Maharashtra State Eligibility Test for Lectureship

Conducted by University of Pune

(AS THE STATE AGENCY)

SYLLABUS AND SAMPLE QUESTIONS

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[30] : MATHEMATICAL SCIENCES
SYLLABUS

PAPER II

General Information : Units 1, 2, 3 and 4 are compulsory for all candidates. Candidates with Mathematics background may omit units 10-14 and units 17,18. Candidates with Statistics background may omit units 6,7,9,15 and 16. Adequate alternatives would be given for candidates with O.R. background.


2. Basic Concepts of Linear Algebra : Space of n-vectors, Linear dependence, Basis, Linear transformation, Algebra of matrices, Rank of a matrix, Determinants, Linear equations, Quadratic forms, Characteristic roots and vectors.

3. Basic Concepts of Probability : Sample space, Discrete probability, Simple theorems on probability, Independence of events, Bayes Theorem, Discrete and continuous random variables, Binomial, Poisson and Normal distributions; Expectation and moments, Independence of random variables, Chebyshev’s inequality.


5. Real Analysis : Finite, countable and uncountable sets, Bounded and unbounded sets, Archimedean property, ordered field, completeness of R, Extended real number system, limsup and liminf of a sequence, the epsilon-delta definition of continuity and convergence, the algebra of continuous functions, monotonic functions, types of discontinuities, infinite limits and limits at infinity, functions of bounded variation. elements of metric spaces.


9. **Differential Equations**: First order ODE, singular solutions initial value Problems of First Order ODE, General theory of homogeneous and non-homogeneous Linear ODE, Variation of Parameters. Lagrange’s and Charpit’s methods of solving First order Partial Differential Equations. PDE’s of higher order with constant coefficients.


11. **Probability**: Axiomatic definition of probability. Random variables and distribution functions (univariate and multivariate); expectation and moments; independent events and independent random variables; Bayes theorem; marginal and conditional distribution in the multivariate case, covariance matrix and correlation coefficients (product moment, partial and multipal), regression.

   Moment generating functions, characteristic functions; probability inequalities (Tehebyshef, Markov, Jensen). Convergence in probability and in distribution; weak law of large numbers and central limit theorem for independent identically distributed random variables with finite variance.

12. **Probability Distribution**: Bemoulli, Binomial, Multinomial. Hypergeometric, Poisson, Geometric and Negative binomial distributions, Uniform, exponential, Cauchy, Beta, Gamma, and normal (univariate and multivariate) distributions. Transformations of random variables; sampling distributions. t, F and chi-square distributions as sampling distributions, as sampling distributions, Standard errors and large sample distributions. Distribution of order statistics and range.


14. **Statistical Methods and Data Analysis**: Tests for mean and variance in the normal distribution: one-population and two-population cases; related confidence intervals. Tests for product moment, partial and multiple correlation coefficients; comparison of k linear regressions. Fitting polynomial regression; related test Analysis of discrete data: chi-square test of goodness of fit, contingency tables. Analysis of variance: one-way and two-way classification (equal number of observations per cell). Large sample tests through normal approximation.

   Nonparametric tests: sign test, Median test, Mann-Whitney test, Wilcoxon test for one and two-samples, rank correlation and test of independence.
15. **Operational Research Modelling**: Definition and scope of Operational Research. Different types of models. Replacement models and sequencing theory. Inventory problems and their analytical structure. Simple deterministic and stochastic models of inventory control. Basic characteristics of queueing system, different performance measures, steady state solution of Markovian queueing models: M/M/1, M/M/1 with limited waiting space M/M/C, M/M/C with limited waiting space.


17. **Finite Population**: Sampling Techniques and Estimation: Simple random sampling with and without replacement. Stratified sampling; allocation problem; systematic sampling Two stage sampling. Related estimation problems in the above cases.


**SYLLABUS**

**PAPER III**

1. **Real Analysis**: Riemann integrable functions; improper integrals, their convergence and uniform convergence. Euclidean space R”, Bolzano-Weierstrass theorem, compact Subsets of R”, Heine-Borel Tttheorem, Fourier series.


   Integral functions, line and surface integrals, Green’s theorem, Stoke’s theorem.


3. **Algebra**: Symmetric groups, Alternating groups, Simple groups, Rings, Maximal Ideals, Prime Ideals, Integral domains Euclidean domains, principal Ideal domains, Unique Factorisation domains, quotient fields, Finite fields, Algebra of Linear Transformations, Reduction of matrices to Canonical Forms, Inner Product Spaces, Orthogonality, Quadratic Forms, Reduction of quadratic forms.

4. **Advanced Analysis**: Elements of Metric Spaces, Convergence, continuity, compactness, Connectedness, Weierstrass’s approximation Theorem, Completeness, Bare category theorem, Labesgue measure, Labesgue Integral, Differentiation and Integration.
5. **Advanced Algebra**: Conjugate elements and class equations of finite groups, Sylow theorems, solvable groups, Jordan Holder Theorem, Direct Products, Structure Theorem for finite abelian groups, Chain conditions on Rings; Characteristic of Field, Field extensions, Elements of Galois theory, solvability by Radicals, Ruler and compass construction.

6. **Functional Analysis**: Banach Spaces Hahn-Banach Theorem, Open mapping and closed Graph Theorems. Principal of Uniform boundedness, Boundedness and continuity of Linear Transformations, Dual Space, Embedding in the second dual, Hilbert Spaces, Projections. Orthogonal Basis, Riesz-representation theorem, Bessel’s Inequality, Parval’s identity, self adjointed operators, Normal Operators.


8. **Discrete Mathematics**: Partially ordered sets, Lattices, Complete Lattices, Distributive lattices, Complements, Boolean Algebra, Boolean Expressions, Application to switching circuits, Elements of Graph Theory, Eulerian and Hamiltonian graphs, planar Graphs, Directed Graphs, Trees, Permutations and Combinations, Pigeonhole principle, principle of Inclusion and Exclusion, Derangements.


10. **Number Theory**: Divisibility; Linear diophantine equations. Congruences. Quadratic residues; Sums of two squares, Arithmetic functions Mu, Tau, and Sigma (and ).

11. **Mechanics**: Generalize coordinates; Lagranges equation; Hamilton’s canonical equations; Variational Principles-Hamilton’s principles and principles of least action; Two dimensional motion of rigid bodies; Euler’s dynamical equations for the motion of rigid body; Motion of a rigid body about an axis; Motion about revolving axes.

12. **Elasticity**: Analysis of strain and stress, strain and stress tensors; Geometrical representation; Compatibility conditions; Strain energy function; Constitutive relations; Elastic solids “Hookes law; Saint-Venant’s principle, Equations of equilibrium; Plane problem-Airy’s stress function vibrations of elastic, cylindrical and spherical media.

13. **Fluid Mechanics**: Equation of continuity in fluid motion; Euler’s equations of motion for perfect fluids; Two dimensional motion complex potential; Motion of sphere in perfect liquid and monition of liquid past a sphere; Vorticity; Navier-Stokes’s equations for viscous flows-some exact solutions.
14. **Differential Geometry**: Space curves-their curvature and torsion; Serret Frehat Formula; Fundamental theorem of space curves; Curves on surfaces; First and second fundamental form; Gaussian curvatures; Principal directions and principal curvatures; Geodesics, Fundamental equations of surface theory.


16. **Linear Integral Equations**: Linear Integral Equations of the first and second kind of Fredholm and Volterra type; solution by successive substitutions and successive approximations; Solution of equations with separable kernels; The Fredholm Alternative; Holbert-Schmidt theory for symmetric kernels.

17. **Numerical analysis**: Finite differences, Interpolation; Numerical solution of algebraic equation; Iteration; Newton-Raphson Method; Solution on Linear system; Direct method; Gauss elimination method; Matrix-Inversion eigenvalue problems; Numerical differentiation and integration.

   Numerical solution of ordinary differential equation; iteration method, Picard’s method, Euler’s method and improved Euler’s method.

18. **Integral Transform**: Laplace transform; Transform of elementary functions, Transform of Derivatives, Inverse Transform, Convolution Theorem, Applications, Ordinary and Partial differential equations; Fourier transform; sine and cosine transform, Inverse Fourier Transform, Application to ordinary and partial differential equations.


   Almost sure convergence, convergence in mean square, Khintchine’s weak law of large numbers; Kolmogorov’s inequality, strong law of large numbers.

22. **Distribution Theory**: Properties of distribution functions and characteristic functions; continuity theorem, inversion formula, Representation of distribution function as a mixture of discrete and continuous distribution functions; Convolutions, marginal and conditional distributions of bivariate discrete and continuous distributions.

Relations between characteristic functions and moments; Moment inequalities of Holder and Minkowski.


Admissible, Bayes and minimax estimators; illustrations. Unbiasedness. UMVU estimators.

Families of distributions with monotone likelihood property, exponential family of distributions. Test of a simple hypothesis against a simple alternative from decision-theoretic viewpoint. Tests with Neyman structure. Uniformly most powerful unbiased tests. Locally most powerful tests.

Inference on location and scale parameters; estimation and tests. Equivariant estimators. Invariance in hypothesis testing.


26. **Linear Models and Regression**: Standard Gauss-Markov models; Estimability of parameters; best linear unbiased estimates(BLUE); Method of least squares and Gauss-Markov theorem; Variance-covariance matrix of BLUES.

Tests of linear hypothesis; One-way and two-way classifications. Fixed, random and mixed effects models (two-way classifications only); variance components, Bivariate and multiple linear regression; Polynomial regression; use of orthogonal polynomials. Analysis of covariance. Linear and nonlinear regression outliers.

28. **Design of Experiments**: Factorial experiments, confounding and fractional replication.

Design of Experiments: Split and strip plot designs; Quasi-Latin square designs; Youden square. Design for study of response surfaces; first and second order designs.

Incomplete block designs; Balanced, connectedness and orthogonality, BIBD with recovery of inter-block information PBIBD with 2 associate classes. Analysis of series of experiments, estimation of residual effects. Construction of orthogonal-Latin squares, BIB designs, and confounded factorial designs.

Optimality criteria for experimental designs.


30. **Stochastic Processes**: Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution; branching processes; Random walk; Gambler’s ruin.

Markov processes in continuous time; Poisson processes, birth and death processes, Wiener process.


32. **Industrial Statistics**: Control charts for variables and attributes; Acceptance sampling by attributes; single, double and sequential sampling plans; OC and ASN functions, AOQL and ATI; Acceptance sampling by varieties. Tolerance limits Reliability analysis: Hazard function, distribution with DFR and IFR; Series and parallel systems. Life testing experiments.

33. **Inventory and Queueing theory**: Inventory $(S,s)$ policy periodic review models with stochastic demand. Dynamic inventory models. Probabilistic re-order point, lot size inventory system with and without lead time. Distribution free analysis. Solution of inventory problem with unknown density function. Warehousing problem. Queues: Imbedded markov chain method to obtain steady state solution of $M/G/1$, $G/M/1$ and $M/D/C$, Network models. Machine maintenance models. Design and control of queueing systems.