलंब केल्यास विद्यार्थ्याला परीक्षेस अपात्र ठरविण्यात येईल.
9. परीक्षा संपल्यानंतर विद्यार्थ्याने मूळ ओ.एम.आर. उत्तरपत्रिका पर्यवेक्षकांकडे परत करणे आवश्यक आहे. तथापी, प्रश्नपत्रिका व ओ.एम.आर. उत्तरपत्रिकेची द्वितीय प्रत आपल्याबरोबर नेण्यास विद्यार्थ्यांना परवानगी आहे.
10. **फक्त निळ्या किंवा काळ्या बॉल पेनचाच वापर करावा.**
11. **कॅलक्युलेटर किंवा लॉग टेबल वापरण्यास परवानगी नाही.**
12. **चुकीच्या उत्तरासाठी गुण कपात केली जाणार नाही.**

Mathematical Science

Paper III

Time Allowed : 2½ Hours] **[Maximum Marks : 150**

Note : This paper consists of **One hundred forty five (145)** multiple choice questions, each question carrying **Two (2)** marks. There are three sections, Section-I, II, III in this paper. Students should attempt all questions from Sections I and II *or* Sections I and III.

Section I

1. Let $h(x) = x^2$, if x is rational, $h(x) = 0$, if x is irrational.

   (A) $h(x)$ is continuous at every point of **R**

   (B) $h(x)$ is discontinuous at every point of **R**

   (C) $h(x)$ is discontinuous at every point of **R** except origin

   (D) $h(x)$ is discontinuous at rational points

2. The eigenvalues of the matrix

   $$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 2 & -3 & 1 \end{pmatrix}$$ are :

   (A) 1, 1, 1

   (B) 1, 2, –3

   (C) 1, 3, –1

   (D) 1, i, $-i$

3. Let M be a square matrix with real positive entries such that each column adds up to 1. Then which of the following statements is *not* true ?

(A) An eigenvalue of M is 1

(B) The sum of the entries of an eigenvector corresponding to an eigenvalue $\lambda \neq 1$ is zero

(C) There is no eigenvalue $\lambda \neq 1$ such that $|\lambda| \geq 1$

(D) The sum of the entries of an eigenvector corresponding to eigenvalue 1 is zero

4. Let $f(x, y) = x^2 y + 7x$. Then the directional derivative of f at the point (1, –2) in the direction $\left(\frac{3}{5}, \frac{4}{5}\right)$ is :

(A) $\frac{12}{5}$

(B) 13

(C) $\frac{13}{5}$

(D) $\frac{-13}{5}$

5. Let $f(x, y) = \frac{x^2 y}{x^2 + y^2}$, if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$, then which of the following statements is *not true* ?

(A) f is continuous at (0, 0)

(B) $\frac{\partial f}{\partial x}$ exists at (0, 0)

(C) $\frac{\partial f}{\partial y}$ exists at (0, 0)

(D) f is differentiable at (0, 0)

Section II

6. Let X be a finite dimensional normed space. Then which of the following is *not* true ?

(A) There are only finitely many subspaces of X

(B) Every subspace of X is finite dimensional

(C) Every subspace of X is closed

(D) There is a subspace of X which is complete

7. Let X be an inner product space, Y an orthonormal set of vectors in x and y an arbitrary vector in X. Then which of the following is *true* ?

(A) For all $x_1, x_2, ..., x_n \in Y$,

$$\sum_{i=1}^{n} |(y, x_i)|^2 = \|y\|^2$$

(B) If $z \in X$, then $\sum_{x \in Y} |(y, x) \overline{(z, x)}| \geq \|y\| \|z\|$

(C) The set $E = \{x \in Y | (y, x) \neq 0\}$ is countable

(D) The set $E = \{x \in Y | (y, x) \neq 0\}$ is dense in X

8. Let $Y = \{y_\alpha\}_{\alpha \in \Lambda}$ be a complete orthonormal set in an inner product space X. Then which of the following is *true* ?

(A) $\|x\|^2 = \sum_{\alpha} |(x, y_\alpha)|^2$ for every $x \in X$

(B) $\overline{[Y]} = X$

(C) For any $x, y \in X$, $(x, y) = \sum_{\alpha} (x, y_\alpha)(y_\alpha, y)$

(D) $x \perp Y \Rightarrow x = 0$, whenever X is complete

9. If M is a closed subspace of an inner product space X, then which of the following is *not* true ?

(A) If X is a Hilbert space then $X = M \oplus M^{\perp}$

(B) If M is complete then $M = M^{\perp\perp}$

(C) If X is a Hilbert space then X/M is isomorphic to $M^{\perp}$

(D) If $X = M \oplus M^{\perp}$ then M is finite dimensional

10. Which of the following statements is *not* true ?

(A) If for every functional on a normed space X, $f(x) = f(y)$, then $x = y$

(B) A closed subspace of a reflexive normed space is reflexive

(C) For an element x in a normed space X, $\| x \| = \sup \{| f(x)| \ / \ f \in \tilde{X}, \| f \| = 1\}$

(D) If f is a linear functional on the Hilbert space X with null space N then f is continuous whenever N is closed

11. Let X be a topological space and $Z \leq Y \leq X$ which of the following statements is *not* true ?

(A) If Z is closed in Y and Y is closed in X, then Z is closed in X

(B) If Z is open in Y and Y is open in X then Z is open in X

(C) If Z is connected in Y then Z is connected in X

(D) If Z open in Y then Z is open in X

12. Let an element x be a limit point of a subset Y of a Hausdorff space X. Then which of the following statements need not be *true* ?

(A) Every neighbourhood of x intersects Y

(B) Every neighbourhood of x intersects Y in at least two points

(C) Every neighbourhood of x intersects Y in at least one point different from x

(D) Every neighbourhood of x intersects Y in infinitely many points

13. Let X and Y be topological spaces and $P : X \rightarrow Y$ be a surjective map. Which of the following statements is *true* ?

(A) If P is a quotient map then P is an open map

(B) If P is an open map then P is a quotient map

(C) If P is a quotient map then P is a closed map

(D) If P is continuous then P is a quotient map

14. Which of the following spaces is *not* locally compact subspace of **R** :

(A) **R**

(B) **Q***-the space of irrationals

(C) (0, 1)

(D) [0, 1]

15. In how many ways can one distribute 10 identical white marbles among six distinct containers ?

(A) 3003

(B) 2995

(C) 6006

(D) 3002

16. Let D_n be the poset of all positive divisors of the given integer $n > 1$ with the partial order, $d_1 \leq d_2$ if and only if d_1 divided d_2. Then which of the following statements is *false* ?

(A) D_n is a distributive lattice

(B) D_n is a direct product of chains

(C) D_n is a Boolean lattice for any square free integer n

(D) D_n contains two maximal chains of different lengths, whenever n is not a power of a prime

17. Find the length of a longest path in the hypercube Q_{10}, the graph obtained by taking Cartesian product of two elements path with itself ten times :

(A) 1000

(B) 1023

(C) 512

(D) 1024

18. Let m be a positive odd integer. Which of the following is always *true* ?

(A) m divides $2^n - 1$ for every n such that $m \leq 2^{n-1}$

(B) m is divisible by $2^n - 1$ for some positive even integer n

(C) m divides $2^n - 1$ for some positive integer n

(D) m is divisible by $2^n - 1$ for every positive even integer n such that $2^n - 1 \leq m$

19. The partial differential equation obtained by eliminating arbitrary function from :

$$z = f(x^2 + y^2 + z^2)$$

is :

(A) $p(y + zp) = q(x + zq)$

(B) $px(y + zq) = qy(x + zp)$

(C) $px(y + zp) = qy(x + zq)$

(D) $p(y + zq) = q(x + zp)$

20. Consider the first order partial differential equations :

(i) $yp - xq = xyz + x$

(ii) $(x^2 + z^2)p - xy\,q = pq$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$

(A) Equation (i) is linear and Equation (ii) is semilinear

(B) Equation (i) is linear and Equation (ii) is non-linear

(C) Both the equations are non-linear

(D) Equation (i) is linear and Equation (ii) is quasilinear

21. Equation of the surface which intersects the surfaces of the system :

$$z(x + y) = c(3z + 1)$$

orthogonally and which passes through the circle $x^2 + y^2 = 1, \; z = 1$ is :

(A) $x^2 + y^2 = 2z^3 + z^2 - 2$

(B) $x + y + z = 2$

(C) $x^2 + y^2 + z^2 = 4$

(D) $x^2 + y^2 + z^2 = 2z^3$

22. The integral curves of the system of equations :

$$\frac{dx}{dt} = y, \; \frac{dy}{dt} = x$$

are :

(A) Circles

(B) Ellipses

(C) Parabolas

(D) Hyperbolas

23. The differential equation :

$$\frac{d^2y}{dx^2} + e^x y = 0$$

has a solution of the form

$$\phi(x) = \sum_{k=0}^{\infty} C_k x^k,$$

which satisfies $\phi(0) = 1$, $\phi'(0) = 0$

then :

(A) $C_0 = 1, C_1 = 0, C_2 = \frac{1}{2}$

(B) $C_0 = 0, C_1 = 1, C_2 = \frac{1}{2}$

(C) $C_0 = 1, C_1 = 0, C_2 = -\frac{1}{2}$

(D) $C_0 = 1, C_1 = 2, C_2 = -\frac{1}{2}$

24. Which of the following linear diophantine equations has no solution ?

(A) $2x + 3 \equiv 0 \pmod{101}$

(B) $3x + 4 \equiv 0 \pmod{123}$

(C) $4x + 5 \equiv 0 \pmod{67}$

(D) $5x + 6 \equiv 0 \pmod{12}$

25. Which of the following natural numbers can be written as a sum of two squares ?

(A) 28

(B) 115

(C) 220

(D) 180

26. If $\mu(n)$ denotes the Möbius function, then :

$$\sum_{d \mid 1000} \mu(d)$$

is equal to :

(A) 1

(B) 1000

(C) 0

(D) –10

27. A particle of mass m is moving in 2-dimensions. Its kinetic energy in terms of polar co-ordinates is given by the expression :

(A) $\frac{1}{2} m (r^2 + r^2 \dot{\theta}^2)$

(B) $\frac{1}{2} m (\dot{r}^2 + r \dot{\theta}^2)$

(C) $\frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2)$

(D) $\frac{1}{2} m (\dot{r}^2 + \theta \dot{\theta}^2)$

28. Consider a simple pendulum having length l and mass m. If θ denotes the angle with vertical direction, then the Lagrangian of the system is given as :

(A) $\frac{1}{2} m l^2 \dot{\theta}^2 - m g l (1 - \cos\theta)$

(B) $m l^2 \dot{\theta}^2 - \frac{1}{2} m g l (1 - \cos\theta)$

(C) $\frac{1}{2} m l^2 \dot{\theta}^2 - m g l (1 + \cos\theta)$

(D) $\frac{1}{2} m l^2 \dot{\theta}^2 + m g l (1 + \cos\theta)$

29. The general displacement of a rigid body with one point fixed is represented by a 3×3 real matrix A, where A is :

(A) orthogonal matrix

(B) orthogonal matrix having eigen value +1

(C) symmetric matrix with determinant –1

(D) symmetric matrix with determinant +1

30. Let Lagrangian of a system be :

$$L(x, \dot{x}) = \frac{1}{2} m \, \dot{x}^2 - \frac{1}{2} k x^2,$$

where m, k are positive constants. If $\omega = \sqrt{k / m}$ and C_1, C_2 denote arbitrary constants, then the solutions of Euler-Lagrange equations are :

(A) $x(t) = C_1 e^{\omega t} + C_2 e^{-\omega t}$

(B) $x(t) = C_1 + C_2 t$

(C) $x(t) = C_1 \cos \omega t + C_2 \sin \omega t$

(D) $x(t) = C_1 \cos \omega^2 t + C_2 \sin \omega^2 t$

31. Let $\alpha : (0, 1) \to \mathbf{R}^3$ be a curve parametrised by arc length such that $\alpha''(s) \neq 0$ for all $s \in (0, 1)$. If for $s \in (0, 1)$, the curve α has curvature 2 and torsion $s + 1$, then :

(A) $\bar{t}\,'(s) = (s+1)\, \bar{n}(s)$ and

$$\bar{b}\,'(s) = 2\, \bar{n}(s)$$

(B) $\bar{t}\,'(s) = 2\, \bar{n}(s)$ and

$$\bar{n}\,'(s) = -2\, \bar{t}(s) - (s+1)\, \bar{b}(s)$$

(C) $\bar{t}\,'(s) = 2\, \bar{n}(s)$ and

$$\bar{n}\,'(s) = -(s+1)\, \bar{t}(s) - 2\, \bar{b}(s)$$

(D) $\bar{b}\,'(s) = (s+1)\, \bar{n}(s)$ and

$$\bar{n}\,'(s) = -(s+1)\, \bar{t}(s) - 2\, \bar{b}(s)$$

32. Let :

$$\alpha : (-1, 1) \to \mathbf{R}^3$$

be defined by $\alpha(t) = (t, t^2, t^3)$. Then the curvature $k(t)$ at $t = 0$ is :

(A) 3

(B) $\sqrt{2}$

(C) 2

(D) 1

33. For $0 < u < 2\pi$, $-\infty < v < \infty$, the helicoid $\bar{x}(u, v)$ is given by :

$$\bar{x}(u, v) = (v \cos u, v \sin u, au)$$

Then the coefficients E, F, G in the first fundamental form of the helicoid are :

(A) E = 1, F = 0, G = 1

(B) E = V^2, F = 1, G = 0

(C) E = $V^2 + a^2$, F = 1, G = 1

(D) E = $V^2 + a^2$, F = 0, G = 1

34. If the integrand f does not depend on y, the Euler-Lagrange's equation of the functional :

$$I(y(x)) = \int_{x_1}^{x_2} f(x, y, y', y'')\, dx$$

under the condition that both y and y' are prescribed at the end points has the first integral as :

(A) $\frac{\partial f}{\partial y''} - \frac{d}{dx}\left(\frac{\partial f}{\partial y''}\right)$ = constant

(B) $\frac{\partial f}{\partial y'} - \frac{d}{dx}\left(\frac{\partial f}{\partial y''}\right)$ = constant

(C) $\frac{\partial f}{\partial y'} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)$ = constant

(D) $\frac{\partial f}{\partial y''} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)$ = constant

35. The curve which extremizes the functional :

$$I(y(x)) = \int_0^{\pi/4} \left[(y'')^2 - y^2 + x^2 \right] dx$$

under the conditions :

$$y(0) = 0, \ y'(0) = 1,$$

$$y\left(\frac{\pi}{4}\right) = y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

is :

(A) $y = 1 - \cos x$

(B) $y = \tan x$

(C) $y = \cos x$

(D) $y = \sin x$

36. The differential equation for the extremal of the functional :

$$I(u(x, y)) = \iint_0 \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \right] dxdy$$

where the values of u are prescribed on the boundary of the domain D is :

(A) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(B) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$

(C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} = 0$

(D) $\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} = 0$

37. The integral equation :

$$x(t) = 1 + t^2 + 4 \int_0^t (t - s)\, x(s)\, ds$$

is a :

(A) Linear Volterra integral equation of second kind with convolution kernel

(B) Linear Fredholm integral equation of second kind with symmetric kernel

(C) Linear Volterra integral equation of first kind with convolution kernel

(D) Linear Fredholm integral equation of first kind with symmetric kernel

38. The equivalent Fredholm integral equation of the boundary value problem :

$y'' + y = x, \quad y(0) = 1, \quad y(1) = 0$

is :

(A) $u(x) = 2x - 1 + \int_0^1 k_1(x,t)\, u(t)\, dt$

where

$$k_1(x, t) = \begin{cases} t(1-x), & 0 \le t \le x \\ x(1-t), & x \le t \le 1 \end{cases}$$

(B) $u(x) = 2x - 1 + \int_0^1 k_2(x,t)\, u(t)\, dt$

where

$$k_2(x, t) = \begin{cases} (1-x), & 0 \le t \le x \\ (1-t), & x \le t \le 1 \end{cases}$$

(C) $u(x) = 2x + 1 + \int_0^1 k_3(x,t)\, u(t)\, dt$

where $k_3(x, t) = \begin{cases} t, & 0 \le t \le x \\ x, & x \le t \le 1 \end{cases}$

(D) $u(x) = 2x + 1 + \int_0^1 k_4(x,t)\, u(t)\, dt$

where

$$k_4(x, t) = \begin{cases} xt(1-x), & 0 \le t \le x \\ x^2(1-t), & x \le t \le 1 \end{cases}$$

39. Solution of the integral equation :

$$x(t) = e^{t^2} + \int_0^t e^{t^2 - s^2}\, x(s)\, ds$$

with the help of resolvent kernel method is :

(A) e^{t^2}

(B) $e^{t^2 + t}$

(C) e^{-t^2}

(D) $e^{t^2 - t}$

40. Which of the following is *incorrect* ?

(A) $E = 1 + \Delta$

(B) $\Delta = \nabla E$

(C) $\nabla = 1 - E^{-1}$

(D) $E = \Delta + \nabla$

Where :

Δ-forward difference operator

∇-backward difference operator

E-shift operator

41. The Lagrange polynomial that passes through the points $(a, f(a))$ and $(b, f(b))$ is :

(A) $\left(\frac{x-b}{a-b}\right) f(a) + \left(\frac{x-a}{b-a}\right) f(b)$

(B) $\left(\frac{x-b}{a-b}\right) f(a) + \left(\frac{x-a}{a-b}\right) f(b)$

(C) $\left(\frac{x-b}{a-b}\right) f(b) + \left(\frac{x-a}{b-a}\right) f(a)$

(D) $\left(\frac{x-b}{a-b}\right) f(b) + \left(\frac{x-a}{a-b}\right) f(a)$

42. For given initial value problem :

$$y' = 1 - y, \quad y(0) = 0$$

the value of $y(0.2)$ by Euler's method (taking $h = 0.1$) is :

(A) 0.10

(B) 0.20

(C) 0.19

(D) 0.18

43. The third divided difference of the function $\frac{1}{x}$ with arguments a, b, c, d is :

(A) $\frac{ab-cd}{abcd}$

(B) $\frac{-1}{abcd}$

(C) $\frac{1}{abcd}$

(D) $\frac{ac-bd}{abcd}$

44. If $\mathrm{L}\{f(t)\} = \mathrm{F}(s)$, then :

$$\mathrm{L}\{e^{at} f(t)\} =$$

is :

(L is the Laplace transformation)

(A) $\mathrm{F}(s-a)$

(B) $\mathrm{F}(s+a)$

(C) $e^{as}\,\mathrm{F}(s)$

(D) $e^{as}\,\mathrm{F}(s+a)$

45. The value of the integral :

$$\int_0^t J_0(\tau)\, J_0(t-\tau)\, d\tau$$

where J_0 be the Bessel function of zero order is :

(A) e^t

(B) e^{-t}

(C) $\sin t$

(D) $\cos t$

46. The inverse Fourier transform of $e^{-|s|a}$ is :

(A) $\sqrt{\frac{2}{\pi}} \frac{a}{(t^2+a^2)}$

(B) $\sqrt{\frac{2}{\pi}} \frac{a}{(t^2-a^2)}$

(C) $\sqrt{\frac{2}{\pi}} \frac{a}{(t+a)}$

(D) $\sqrt{\frac{2}{\pi}} \frac{a}{(t-a)}$

47. Let μ be a measure on a ring **R**, the d defined by :

$$d(A, B) = \mu(A \,\Delta\, B),\ A, B \in \mathbf{R}$$

is :

(A) a pseudometric on **R**

(B) a metric on **R**

(C) $d(A, B) = 0$ does not imply $A = B$

(D) $d(A, B) = d(A, C) + d(C, B)$ holds for all $A, B, C \in \mathbf{R}$

48. Let $\{f_n\}$ be a sequence of measurable functions, $f_n : X \to [0, \infty]$,

I : then $\liminf \int f_n d\mu \geq \int \liminf f_n \, d\mu$

II : and $f_n(x) \uparrow$ for each x, then $\int \lim f_n \, d\mu = \lim \int f_n \, d\mu$

(A) Both I and II are true

(B) Both I and II are not true

(C) Only I is true

(D) Only II is true

49. Let μ be a measure on a ring R and the set function W* defined on H(R) by :

$$\mu^*(E) = \inf\left[\sum_{n=1}^{\infty} \mu(E_n) : E_n \in R,\ n = 1,2,\ldots\ldots, E \subseteq \bigcup_{n=1}^{\infty} E_n\right].$$

Then :

(A) μ* is a measure on H(R)

(B) μ* is an outer measure on H(R)

(C) μ* is σ-finite measure on H(R)

(D) μ* is complete measure on H(R)

50. Let $f, g \in L^P(X, \mu)$, where $P \geq 1$, then :

(A) $\|f+g\|_P = \|f\|_P + \|g\|_P$

(B) $\|f+g\|_P \leq \|f\|_P - \|g\|_P$

(C) $\|f+g\|_P \leq \|f\|_P + \|g\|_P$

(D) $\|f+g\|_P \geq \|f\|_P + \|g\|_P$

51. Consider the function $f(x, y) = xy(12 - 3x - 4y)$. The point $\left(\frac{4}{3}, 1\right)$:

(A) is not a critical point of f

(B) is a local maximum of f

(C) is a local minimum of f

(D) is a saddle point of f

52. If $u = f(x+2y) + g(x-2y)$, then :

(A) $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$

(B) $\frac{\partial^2 u}{\partial x^2} = 4\frac{\partial^2 u}{\partial y^2}$

(C) $2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$

(D) $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial^2 u}{\partial y^2}$

53. $f : \mathbf{R}^2 \to \mathbf{R}^3$ is defined as

$$f(u, v) = (u^2 - 5v, ve^{2u}, 2u - \log(1 + v^2)).$$

Then $Df(0, 0) =$

(A) $\begin{pmatrix} 0 & 1 & 2 \\ -5 & 1 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} -5 & -5 \\ 1 & 1 \\ 2 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & -5 \\ 0 & 1 \\ 2 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 1 & -5 \\ 1 & 1 & 0 \end{pmatrix}$

54. Let $f : \mathbf{R}^2 \to \mathbf{R}^3$ be :

$$f(u, v) = (u^2 - 5v, ve^{2u}, 2u - \log(1 + v^2)).$$

Suppose $g : \mathbf{R}^2 \to \mathbf{R}^2$ is of class C^1 and $g(1, 2) = (0, 0)$ and $Dg(1, 2) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Then $D(f \circ g)(1, 2) =$

(A) $\begin{pmatrix} -15 & -20 \\ 4 & 6 \\ 2 & 4 \end{pmatrix}$

(B) $\begin{pmatrix} -15 & -20 \\ 3 & 4 \\ 2 & 4 \end{pmatrix}$

(C) $\begin{pmatrix} -15 & 4 & 2 \\ -20 & 6 & 4 \end{pmatrix}$

(D) $\begin{pmatrix} -15 & 4 & 2 \\ 20 & -6 & 4 \end{pmatrix}$

55. Let M be a real symmetric positive definite $n \times n$-matrix. Then which of the following statements is false ?

(A) There is an invertible matrix P such that $M = P^T P$

(B) The determinant of M is positive

(C) M is diagonalizable

(D) M is unitary

56. Which of the following fields is *not* a unique factorization domain ?

(A) $\mathbf{Z}$

(B) $\mathbf{R}[x]$

(C) $\mathbf{Z}\left[\sqrt{-5}\right]$

(D) $\mathbf{Z}[i]$

57. In which of the following rings there are infinitely many units ?

(A) $\mathbf{Z}[x]$

(B) $\mathbf{Z}[i]$

(C) $\mathbf{Z}\left[\sqrt{-3}\right]$

(D) $\mathbf{Z}\left[\sqrt{2}\right]$

58. Let p be a prime integer and $\mathbf{Z}_p[x]$ be the polynomial ring over the field $\mathbf{Z}_p$. Then which of the following is true ?

(A) For any positive integer n, there is an irreducible polynomial in $\mathbf{Z}_p[x]$ of degree n

(B) There is no field containing $\mathbf{Z}_p[x]$ as a subring

(C) If $f(x) \in \mathbf{Z}_p[x]$ is reducible then $f(\alpha) = 0$ for some $\alpha \in \mathbf{Z}_p$

(D) For some integer n, there are infinitely many reducible polynomials in $\mathbf{Z}_p[x]$ with degree n

59. Find the matrix of the linear transformation $\frac{d}{dt} : \mathbf{R}_2[t] \to \mathbf{R}_2[t]$, the derivative map with respect to the basis :

$b_1(t) = t^2 + t + 1, \quad b_2(t) = t^2 + 3t + 2$

and $b_3(t) = t^2 + 2t + 1$

($\mathbf{R}_2[t]$ is the space of real polynomials of degree $\leq$ 2)

(A) $\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} -1 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & -4 & -2 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix}$

60. The radius of convergence of the power series $\sum_{n=0}^{\infty} z^{n^2}$ is :

(A) 0

(B) 1

(C) ∞

(D) 2

61. For which of the following functions f, there is a sequence $\{z_n\}$ in $\mathbb{C} - \{1\}$ such that :

$$\lim_{n \to \infty} z_n = 1, \text{ but } f(z_n)$$

has no limit in $\mathbb{C} \cup \{\infty\}$?

(A) $f(z) = \frac{1}{(z-1)^5}$

(B) $f(z) = \frac{\sin(z-1)}{(z-1)^4}$

(C) $f(z) = \cos\left(\frac{1}{z-1}\right)$

(D) $f(z) = e^{(z-1)^2} + \frac{1}{z-1} + \cos(z - z^3)$

62. Which of the following functions is an open mapping from $\mathrm{C} - \{0\}$ to $\mathbb{C}$?

(A) $f(z) = \frac{1}{z}$

(B) $f(z) = |z| + 1$

(C) $f(z) = x^2 + xy + y^2$, (where $z = x + iy$)

(D) $f(z) = \begin{cases} \frac{1}{z-1} & \text{if } z \neq 1, \\ 1 & \text{if } z = 1. \end{cases}$

63. A bilinear transformation other than the identify mapping has :

(A) No fixed piont

(B) Infinitely many fixed points

(C) At most 2 fixed points

(D) At most 1 fixed point

64. Let $f(z)$ be a non-constant analytic function defined on the disc $\mathrm{D} : |z - z_0| < r$. Then $|f(z)|$ has :

(A) maximum value at z_0

(B) minimum value at z_0

(C) no maximum value at any $z \in \mathrm{D}$

(D) a local maximum at some $z \in \mathrm{D}$

65. Let E and F denote the following subsets of **R**.

$\mathrm{E} = \{0\} \cup \{1/n \,/\, n \in \mathbf{N}\}$

$\mathrm{F} = \{1/n \,/\, n \in \mathbf{N}\}$

then :

(A) E is a compact subset whereas F is not compact

(B) E and F are compact subsets of **R**

(C) F is compact subset whereas E is not compact

(D) Neither E nor F is compact

66. Define :

$$f_1(x) = \sin\frac{1}{x},\ x \neq 0\,,$$

$$f_1(0) = 0$$

and $$f_2(x) = x\sin\frac{1}{x}$$

$$f_2(0) = 0\,.$$

(A) f_1 and f_2 are both continuous at 0

(B) Neither f_1 nor f_2 is continuous at 0

(C) f_2 is continuous at 0 but f_1 is not continuous at 0

(D) f_1 is continuous at 0 but f_2 is not

67. Consider the following statements.

(*i*) A path connected set is connected.

(*ii*) A connected set is path connected.

(*iii*) Union of connected sets is connected which of them are *correct* ?

(A) (*i*)

(B) (*ii*)

(C) (*i*), (*iii*)

(D) (*ii*), (*iii*)

68. Consider the following statements :

(I) Every Borel set is measurable

(II) Every measurable set is Borel.

Then :

(A) Only (I) is true

(B) Only (II) is true

(C) Both (I) and (II) are true

(D) Both (I) and (II) are not true

69. Which of the following is *incorrect* ?

(A) Continuous functions are measurable

(B) The characteristic function χ_A of the set A is measurable iff A is measurable

(C) Let f be a continuous function and g a measurable function then the composit function fog is measurable

(D) If $|f|$ is measurable then f is also measurable

70. If the function f is of bounded variation on $[a . b]$, then :

(A) f is differentiable

(B) f is differentiable *a.e.*

(C) f is monotone

(D) f is continuous

71. For a non-abelian group G of order P^3 :

(A) The derived subgroup G' of G is abelian

(B) The centre of G is {identify}

(C) The centre of G is not a normal subgroup of G

(D) The derived subgroup G' of G is equal to G

72. Which of the following groups is *not* a simple group ?

(A) $\mathbf{Z}_7$

(B) A_5

(C) A_{100}

(D) A_4

73. How many abelian groups of order 81 are there upto isomorphism ?

(A) 5

(B) 9

(C) 3

(D) 4

74. For which of the following complex numbers α, $\frac{1}{1+\alpha}$ can be written as a polynomial in α ?

(A) $\alpha = \sqrt[3]{7} + \sqrt[5]{2} + i$

(B) $\alpha = \pi + i$

(C) $\alpha = e$

(D) $\alpha = (\pi - 1) + 2i$

75. Which of the following numbers is *not* constructible by ruler and compass ?

(A) $\sqrt[4]{2}$

(B) $\frac{1}{7} + \frac{i}{9}$

(C) $\sqrt[3]{2}$

(D) $\sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt[i]{7}$

Section III

76. Let X_1, X_2, ..., X_n be a random sample observed from $U(-\theta, \theta)$.

Define :

$$T_1 = X_{(1)} \text{ and } T_2 = X_{(n)}$$

Then :

(A) T_1 and T_2 both are consistent estimators for θ

(B) $T_2 - T_1$ is consistent estimator for 2θ

(C) $(T_1 + T_2)/2$ is consistent estimator for θ

(D) $T_2 - T_1$ is consistent estimator for θ

77. The sum of the variances of the principal components is equal to the :

(A) Sum of the variances of the original variates

(B) Product of the variances of the original variates

(C) Twice of the sum of the variances of the original variables

(D) The sum of the variances of original variables subtracted from their product

78. If $A \sim W_p (n-1, \Sigma)$ with $n > p$ and $|\Sigma| > 0$ and let $\underset{\sim}{U}$ be distributed independently of A. Then the $\dfrac{\underset{\sim}{u}' \Sigma^{-1} \underset{\sim}{U}}{\underset{\sim}{u}' A^{-1} \underset{\sim}{u}}$ follows :

(A) χ^2_p

(B) χ^2_n

(C) χ^2_{n-p}

(D) Beta type-II distribution

79. If $A \sim W_p (n-1, \Sigma)$, then the distribution of $\underset{\sim}{l}' A \underset{\sim}{l}$, $\underset{\sim}{l}$ is a known vector, would be :

(A) χ^2_{n-1}

(B) $n\ \chi^2_{n-1}$

(C) Wishart

(D) $(\underset{\sim}{l}' \Sigma \underset{\sim}{l})\ \chi^2_{n-1}$

80. If $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, then the mgf of $\underset{\sim}{Y} = (\underset{\sim}{X} - \underset{\sim}{\mu})' \Sigma^{-1} (\underset{\sim}{X} - \underset{\sim}{\mu})$ would be :

(A) $(1-2t)^{-p/2}$

(B) $e^{\underset{\sim}{\mu}' \underset{\sim}{t} + \frac{1}{2} \underset{\sim}{t}' \Sigma \underset{\sim}{t}}$

(C) $e^{\underset{\sim}{\mu}' \underset{\sim}{t} - \frac{1}{2} \underset{\sim}{t}' \Sigma \underset{\sim}{t}}$

(D) $(1-2t)^{p/2}$

81. If the joint pdf of (X, Y) is :

$$f(x, y) = \frac{1}{2.4\pi} e^{-\frac{1}{0.72}\left[\frac{X^2}{4} - 1.60 \frac{X^y}{2} + Y^2\right]}$$

then the variance covariance matrix Σ is :

(A) $\begin{bmatrix} 4 & 1.60 \\ 1.60 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1.60 \\ 1.60 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{pmatrix} 1.6 & 0 \\ 0 & 1.6 \end{pmatrix}$

82. If $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$ and let $\underset{\sim}{X}$ is partitioned into $\underset{\sim}{X}^{(1)}$ of q-components and $\underset{\sim}{X}^{(2)}$ of $(p-q)$ components, then the regression line of $\underset{\sim}{X}^{(1)}$ on $\underset{\sim}{X}^{(2)}$ will be equal :

(A) the mean of the conditional of $\underset{\sim}{X}^{(1)}$ given $\underset{\sim}{X}^{(2)}$

(B) the mean of the distribution of $\underset{\sim}{X}^{(1)}$

(C) the mean of the conditional distribution of $\underset{\sim}{X}^{(2)}$ given $\underset{\sim}{X}^{(1)}$

(D) $\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$

83. Consider the linear regression model $\log y_i = \alpha + \beta x_i + \gamma z_i + \varepsilon_i,\ 1 \le i \le n$, $\varepsilon_i \sim (0, \sigma^2)$. It is found that :

$\hat{\beta} = 1.5,\ se(\hat{\beta}) = 0.75,\ \hat{\gamma} = 4,$

$se(\hat{\gamma}) = 0.75$

and $cov(\hat{\beta}, \hat{\gamma}) = 0.2$.

What will be $se(\hat{\beta}, \hat{\gamma})$?

(A) 0.5525

(B) 2.25

(C) 1.50

(D) 3.56

84. For a simple multiple linear regression $Y_i = \alpha + \beta X_i + \varepsilon_i,\ i = 1, 2, ..., n$ with $\varepsilon_i \sim iid\ (0, \sigma^2)$, which of the following statements is *not* true ?

($\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$, $\hat{\alpha}, \hat{\beta}$ are least squares estimators and $e_i = Y_i - \hat{Y}_i$)

(A) $\sum_{i=1}^{n} e_i X_i = 0$

(B) $\sum_{i=1}^{n} e_i \hat{Y}_i = 0$

(C) $\sum_{i=1}^{n} \hat{Y}_i = \sum_{i=1}^{n} Y_i$

(D) $V(\hat{\beta}) = \sigma^2 / \sum_{i=1}^{n} X_i^2$

85. Consider a simple linear regression :
$Y_i = \alpha + \beta X_i + \varepsilon_i \quad i = 1, 2, ..., n$

with $\varepsilon_i \sim f(\varepsilon_i) = \frac{1}{\overline{|(\theta)}} e^{-\varepsilon_i'} \varepsilon_i^{\theta-1},\ \varepsilon_i > 0$

then, which of the following statements is *true* ?

(A) α can be estimated by OLS

(B) $\hat{\alpha}_{OLS}$ is an unbiased estimate of α

(C) β can be estimated by OLS $\left(= \hat{\beta}_{OLS}\right)$ and $\hat{\beta}_{OLS}$ is an unbiased estimate of β

(D) Both α and β cannot be estimated by OLS

86. Consider a standard multiple regression model $\{Y, X\beta, \sigma^2 I\}$ with restriction $R\beta = \gamma$. Then, which of the following statements is *true* ?

(A) $\hat{\beta}_{RLS}$ (restricted least-squares estimator) is a biased estimate of β

(B) $\hat{\beta}_{RLS}$ is an unbiased estimate of β

(C) $V(\hat{\beta}_{RLS}) - V(\hat{\beta}_{OLS})$ is a positive semidefinite matrix

(D) $V(\hat{\beta}_{RLS}) = \sigma^2 (X'X)^{-1}$

87. Consider a standard regression model $\{Y, X\beta, \sigma^2 I\}$. Let $\hat{\beta}_{OLS}$ be the ordinary least squares estimator of β. Which of the following statements about $\hat{\beta}_{OLS}$ is not true in general ?

(A) $\hat{\beta}_{OLS}$ is consistent

(B) $\hat{\beta}_{OLS}$ is unbiased

(C) $V\left(\hat{\beta}_{OLS}\right) = \sigma^2 (X'X)^{-1}$

(D) $\hat{\beta}_{OLS}$ is BLUE

88. Consider a regression setup $\{Y, X\beta, \Omega\}$. Which of the following statements is *not* true in general ?

(A) $\hat{\beta}_{GLS} = (X^1\Omega^{-1}X)^{-1}\ X^1\Omega^{-1}\ Y$

(B) $\hat{\beta}_{GLS} = (X^1\Omega X)^{-1}\ X^1\Omega Y$

(C) $V\left(\hat{\beta}_{GLS}\right) = (X^1\Omega^{-1}\ X)^{-1}$

(D) $E\left(\hat{\beta}_{GLS}\right) = \beta$

89. The ratio estimator $\left(\hat{y}_k\right)$ is more precise estimator than mean per unit estimator $(\bar{y})$ if :

(A) $\rho < \frac{CV(x)}{CV(y)}$

(B) $\rho < \frac{CV(x)}{2\ CV(y)}$

(C) $\rho = \frac{CV(x)}{2\ CV(y)}$

(D) $\rho > \frac{CV(x)}{2\ CV(y)}$

90. The regression estimator is equally efficient to the mean per unit estimator if :

(A) $0 < \rho < 1$

(B) $\rho = 0$

(C) $-1 < \rho < 0$

(D) $\rho = 1$

91. If π_i and π_{ij} are respectively the first order and second order inclusion probabilities of a sampling design in PPSWOR, then which of the following relations is *true* ?

(A) $\sum_i \pi_i = (n-1)$

(B) $\sum_i \pi_i = n\ (n-1)$

(C) $\sum_i \pi_i = (n+1)$

(D) $\sum_i \pi_i = n$

92. Suppose NM units in the population are grouped at random into N clusters of M units each; then both the clustor sampling and SRSWOR of M elements are equally efficient if :

(A) NM = M

(B) M > 1

(C) M = N

(D) M = 1

93. In PPSWR, the probability of selection of unit i at each draw remains :

(A) p_i

(B) 1/N

(C) $1 / N_{Cn}$

(D) p_i/N

94. The statistical model correspond to a $p \times p$ Latin-square design is :

(A) $y_{ijk} = \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk}$,

$i, j, k = 1, 2, ... p$

(B) $y_{ijk} = \mu + \alpha_i + \tau_j + \varepsilon_{ijk}$,

$i, j, k = 1, 2, ... p$

(C) $y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk}$,

$i, j, k = 1, 2, ... p$

(D) $y_{ijk} = \mu + \alpha_{ij} + \beta_k + \varepsilon_{ijk}$,

$i, j, k = 1, 2, ... p$

95. With the usual notation in BIBD the i-th treatment effect is :

(A) $\tau_i = Q_i$

(B) $\tau_i = \dfrac{Q_i}{\lambda\nu}$

(C) $\tau_i = \dfrac{Q_i}{rE}$, where $E = \dfrac{\lambda\nu}{kr}$

(D) $\tau_i = \dfrac{Q_i}{E}$ where $E = \dfrac{\lambda\nu}{kr}$

96. In a 2^3 factorial experiment with ABC contained in two blocks of a single replicate :

(A) Contrast of ABC will be same as the block effect

(B) Contrast of ABC will be same as the total effect

(C) Contrast of ABC will be same as the difference of block and main effect

(D) Contrast of ABC will be same as the difference of block and interaction effect

97. Consider the following statements :

1. A first order design is orthogonal if the off-diagonal elements of the (X' X) matrix are all zero.
2. A response surface design is always rotatable.
3. The method of steepest ascent is a procedure for moving sequentially in the direction of the maximum increase in the response.

Which of the above are *correct* ?

(A) Both 1 and 2 are correct

(B) Both 1 and 3 are correct

(C) Both 2 and 3 are correct

(D) All are correct

98. Suppose two factors A and B are considered for a split-plot design, where A is applied to whole plots and factor B is applied to strips. What is the expected mean square of factor A ?

(A) $\sigma_\varepsilon^2 + b\sigma_{\tau\beta}^2$

(B) $\sigma_\varepsilon^2 + b\sigma_{\tau\beta}^2 - \frac{rb\sum\beta_j^2}{a-1}$

(C) $\sigma_\varepsilon^2 + b\sigma_{\tau\beta}^2 + \frac{rb\sum\beta_j^2}{a-1}$

(D) $\sigma_\varepsilon^2 - b\sigma_{\tau\beta}^2 + \frac{rb\sum\beta_j^2}{a-1}$

99. Let $X_t = \mu + \phi X_{t-1} + Z_t$ where $Z_t \sim$ iid Normal (0, 1). Assume that the process was started at time 0 and that $X_0 = 0$. Then which of the following statements are *correct* ?

(*i*) $X_t = \frac{1 - \phi^t}{1 - \phi} \mu + \sum_{j-1}^{t} \phi^{t-j} Z_j$

(*ii*) $E(X_t) = \mu$

(*iii*) $V(X_t) = \frac{1}{1 - \phi^2}$

(A) (*i*) and (*ii*)

(B) (*ii*) and (*iii*)

(C) (*i*) and (*iii*)

(D) Only (*ii*)

100. The time series model $X_t = \phi$, $X_{t-1} + \phi_2$, $X_{t-2} + Z_t$, $Z_i \sim$ iid $N(0, \sigma^2)$ is stationary if :

(A) $\phi_1 + \phi_2 < 1$, $\phi_1 - \phi_2 < 1$, $-1 < \phi_1 < 1$

(B) $\phi_1 + \phi_2 > 1$, $\phi_2 - \phi_1 < 1$, $-1 < \phi_2 < 1$

(C) $\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, $-1 < \phi_2 < 1$

(D) $\phi_1 + \phi_2 > 1$, $\phi_1 - \phi_2 < 1$, $-1 < \phi_1 < 1$

101. Let $X_t = Z_t + \theta Z_{t-1}$ and $Y_t = Z_t + 1/\theta\ Z_{t-1}$, where $Z_t \sim$ iid Normal (0, 1). Which of the following statements is *true* ?

(A) ACF of $\{X_t\}$ process is same as that of $\{Y_t\}$

(B) ACF of $\{X_t\}$ process is different from that of $\{Y_t\}$

(C) Both the processes $\{X_t\}$ and $\{Y_t\}$ are invertible for a given value of θ

(D) Both $\{X_t\}$ and $\{Y_t\}$ are non-stationary

102. The Ψ-weights and π-weights of the process $X_t - 0.5\, X_{t-1} = Z_t - 0.3\, Z_{t-1}$, $Z_t \sim$ iid Normal (0, 1) are :

(A) $\Psi_i = 0.2 \times 0.5^{i-1}$, $\pi_i = 0.2 \times 0.3^{i-1}$

(B) $\Psi_i = 0.2 \times 0.3^{i-1}$, $\pi_i = 0.2 \times 0.5^{i-1}$

(C) $\Psi_i = 0.3 \times 0.2^{i-1}$, $\pi_i = 0.5 \times 0.2^{i-1}$

(D) $\Psi_i = 0.5 \times 0.3^{i-1}$, $\pi_i = 0.2 \times 0.3^{i-1}$

103. Consider the following statements :

1. If i lead to j and j do not lead to i, then i is inessential
2. An inessential state is always a periodic
3. All inessential states will be transient.

Which of the above statements are *correct* ?

(A) Only 1 is correct

(B) Only 3 is correct

(C) 1 and 2 are correct

(D) 2 and 3 are correct

104. Let $\{X_n, n \geq 0\}$ be a MC with states 0, 1, 2 and the transition probability matrix is :

$$p = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

with initial probability as $p(X_0 = i) = \frac{1}{3}, i = 0, 1, 2.........$ What is the value of $p(X_2 = 2 | x_1 = 1)$?

(A) 1/4

(B) 1/2

(C) 3/4

(D) 1

105. Let $\{X_n\}_0^\infty$ be a doubly stochastic Markov chain such that :

$$p[X_{n+1} = 1 | X_n = 1] = p_1 = 1 - p[X_{n+1} = 0 | X_n = 1]$$

$$p[X_{n+1} = 1 | X_n = 0] = p_0 = 1 - p[X_{n+1} = 0 | X_n = 0]$$

and $p[X_1 = 1] = \pi_1 = 1 - p(X_1 = 0)$.

What is the value of π_1 ?

(A) $\pi_1 = p_0$

(B) $\pi_1 = p_1$

(C) $\pi_1 = \frac{p_1 - p_0}{2}$

(D) $\pi_1 = 1/2$

106. The number of persons dying at age 75 is 476 and the complete expectation of life at 75 and 76 years are 3.92 and 3.66 years. What is the number of people living at age 75 ?

(A) 2588

(B) 2676

(C) 2700

(D) 2750

107. Given that the complete expectation of life at ages 30 and 31 for a particular group are respectively 21.39 and 20.91 years and that the number living at age 30 is 41176. What is the number of people that attains the age 31 ?

(A) 38144

(B) 39012

(C) 39284

(D) 40176

108. Suppose that customers arrive at a Bank according to a Poisson process with a mean rate of 3 per minute. In an interval of 2 minutes, what is the probability that the number of customer arriving is greater than 4 ?

(A) 0.133

(B) 0.152

(C) 0.714

(D) 0.999

109. Let X and Y be any two random variables then correlation between X and $Y - E(Y|X)$ is :

(A) –1

(B) +1

(C) 0

(D) not defined unless distributions of X and Y are specified

110. Suppose mean of the quality characteristic is being monitored by X-bar chart. The chart has in-control ARL = 200. It means :

(A) The chart will take on an average 200 subgroup samples to detect shift in the mean

(B) The chart will take on an average 200 subgroup samples to give an out-of-control signal, when there is no shift in the mean

(C) The chart will require at least 200 subgroup samples to detect shift of any magnitude in the mean

(D) 200 subgroup samples are required to implement the chart

111. If the unit cost rises then optimum order quantity :

(A) decreases

(B) increases

(C) constant

(D) may decrease or increase

112. In a service department manned by one server on an average one customer arrives every 10 minutes. Every customer requires 6 minutes to be served then probability that there would be two customers in the queue is :

(A) 1/2

(B) .157

(C) .144

(D) .6

113. Suppose the lot size is N, quality of the lot is 'p' and probability of lot acceptance is p_a. If sample of size n is taken under rectifying inspection policy, the average total inspection per lot will be :

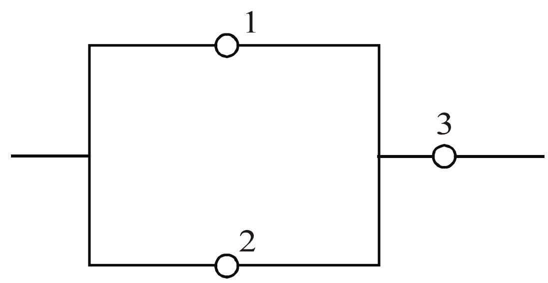
(A) $(1 - p_a)(N - n)$

(B) $n + (1 - p_a)(N - n)$

(C) $n + p_a(N - n)$

(D) $n\,p_a$

114. Consider the three component system with equal probability (p) of functioning for each of the components :



Then the system reliability is :

(A) $p(1 - p)^2$

(B) $2p(1 - p)$

(C) $p^2(2 - p)$

(D) $p(2 - p)$

115. Let X and Y be uniformly distributed over (0, 1). Then E|X − Y| :

(A) ≥ 1/2

(B) ≤ 1/2

(C) ≥ 3/4

(D) ≥ 1

116. In the Branch and Bound approach to maximization problem, a node is terminated if :

(A) A node has beasible solution

(B) A node yields a solution that is feasible but not an integer

(C) Upper bound is more than the current sub-problem's lower bound

(D) No upper bound is attained

117. Shadow price indicates how much one unit change in the resource value will change the :

(A) Optimality range of an objective function

(B) Optimal value of the objective function

(C) Value of the basic variable in the optimal solution

(D) The price that is paid for purchase of resources

118. According to Beale's method, the solution to the following quadratic programming :

Max : $Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

Subject to : $x_1 + 2x_2 + x_3 = 10$

$$x_1 + x_2 + x_4 = 9$$

$$x_1, x_2, x_3, x_4 \geq 0$$

is :

(A) $x_1 = 0$, $x_2 = 5$, $x_3 = 0$, $x_4 = 4$

(B) $x_1 = 0$, $x_2 = 4$, $x_3 = 2$, $x_4 = 0$

(C) $x_1 = 2$, $x_2 = 2$, $x_3 = 0$, $x_4 = 4$

(D) $x_1 = 1$, $x_2 = 4$, $x_3 = 1$, $x_4 = 4$

119. Consider the following statements regarding Revised Simplex method :

(1) It provides inverse and simplex multipliers in every iteration.

(2) The basis matrix is obtained after introducing slack variables

(3) $z_j - c_j \geq 0$ in all cases

Which of the above is *correct* ?

(A) Only (1) is correct

(B) Only (2) is correct

(C) Only (3) is correct

(D) None is correct

120. Let the random variable X has characteristic function $\phi(t)$. Which property $\phi(t)$ does *not* hold ?

(A) – X has characteristic function $-\phi(t)$

(B) $aX + b$ has characteristic function $e^{itb}\ \phi(at)$

(C) If X_i's are independent copies of X, for $i = 1, 2, \ldots\ldots, n$, then $E\left(e^{it}\sum_{1}^{n} X_i\right) = [\phi(t)]^n$

(D) If X is to have a symmetric distribution, then its characteristic function should be real

121. Suppose that $\{y_n\}$ is a sequence of independent random variables assuming values 1/2 and 3/2 having probability 1/2 each. Let

$X_n = y_1\ y_2\ y_3\ .....\ y_n$ and

$F_n = \sigma\ \{y_1, y_2, ..., y_n\}$.

Then $E(X_{n+1} / F_n)$ will be almost surely equal to :

(A) 1

(B) X_n

(C) X_{n-1}

(D) n

122. Suppose Z is an integrable random variable on (Ω, F, P) and F_n are non-decreasing σ-fields in F. Let $X_n = E(Z|\ F_n)$. Then, $E(X_{n+1}|\ F_n) =$

(A) $E(X_{n+1}) + X_n$

(B) X_{n-1}

(C) X_n

(D) $E(Z) + X_n$

123. Let X be a discrete random variable taking only integer values. Then, the characteristic function $\phi(t)$ of X will not have the following properties :

(A) $\phi(t+2\pi) = \phi(t)$

(B) $|\ \phi(t)| \le 1$

(C) $\phi(t+\pi) = \phi(t)$

(D) $\phi_n(t) \to \phi(t) \Leftrightarrow X_n \xrightarrow{d} X$

(ϕ_n and ϕ are the characteristic functions of X_n and X respectively)

124. Let X be a zero-mean unit variance Gaussian random variable. Define Y as :

$$Y = \begin{cases} X & \text{if} \quad |\ X| \le \alpha \\ -X & \text{if} \quad |\ X| > \alpha \end{cases}$$

α is some positive real number. Then, for any Borel set B :

(A) $P(Y \in B) < P(X \in B)$

(B) $P(Y \in B) = P(X \in B)$

(C) $P(Y \in B) > P(X \in B)$

(D) $P(Y \in B) = 0$

125. Let X_n be the amount that someone wins on a trial n, in a game of chance. Assume that X_i's are independent random variables, each with mean m (assume $m < 0$) and variance σ^2. Then, as $n \to \infty$,

$$P\left(\frac{X_1 + X_2 + ... + X_n}{n} < m / 2\right)$$

will tend to :

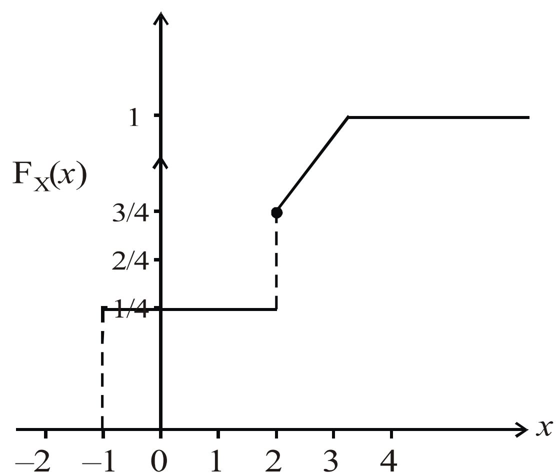
(A) 0

(B) 1

(C) 1/2

(D) 1/4

126. Let X be a random variable, whose cdf is plotted below. What will be $E(X^3)$?



(A) 3/16

(B) 25/16

(C) 125/16

(D) 115/16

127. Let $F(x)$ be a cdf of a random variable X, where :

$$F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ \left(\frac{x+1}{3}\right) & \text{if} \quad 0 \le x < 1 \\ 1 & \text{if} \quad x \ge 1 \end{cases}$$

Then variance of X is given by :

(A) 1/9

(B) 1/36

(C) 1/6

(D) 7/36

128. There are two books having certain misprints. The number of misprints in the first book is Poisson variate with mean 5, while the other book has number of misprints distributed as Poisson with mean 7. A book is chosen by reader, based on a uniform random number over (0, 1). If random number is below 0.4 then the first book is chosen, otherwise the second book is chosen. What is the expected number of misprints that a reader will come across ?

(A) 6

(B) 5

(C) 6.2

(D) 7

129. The characteristic function of exponential distribution with mean 1, evaluated at 1 is :

(A) exp (i^2)

(B) exp $(-i)$

(C) $(1 - i)$

(D) $(1 - i)^{-1}$

130. Let X be Bernoulli variate with parameter 0.6. Then distribution function of 1 – X evaluated at .65 is :

(A) .65

(B) .6

(C) .4

(D) 1

131. Let (X, Y) be a random vector with joint distribution :

$$f(x, y) = \begin{cases} (x + y) & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then :

(A) X and Y are uncorrelated

(B) X and Y have the same mean but different variance

(C) X and Y are negatively correlated

(D) X and Y are independent

132. Let X_1, X_2 and X_3 be iid r v having normal distribution with mean θ and variance 1. Then conditional distribution of $(X_1 + X_3)/2$ given $X_1 + X_2 + X_3$ is :

(A) $X_2/2$

(B) $X_1 + X_2 + X_3$

(C) $(X_1 + X_2 + X_3)/3$

(D) $(X_1 + X_3)/2$

133. Let $\underline{X} = (X_1, X_2, ..., X_k)$ be multinomial random vector with parameters :

$$n, p_1, p_2, ..., p_k, \sum_{i=1}^{k} p_i = 1.$$

Then marginal distribution of X_1 is :

(A) binomial with parameters n, $1 - p_1$

(B) not binomial

(C) binomial with parameters n, p_1

(D) binomial provided $p_1 = p_2, ... = p_k = 1/k$

134. Let $X_1, X_2, ..., X_n$ be iid rvs from the pdf $f(x/\mu)$,

$$f(x/\mu) = \begin{cases} \exp[-(x-\mu)] & ; \quad x \geq \mu, \mu \in R \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Then, which of the following statements is *correct* ?

(A) $X_{(n)}$ is complete statistic

(B) $X_{(1)}$ and $s^2 = \sum_{1}^{n} (X_i - \bar{X})^2$ are not independent

(C) $X_{(1)}$ is complete sufficient statistic

(D) $\sum_{i=1}^{n} [(X_{(i)} - X_{(1)})]$ is complete sufficient statistics

where $X_{(1)} = \text{Min } X_i$ and $X_{(n)} = \text{Max } X_i$

135. Let $X_1, X_2, ..., X_n$ be iid rvs with Poisson (λ). The UMVUE of $P[X = x]$ is :

(A) $\binom{t}{x}\left(\frac{1}{n}\right)^x \left(\frac{n-1}{n}\right)^{t-x}$; $x = 0, 1, 2 .... t$

(B) $\binom{t}{x}\left(\frac{1}{n}\right)^x \left(\frac{n-1}{n}\right)^{t-x}$; $x = 0, 1, 2 .... n$

(C) $\binom{t}{x-1}\left(\frac{1}{n}\right)^{x-1} \left(\frac{n-1}{n}\right)^{t-x+1}$; $x = 1, 2 .... n$

(D) $\binom{t-1}{x}\left(\frac{1}{n}\right)^x \left(\frac{n-1}{n}\right)^{t-1-x}$; $x = 1, 2 .... t$

136. Let $X_1, X_2, ..., X_n$ be iid rvs with the following pdf $f(x/\lambda)$,

$$f(x/\lambda) = \frac{\lambda}{(1+x)^{\lambda+1}}; \quad x > 0, \ \lambda > 0$$

UMVUE of λ is given by :

(A) $\frac{T}{n-1}$

(B) $\frac{T}{n}$

(C) $\frac{n}{T}$

(D) $\frac{n-1}{T}$

where $T = \sum_{i=1}^{n} Y_i$, $\quad Y_i = \log 1 + X_i$

137. Let $X_1, X_2, ... X_n$ be iid rvs with $U(\theta, 2\theta)$. The maximum likelihood estimator of θ is given by :

(A) $X_{(n)}$

(B) $\frac{X_{(n)}}{2}$

(C) $X_{(1)}$

(D) $\frac{X_{(1)} + X_{(n)}}{2}$

where $X_{(1)} = \text{Min } X_i$ and $X_{(n)} = \underset{i}{\text{Max}} \ X_i$

138. Let X_1 and X_2 be iid rvs with $U(0, \theta_i)$; $i = 1, 2$. The UMP test of size α to test $H_0 : \theta_1 = \theta_2$ against $H_1 : \theta_1 \neq \theta_2$ is :

(A) $\phi(x) = \begin{cases} 1 & ; \ x_{(2)} < \theta_1 \alpha^{1/2} \text{ or } x_{(2)} > \theta_1 \\ 0 & ; \ \text{otherwise} \end{cases}$

(B) $\phi(x) = \begin{cases} 1 & ; \ x_{(2)} < \theta_1 \text{ or } x_{(2)} > \theta_1 \alpha^{1/2} \\ 0 & ; \ \text{otherwise} \end{cases}$

(C) UMP test does not exist

(D) $\phi(X) = \begin{cases} 1 & ; \ X_{(2)} > \theta_1 (1-\alpha)^{1/2} \\ 0 & ; \ \text{otherwise} \end{cases}$

where $X_{(2)} = \text{Max}(X_1, X_2)$

139. Let X_1, X_2 be iid rvs with $N(\theta, 1)$. The uniformly most powerful unbiased (UMPU) test of size α to test $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ is given as :

(A) $\phi(X) = \begin{cases} 1 \ ; \ (X_1 + X_2) < 2\,\theta_0 - \sqrt{2}\, z_{\alpha/2} \\ 0 \ ; \ \text{otherwise} \end{cases}$

(B) $\phi(X) = \begin{cases} 1 \ ; \ (X_1 + X_2) < 2\,\theta_0 - \sqrt{2}\, z_{\alpha/2} \text{ or} \\ \quad X_1 + X_2 > 2\,\theta_0 + \sqrt{2}\, z_{\alpha/2} \\ 0 \ ; \ \text{otherwise} \end{cases}$

(C) $\phi(X) = \begin{cases} 1 \ ; \ (X_1 + X_2) > 2\,\theta_0 + \sqrt{2}\, z_{\alpha/2} \\ 0 \ ; \ \text{otherwise} \end{cases}$

(D) UMPU test does not exist

140. Let $X_1, X_2, ..., X_n$ be iid rvs with Cauchy distribution $C(\theta)$. The locally most powerful test of size α for testing $H_0 : \theta = 0$ against $H_1 : \theta > 0$ is given by :

(A) $\phi(X) = \begin{cases} 1 \ ; \ \sum_{i=1}^{n} y_i > \sqrt{n}\, z_\alpha \\ 0 \ ; \ \text{otherwise} \end{cases}$

(B) $\phi(X) = \begin{cases} 1 \ ; \ \sum_{i=1}^{n} y_i > \sqrt{\frac{n}{8}} \cdot z_\alpha \\ 0 \ ; \ \text{otherwise} \end{cases}$

(C) $\phi(X) = \begin{cases} 1 \ ; \ \sum_{i=1}^{n} y_i < \frac{z_\alpha}{\sqrt{n}} \\ 0 \ ; \ \text{otherwise} \end{cases}$

(D) $\phi(X) = \begin{cases} 1 \ ; \ \sum_{i=1}^{n} y_i > \sqrt{\frac{8}{n}} \cdot z_\alpha \\ 0 \ ; \ \text{otherwise} \end{cases}$

where $Y_i = \frac{X_i}{1 + X_i^2}$

141. Let the rv X be distributed as $B(n, p);\ 0 < p < 1$. The minimax estimator of p is given by :

(A) $\frac{X}{\sqrt{n}\,(1 + \sqrt{n})} + \frac{1}{1 + \sqrt{n}}$

(B) $\frac{X}{\sqrt{n} + 1} + \frac{1}{2(\sqrt{n} + 1)}$

(C) $\frac{X}{\sqrt{n}} + \frac{1}{2(1 + \sqrt{n})}$

(D) $\frac{X}{\sqrt{n}\,(1 + \sqrt{n})} + \frac{1}{2(1 + \sqrt{n})}$

142. Suppose $\bar{X}_n$ and X_{med} are respectively sample mean and sample median based on a random sample of size n observed from $N(\theta, 1)$. Then which of the following is *correct* ?

(A) $\bar{X}_n$ is an unbiased but X_{med} is not an unbiased estimator of θ

(B) $\bar{X}_n$ is a consistent estimator for θ, but X_{med} is not a consistent estimator for θ

(C) $\bar{X}_n$ and X_{med} both are unbiased and consistent estimators for θ

(D) Asymptotic distribution of $\sqrt{n}\,(X_{med} - \theta)$ is not normal

143. Let $X_1, X_2, ..., X_n$ be a random sample from $U(0, \theta)$. Then asymptotic distribution of $3n\,(2\bar{X} - \theta)^2$ is :

(A) chi-square with n d.f.

(B) chi-square with 1 d.f.

(C) normal with mean 0 and variance 1

(D) exponential distribution with mean 1

144. Let $X_1, X_2, ..., X_n$ be a random sample of size n from exponential distribution with scale parameter θ. Define $T_1 = \bar{X}_n$;

T_2 : mean based on the first $(n - 1)$ observations.

T_3 : mean based on the $(n - 2)$ observations, excluding the first and the last observation.

Then :

(A) T_1 is consistent but T_2 and T_3 are not consistent estimators for θ

(B) All T_1, T_2 and T_3 are consistent estimators for θ

(C) T_1 and T_2 are consistent but T_3 is not consistent estimator for θ

(D) T_1 is the only consistent estimator and there does not exist any other consistent estimator for θ

145. Based on a random sample of size n from $N(\theta, 1)$, asymptotic distribution of $n\bar{X}^2$ is :

(A) $\chi^2_n,\ \theta \in \mathbf{R}$

(B) $\chi^2_1,\ \theta = 0$

(C) $\chi^2_1,\ \theta \neq 0$

(D) $N(\theta^2, 1),\ \theta \in \mathbf{R}$

ROUGH WORK

ROUGH WORK